

Back Stepping SMC for Blood Glucose Control of Type-1 Diabetes Mellitus Patients

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Abstract—Diabetes is a chronic condition marked by abnormally high levels of sugar (glucose) in the blood. People with diabetes either do not produce enough insulin, a hormone that is needed to convert sugar, starches and other food into energy, needed for daily life or cannot use the insulin that their bodies produce. As a result, glucose builds up in the bloodstream. If left untreated, diabetes can lead to blindness, kidney disease, nerve disease, heart disease, and stroke. Therefore, strict glycemic control is required for the diabetes patients. The aim of this paper is to control blood glucose, and for this purpose the Bergman minimal mathematical body model is used to develop the nonlinear controller. A resilient back-stepping based sliding mode control strategy is proposed as a solution, which assures practical tracking of the desired glucose concentration. The performance of the proposed design is compared with the conventional state feedback controller. The numerical simulation results showcases the advantages of back stepping sliding mode controller design to the state feedback controller design.

Index Terms—Back stepping, Bergman minimal model, Blood glucose.

I. INTRODUCTION

During the last few decades, control technology has been applied in a wide variety of systems such as medical, bio-medical, industrial and other fields that require monitoring and adjusting the input of a system to get the desired output. Also, it has been utilized to improve the performance of different types of systems. Diabetes is one of the growing medical problems that need to be addressed. According to the World Health Organization, in 2011 approximately 346 million people suffered from diabetes world-wide. India, China and the

USA rank among the top three countries with the largest numbers of diabetic patients. By 2030, an increase up to 552 million patients is predicted by the International Diabetes Federation [1].

In the human body, pancreas contains beta cells that release insulin. Insulin is responsible for the tight control of blood glucose levels. By producing and releasing the counteracting hormones insulin and glucagon, blood glucose concentration can be decreased or increased, respectively, and can be stabilized within the physiological range of 70-120 mg/dl [2], [3]. Hyperglycemia occurs if the glucose concentration is much higher than normal level (higher than 180 mg/dl). The condition causes some complications such as micro vascular damages in the retina and kidneys and neural damages, leading to blindness [4]. Hypoglycemia occurs when the glucose concentration is lower than normal level (lower than 60 mg/dl). The condition causes loss of consciousness, coma and even death.

The diabetes mellitus is classified into three types. Type 1 Diabetes Mellitus (T1DM) is caused by the autoimmune destruction of the beta cells in pancreas which results in insulin deficiency. Therefore, the patients suffering from T1DM are insulin dependent and they require exogenous insulin injection to regulate their glucose concentration to a normal level [5]. Type 2 diabetes begins with insulin resistance, a condition in which cells fail to respond to insulin properly. As the disease progresses a lack of insulin may also develop. This form is a non-insulin dependent diabetes mellitus. The most common cause is excessive body weight and not

enough exercise. Other forms of diabetes which are frequently non-permanent are caused by metabolic stress in critically ill patients, drug-induced hypoglycemia or by pregnancy [6]. In the current glucose management process, a diabetic person performs the procedures of blood glucose regulation manually. This causes great inconvenience. Therefore, a system that automatically monitors and controls the blood glucose level of a diabetic individual without interfering in his routine activities and also which reduces the chronic effects is required.

Many mathematical models have been developed to describe the glucose-insulin system. In 1981, Bergman proposed a small nonlinear 3rd-order model [7] followed by Cobelli's more complex proposal in 1984 [8]. In 1985, Sorensen presented a patient model consisting of 19 differential equations including the insulin and glucagon subsystems [9]. In 2004, Hovorka *et al.* introduced a model based on 8 differential equations [2] which describes the glucose-insulin system behavior in a type-1 diabetic patient with exogenous insulin administration. More recently, in 2007, Dalla Man *et al.* published a 12th-order mathematical model of healthy humans [10]. That model was later extended to diabetic behavior by including subcutaneous insulin administration [11].

Previously various methods have been employed to design the feedback controls for insulin delivery, such as, the classical methods, like proportional-integral-derivative (PID) control [12], [13], pole placement [14] etc. These control methods require a linearized model for the design. In order to obtain a closed-loop control, the algorithm has to be altered to completely avoid hypoglycemia and to reduce the time of blood glucose settling to the basal value. Hence, Marchetti *et al.* proposed a switching PID control algorithm with a time-varying intravenous glucose set-point [15]. Model predictive control (MPC) is also used because of its capability to incorporate hard constraints on the state and control variables [16]–[18].

In order to provide more robust control, Garcia-Gabin *et al.* [19] proposed a two degree of freedom sliding mode control (SMC). It includes a sliding mode feedback controller tracking a desired glucose

level in conjunction with a feed-forward controller which predicts the bolus insulin in response to a scheduled meal. Traditional SMC has some problems, such as discontinuous control that often yields chattering. To cope with this problem and achieve higher accuracy, higher order sliding mode (HOSM) is proposed by Kaveh and Shtessel [20] where the system model is augmented with an integrator. Another new feature of the SMC is explored in [21] where a double loop HOSMC is used. Since the true glucose regulation model is incompletely defined and approximate, and uncertainty is always present, a high-order sliding mode control mentioned in [22] offers an optimal control solution because of its insensitivity to changing dynamics, input conditions and system structure. By using the fuzzy HOSMC [23], the amount of insulin delivered to the patient at the meal-induced moment has been decreased to its half, in comparison to that in HOSM controller, and as a result it leads to less likelihood of hypoglycemia. But this controller has a disadvantage that, the effect of noise in measuring blood glucose should be estimated and alleviated. For using the sliding mode control the main criterion is to normalize the system equations since the blood glucose system has uncertainties. In order to avoid this problem, in this paper, a back stepping sliding mode control (BSMC) is introduced which possess the ability to control each parameter separately. In this method the need of normalizing the system is eliminated.

The paper is organized as follows. In Section II, brief overview of mathematical Bergman body model is presented, whereas the control design procedure is discussed in Section III. Numerical simulations are given in Section IV followed by conclusion in Section V.

II. BERGMAN MINIMAL MODEL

The Bergman's model is a nonlinear three-compartment model. One compartment each is assigned to glucose and to insulin concentrations in the blood, and the third to the non-observable auxiliary variable $X(t)$, which creates the delay in the action of insulin on glucose. This three-compartment model, is the best model in terms

of a) identifiability of parameters, b) meaning of parameter values and c) goodness-of-fit. Therefore, in this paper the Bergman's model is considered [24]–[26]:

$$\dot{G}(t) = -p_1(G(t) - G_b) - X(t)G(t) + D(t), \quad (1)$$

$$\dot{X}(t) = -p_2X(t) + p_3(I(t) - I_b), \quad (2)$$

$$\dot{I}(t) = -n(I(t) - I_b) + \gamma(G(t) - H)^+ + u(t), \quad (3)$$

where, $G(t)$ is the plasma glucose concentration in mg/dl, $I(t)$ is the plasma insulin concentration in mU/L, $X(t)$ is the variable proportional to the insulin in the remote compartment in mU/L, $u(t)$ is the injected insulin rate in mU/min. G_b is the basal glucose level in mg/dl, I_b is the basal insulin level in $\mu\text{U/ml}$, p_1 is the glucose effectiveness (min^{-1}), p_2 is the decreasing level of insulin action with time (min^{-1}), p_3 is the rate in which insulin action is increased as the level of insulin deviates from the corresponding baseline ($(\mu\text{U/ml})/\text{min}^2$), γ is the rate at which insulin is produced as the level of glucose rises above a target glycerin level ($(\mu\text{U/ml})/\text{min}^2/(\text{mg/dl})$), n is the first order decay rate for insulin in blood (min^{-1}) and H is the pancreatic target glycemia level (mg/dl).

The modification of Bergman's model to a type 1 diabetic behavior became generally known as the Bergman's Minimal (BeM) model. In the BeM model, a fourth differential equation is added. It represents a first order insulin infusion rate of the pump. The role of pump is to inject the insulin into the system when the glucose level goes above the normal basal level. The pump is modeled as a first order delay given by

$$\dot{u}(t) = \frac{1}{\tau}(-u(t) + w(t)), \quad (4)$$

where, $w(t)$ is the manipulated insulin infusion rate (mU/min) and τ is the pump time constant. The term $D(t)$ in (1) represents the rate at which glucose enters the blood from intestinal absorption following a meal. This disturbance can be modeled [27] as

$$D(t) = A \exp(-Bt), \quad B > 0, \quad (5)$$

provided appropriate values are chosen for the constants A and B . If one uses the values as $A = 0.5$ and $B = 0.05$ then, with the parameters given in the Table I, one can get the desired effect. It must be noted that the system output is $G(t)$ and the pump input $w(t)$ is the control input. This 4th order nonlinear model, given by (1)-(4) and disturbance (5), is simulated in open loop with the parameter values mentioned in Table I and result for all the variables are generated. From Fig 1, it is observed that the $G(t)$ for the normal person reaches to the basal value in 200 min whereas that for the unhealthy person it attains 150 mg/dl which is away from the basal value in less than 100 min. Therefore, a control method unlike the sliding mode control, which is capable of providing robust tracking of the desired output, in the presence of various uncertain dynamics appearing in the system must be chosen. Taking these factors into consideration a back stepping sliding mode control algorithm is designed.

III. BACK STEPPING SLIDING MODE CONTROL DESIGN

Back stepping sliding mode controller is capable of controlling each system variable separately so that the output tracks the desired value. For this, above BeM model can be decoupled into four dynamic models. Each state equation is considered as a separate dynamic part. The first dynamic part is

TABLE I
PARAMETERS OF THE BERGMAN MINIMAL MODEL

Parameter	Normal	Patient
G_b	90	90
G_0	140	300
H	79.0353	0
I_b	7	7
I_0	20	20
X_0	0	0
n	0.2659	0.3
p_1	0.0317	0
p_2	0.0123	0.02
p_3	8.2×10^{-8}	5.3×10^{-6}
u_0	0	0
γ	6.5×10^{-5}	0

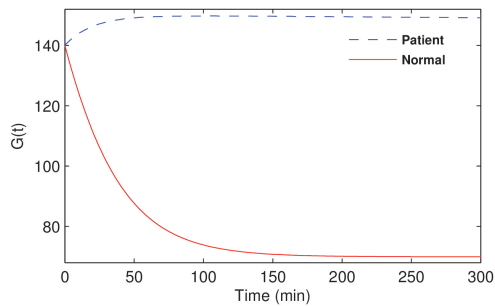


Fig. 1. Glucose response of open loop system.

considered as the (1). For convenience it is rewritten as

$$\dot{G}(t) = -p_1(G(t) - G_b) - X(t)G(t) + D(t). \quad (6)$$

Here, $X(t)$ is considered as the pseudo control input that controls $G(t)$ to track the desired G_b in presence of uncertainties $D(t)$. The sliding surface for the first dynamic part is introduced as

$$S_1(t) = G(t) - G_b. \quad (7)$$

According to the sliding mode control theory [28], the control law to reach the sliding surface S_1 is given as

$$X(t) = X_{eq} + X_c. \quad (8)$$

The control law has two parts namely a) equivalent control and b) corrective control. These are discussed in the following subsections.

A. Equivalent Control

The equivalent control makes the derivative of the sliding surface equal to zero to stay on the sliding surface. When there are no uncertainties all the paths are in the sliding surface. Therefore, X_{eq} is obtained by making $\dot{S}_1 = 0$ and solving as

$$X_{eq}(t) = \frac{1}{G(t)}[-p_1(G(t) - G_b)]. \quad (9)$$

B. Corrective Control

The corrective control compensate the deviations from the sliding surface to reach the sliding surface. This control part is obtained using the reaching

law [29]. The reaching law is used to reduce the chattering of SMC in the premise of ensuring the condition of sliding existence $S\dot{S} < 0$. The general reaching law is given by

$$X_c = -k_1 \text{sgn}(S_1), k_1 > 0. \quad (10)$$

Constant plus proportional reaching law is represented by

$$X_c = -k_1 \text{sgn}(S_1) - q_1 S_1, k_1 > 0, q_1 > 0. \quad (11)$$

1) *Stability Analysis:* Stability analysis is done using the Lyapunov stability criterion. The Lyapunov function is chosen as

$$V = \frac{1}{2} S_1^2. \quad (12)$$

According to Lyapunov theorem, to prove the stability of the control design, \dot{V} should be negative definite. For ensuring the limited time stability, the reachability condition [30] must be maintained

$$\dot{V} \leq -cV^\beta, \forall V(0) \geq 0, c > 0, \beta \in (0, 1). \quad (13)$$

Then the rising time for achieving sliding surface will be adjusted as:

$$t_r \leq \frac{V(0)^{1-\beta}}{c(1-\beta)}. \quad (14)$$

Here, the values of c and β are adjusted such that the sliding condition is obtained. By taking $\beta = 0.5$, the amount of k_1 can be determined by using (7), (9), (12) and (13) as

$$\begin{aligned} \dot{V} &= S_1 \dot{S}_1 \\ &= S_1[-p_1[G(t) - G_b] - [X(t)_{eq} - k_1 \text{sgn}(S_1)]G(t) + D(t)] \\ &= S_1[k_1 \text{sgn}(S_1)G(t) + D(t)] \leq -\frac{c}{\sqrt{2}} |S_1| \Rightarrow \\ &-k \geq \frac{1}{G(t)} \left[D(t) \frac{S_1}{|S_1|} + \frac{c}{\sqrt{2}} \right]. \end{aligned} \quad (15)$$

By choosing $-k$ as the largest possible amount in equation (15), the sliding condition in (13) will always be maintained. As a result

$$-k = \frac{1}{G(t)} \left[D(t) \frac{S_1}{|S_1|} + \frac{c}{\sqrt{2}} \right]. \quad (16)$$

Hence, the desired value of $X(t)$ which is the control input of the second dynamic part of the system is obtained as

$$X(t)_d = \frac{1}{G(t)} [-p_1(G(t) - G_b)] - \frac{1}{G(t)} \left[D(t) \frac{S_1}{|S_1|} + \frac{c}{\sqrt{2}} \right] \text{sgn}(S_1). \quad (17)$$

In a similar pattern the control of the rest of the dynamic parts are designed. The sliding variables taken are :

$$S_2 = X(t) - X(t)_d, \quad (18)$$

$$S_3 = I(t) - I(t)_d, \quad (19)$$

$$S_4 = u(t) - u(t)_d. \quad (20)$$

Solving all these the desired values of the state variables are calculated as:

$$I(t)_d = I_b + \frac{p_2}{p_3} X - \frac{c}{\sqrt{2}p_3} \text{sgn}(S_2). \quad (21)$$

$$u(t)_d = n [I(t) - I_b] - \gamma [G(t) - H]^+ t - \frac{c}{\sqrt{2}} \text{sgn}(S_3). \quad (22)$$

$$w(t)_d = u(t) - \frac{c\tau}{\sqrt{2}} \text{sgn}(S_4). \quad (23)$$

Exerting this signal to the system, the system state variables will converge quickly to the desired values without the normalization of system uncertainties.

IV. NUMERICAL SIMULATION

The designed control system is used as a blood glucose controller for the Type-1 diabetes patients. The evaluation of the design is done in this section with numerical simulation using MATLAB. To validate the designed back stepping algorithm, the control function is applied to the body model and the response of a sick person in presence of the meal disturbance is examined and its performance is then compared with the performance of a state feedback controller designed using pole placement technique [14]. Figs 2, 3, 4 and 5 show the results obtained from the simulations for the above control structure. It is seen that the BSMC brings down the glucose and insulin values to the basal level, and it takes less time to converge. Whereas, the state

feedback controller does not bring the glucose and insulin to the basal levels and there is also a chance of hypoglycemia.

From Fig 2, it is seen that the glucose concentration of the patient settles down to the basal value (90 mg/dl) in 225 min when BSMC is used. Whereas in the case when the state feedback controller is used, the glucose concentration reaches the basal value in 100 min but there is a steady state error in the response.

From Fig 3, it is observed that the insulin concentration in the remote compartment for the BSMC remains null and therefore, there is no delay in the action of insulin on glucose. But in the case of state feedback controller, it is seen that there is a significant overshoot in the remote insulin concentration to a peak value of 3×10^{-3} in 50 min causing a significant delay in the action of plasma insulin on glucose, after which it falls down to zero in 150 min.

It is clear from Fig 4 that when BSMC is used, the plasma insulin concentration of the patient stabilizes to the basal value (7 mU/L) without any delay. Whereas when the state feedback control is provided, the insulin concentration first rises to a peak value of 110 mU/L which will in turn result in significant reduction of glucose levels causing hypoglycemia. The insulin concentration is stabilized at the basal level after a remarkable time delay in 200 min. Fig 5 shows the insulin delivery rate for both the control methods.

Overall dynamic performance of the suggested design for glucose control is found to be satisfactory compared to state feedback control. It is assumed that the simulations done in this paper are extremely helpful to control system engineers and bio-medical engineers to examine the behavior of different system variables and to explore appropriate control method for similar other systems.

V. CONCLUSION

This paper presents a back-stepping sliding mode controller in order to regulate the glucose concentration and the insulin delivery rate more accurately in a close loop configuration, for type 1 diabetes. It can stabilize the blood glucose concentration

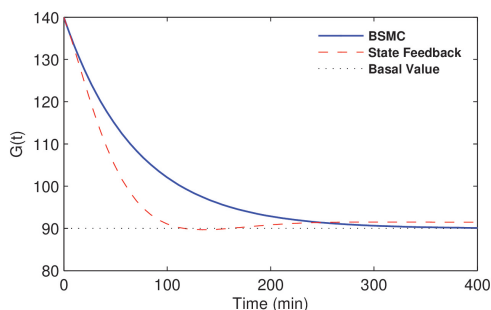


Fig. 2. Glucose concentration.

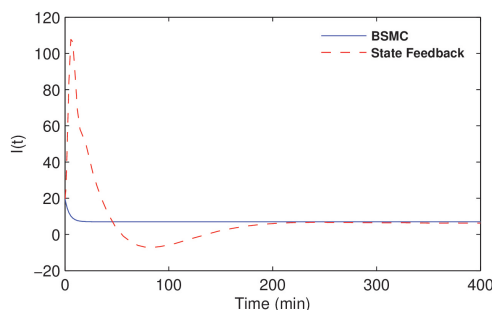


Fig. 4. Plasma insulin concentration.

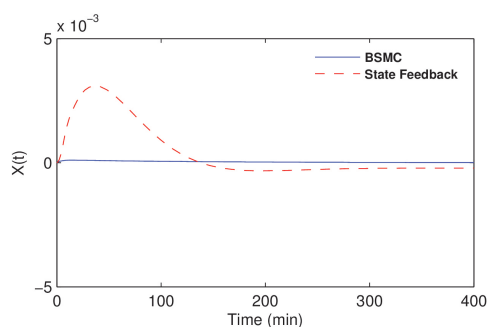


Fig. 3. Insulin concentration in remote compartment.

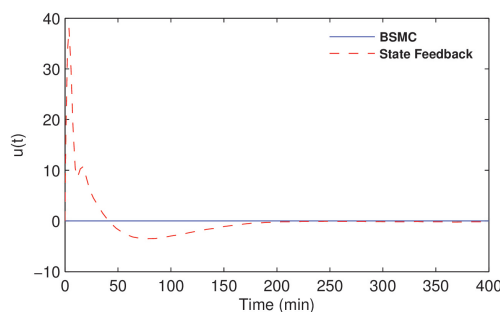


Fig. 5. Insulin injection rate.

of a type 1 diabetic patient at the desired level. This stabilization has been done in presence of a meal disturbance. The robustness of algorithm has been confirmed through simulations in MATLAB. This method is compared with the state feedback controller designed using pole placement technique and the advantage and superiority of it has been shown in terms of reducing the control effort and complexity as well as there is no need for normalizing the system.

In future studies an adaptive back stepping algorithm is to be designed to estimate and alleviate the effect of noise in measuring blood glucose and also to accommodate provision for handling any uncertainty like meal disturbance or physical stress at any time of the simulation.

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