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Flow Shop Scheduling Problem with Five Machines N-Jobs with Transporting Agent

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Abstract: *In every field of production scheduling plays an important role along with sequencing. Usually it is considered as one of the major step in real life applications and also in high aim production. In flow shop scheduling problems total completion time of process that is make-span depends on the sequence of jobs selected in a schedule. In this paper we try to find the algorithm for flow shop scheduling problem of five machines arranged in series. Setup time, processing time and transporting time are considered separately. Also effect of breakdown interval is observed on the schedule. Johnson's algorithm is used for finding the sequence of jobs so that the total make-span gets reduced. Also minimum weighted flow time is calculated.*

1.1 Introduction

Using available information regarding manufacturing,number of steps involved and their interdependence, existing raw material as well as number of machines, preparation of schedule are the most essential part of optimization process. Various scheduling techniques, new algorithms are developed to reduce total make-spanas well as total cost function.

Flow shop scheduling provides the particular sequence of job to minimize the determined constraint related to the production. One of the important aspects of flow shop scheduling problem is that the machines are continuously in working. In most of the centres machines are more in number and jobs are started processing from first machine. All machines are arranged in series so that after finishing at first it will carry on second machine and so on. Scheduling by using Johnson's rule is the method applied in steps. Every job must be activated on every machine and it is assigned by processing time, due date, weightage of job etc. There are many rules of scheduling like Earliest due date, shortest processing time, first come first served, longest processing time etc. But all these rules are more effective in single machine scheduling. So when n-number of machines are involved in the production then proper scheduling is required so that every machine will be busy and jobs will be processed in particular sequence. Use of available resources effectively and achievement of all objectives is the main aim of every optimization process. For this various algorithms and heuristics were developed.Famous Johnson's rule was proposed by Johnson for the scheduling of flow shop problem of two machines n-job and three machine n-job both for reducing make-span. Also this algorithm is useful in reducing the elapsed time of the process.

Adeyanju Sosimi[1] applied Johnson's rule along with genetic algorithm for getting proper schedule of of m-machines for production of Gari in Nigeria. It is nothing but a flow shop scheduling problem with n-number of jobs applied on m-number of machines.Herbert Campbell et.al[2] proposed the algorithm which is applicable for m-machines n-jobs problem. Without applying computer an algorithm can be generated with objective of reducing total flow time. For better optimization both exact and estimated methods are available. Both methods required to concentrate on realistic as well as economical purpose of the result proved. S. Agrawal et.al[3] find the algorithm for flow shop scheduling problem of three machines 5-jobs by treating setup time and processing time separately. The author emphasised on importance of set up time in scheduling as well as in minimizing total make-span of the process. Johnson's algorithm is used for finding the sequence of the problem. Deepak Gupta et.al [4] solved the flow shop scheduling problem of two machines with transportation time where processing times are assigned with particular probabilities of the jobs by using Branch and bound algorithm. Numerical problem solved for showing the utility of the algorithm.

Deepak Gupta et.al [5] worked on three machine specially structured with n job flow shop scheduling problem to decrease the hire cost by implementing particular rental policy where setup time and processing time with transportation time are separately considered. Deepak Gupta et.al[6] considered setup time and transportation time separately with probabilities followed by job block criteria. Branch and bound algorithm is used to minimize make-span. P. Pandian et.al [7] proposed an algorithm for flow shop scheduling problem of three machines involving transporting time, breakdown interval and job weightage.Valerie Botta-Genoulaz[8] worked on hybrid V-type flow shop problem of parallel indistinguishable machines with n-jobs with consideration of time delay due to setup time, transportation time, processing time of machines etc. The author represents 6 new algorithms for minimization of lateness. Ichiro Nabeshima[9] studied two and three machine flow shop scheduling problem with set up time, processing time, transportation time, removal time, and start and end lags time. Sameer Sharma et al.[10] developed an algorithm for 3 machine flow shop scheduling problem taking processing time and set up of machines independently with probabilities together with breakdown interval and transportation time. Deepak Gupta et.al[11] worked on the non idleness of machines in working of jobs when there is no breakdown interval in process in n-job m-machines flow shop scheduling problem. The heuristic provides an algorithm for minimizing the rental cost of machines for sequence of jobs chosen. JohannHurink et.al[12]solved three machines flow shop scheduling problem of n-jobs with consideration of single transportation agent(robot) and infinite buffer space for time lagging of machines. The objective defined is to find a schedule to minimize make-span. The exceptional cases where the transportation time and processing time occurred same are solved separately.

In current years maximum research work has been done where relation between transportation of jobs and scheduling of classical jobs taken in to consideration. Also in most of the research work setup time is not considered separately. It is considered as a part of processing time. But this will not give exact solution of the problems where it is required to consider setup time. Ali Allahverdiet al.[13]have reviewed the papers involving setup times. The applicationswith the concept of grouping of jobs, different parameters related time have studied. It dividesthe scheduling problems into sequence dependant and independent setup time, batch and non-batch and categorizes the literature into job shop, flow shop, single machine, parallel machine scheduling problem. This provides the base for researchers. Dar-LiYang et al.[14] used an algorithm of time polynomial for solving two machine flow shop scheduling problem with setup time and time required for removing job from one machine and transporting to another machine i.e. transportation time. The concept of grouping of jobs is applied. The objective is to minimize the working time.A.Khodadadi[15]discussed three machines FSSP by considering transportation time of jobs from one machine to other and expands an algorithm for it. Amit Mittal et al.[16] solved 3-machine flow shop scheduling problem with transportation time and importance of jobs processing using Johnson's rule for minimization of total make-span.

When any job is ready for processing on a machine it includes setup time of machine, processing time and transportation time of an article from one machine to another machine. Set up time is very affective factor when the job transferred on second machine. Hence set up time of machine cannot be ignoredin entire process. Another important factor is breakdown interval which occurs because of availability of material and machines at proper time, may be change in release date, due date, accessibility of equipments, shut down of electricity, variation in processing time of machines, technical problem etc. Also importance of jobs processing is also one of the important point. First time this concept of weights of jobs is considered by Miyazaki in 1980 [80]. In this chapter we try to find the algorithm for flow shop scheduling problem of five machines arranged in series. Setup time, processing time and transporting time are considered separately. Also effect of breakdown interval is observed on the schedule. Johnson's rule is used for finding the sequence of jobs. Five machines problem has reduced into four and three, machines and finally into two machines so that Johnson's rule can be used for preparing schedule. Importance of jobs is taken into consideration by applying job weightage to each job. Theorem is proved for five machines for getting optimum solution. With the help of thetheorem, numerical problem has been solved to show the utility of the theorem[17,18].

1.2 Statement of Theorem for optimal solution

When the articles i-1, i, i+1 arearranged in sequence, the optimal solution can be obtained so that

$$\mathrm{Min}(p_i + P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i + d_i, a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1} + d_{i+1} + v_{i+1} + V_{i+1}) < Min(p_{i+1} + P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1} + d_{i+1}, a_i + q_i + Qi+bi+ri+Ri+ci+si+Si+di+vi+Vi)$$

1.3 Proof

Let X and X' denotes sequences of items.

$X = \{T_1, T_2, - - - - - - T_{i-1}, T_i, T_{i+1}, T_{i+2}, - - - T_n\}$

$X' = \{T'_1, T'_2, - - - - - - T'_{i-1}, T'_i, T'_{i+1}, T'_{i+2}, - - - T'_n\}$

Let processing time of any item j on machine Y (ie. P, Q , R, S,V) for sequences X , X' be given by (Y_j, Y'_j)

Similarly, completion time of any item p on machine Y (ie. P, Q, R, S,V) for sequences X, X' be given by (CY_j, CY'_j)

Let (a_j, a'_j) be the transportation time of item j from machine P to Q,

(b_j, b'_j) be the transportation time of item j from machine Q to R, and

(c_j, c'_j) be the transportation time of item j from machine R to S.

(d_j, d'_j) be the transportation time of item j from machine S to V.

Let p_j, p'_j be the set up time of machine P,

q_j, q'_j be the set up time of machine Q

r_j, r'_j be the set up time of machine R

s_j, s'_j beset up time of item on machines S resp.

v_j, v'_j beset up time of item on machines V resp. for sequences X and X'.

The time required on machines Q ,R & S for completion of j^{th} item is given by

$CQ_j = \max(cP_j + a_j, cQ_{j-1}) + q_j + Q_j$

$= cP_j + a_j + q_j + Q_j$

$CR_j = \max(cQ_j + b_j, cR_{j-1}) + r_j + R_j$

$= cQ_j + b_j + r_j + R_j$

$CS_j = \max(cR_j + c_j, cS_{j-1}) + s_j + S_j$

$= cR_j + c_j + s_j + S_j$

$CV_j = \max(cV_j + d_j, cV_{j-1}) + v_j + V_j$

$= \max(cP_j + a_j + q_j + Q_j + b_j + r_j + R_j + c_j + s_j + S_j + d_j, cV_{j-1}) + v_j + V_j$ ----(1)

We will choose the sequence X such that $cV_n < c'V_n$ --------- (2)

$\mathrm{Max}(cP_m + a_m + q_m + Q_m + b_m + r_m + R_m + c_m + s_m + S_m + d_m, cV_{j-1}) + v_m + V_m$

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$< \max(c'P_m + a'_m + q'_m + Q'_m + b'_m + r'_m + R'_m + c'_m + s'_m + S'_m + d'_m, cV'_{j-1}) + v'_m + V'_m$ But

$cP_m + a_m + q_m + Q_m + b_m + r_m + R_m + c_m + s_m + S_m + d_m$
$= c'P_m + a'_m + q'_m + Q'_m + r'_m + R'_m + c'_m + s'_m + S'_m + d'_m$

Also $v_m = v'_m$, $V_m = V'_m$ Therefore equation 2 will be true if $cV'_{m-1} < c'V_{m-1}$

Proceeding in this way we get that inequality 2 is true if

$cV_j < c'V_j$ (j= i+1,i+2-----,m&i= 1,2,------m-1) ------------- (4)

We now calculate the values of cV_{i+1} & $c'V_{i+1}$

$cV_{i+1} = max\ (cS_{i+1} + d_{i+1}, cV_i) + v_{i+1} + V_{i+1}$

$= \max(cP_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1} + d_{i+1}, cV_i) + v_{i+1} + V_{i+1}$

$= \max\{c P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1} + d_{i+1}, \max(cS_i + d_i, cV_{i-1} + v_i + V_i\} + v_{i+1} + V_{i+1}$

$= \max\{c P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1} + d_{i+1}, cS_i + d_i + v_i + V_i + v_{i+1} + V_{i+1}$

$= \max\{c P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1} + d_{i+1}, \max(c P_i + a_i, cQ_{i-1}) + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i + d_i + v_i + V_i, cV_{i-1} + v_i + V_i\} + v_{i+1} + V_{i+1}$

$= \max\{c P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1} + d_{i+1}, c P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i + + d_i + v_i + V_i, cV_{i-1} + v_i + V_i\} + v_{i+1} + V_{i+1}$

$cV_{i+1} = \max\{c P_{i-1} + p_i + P_i + p_{i+1} + P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1} + d_{i+1} + v_{i+1} + V_{i+1}, c P_{i-1} + p_i + P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i + d_{i+1} + v_{i+1} + V_{i+1}, cV_{i-1} + v_i + V_i + v_{i+1} + V_{i+1}\}$ -------------(5)

Similarly,

$C'V_{i+1} = \max\{c' P_{i-1} + p'_i + P'_i + p'_{i+1} + P'_{i+1} + a'_{i+1} + q'_{i+1} + Q'_{i+1} + b'_{i+1} + r'_{i+1} + R'_{i+1} + c'_{i+1} + s'_{i+1} + S'_{i+1} + d'_{i+1} + v_{i+1} + V'_{i+1}, c' P_{i-1} + p'_i + P'_i + a'_i + q'_i + Q'_i + b'_i + r'_i + R'_i + c'_i + s'_i + S'_i + d'_{i+1} + v'_{i+1} + V'_{i+1}, cV'_{i-1} + v'_i + V'_i + v'_{i+1} + V'_{i+1}\}$ --------------------(6)

Comparing sequences S & S', we get $c P_{i-1} = c'P_{i-1}$ & $c V_{i-1} = c'V_{i-1}$

$Y_i = Y'_{i+1}$, $Y_{i+1} = Y'_i$ Where Y = P, Q, R,S or V ------------(7)

Also 1) $a_i = a'_{i+1}$, $a_{i+1} = a'_i$ 2) $b_i = b'_{i+1}$, $b_{i+1} = b'_i$

3) $c_i = c'_{i+1}$, $c_{i+1} = c'_i$ 4) $d_i = d'_{i+1}$, $d_{i+1} = d'_i$

Writing eq. 2 for j=i+1& using eq. 7, we get

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$$\text{Max}\{c\,P_{i-1}+p_i+P_i+p_{i+1}+P_{i+1}+a_{i+1}+q_{i+1}+Q_{i+1}+b_{i+1}+r_{i+1}+R_{i+1}+c_{i+1}+s_{i+1}+S_{i+1}+d_{i+1}+v_{i+1}+V_{i+1},\ c\,P_{i-1}+p_i+P_i+a_i+q_i+Q_i+b_i+r_i+R_i+c_i+s_i+S_i+d_i+v_i+V_i+v_{i+1}+V_{i+1},\ cV_{i-1}+v_i+V_i+v_{i+1}+V_{i+1}\}<$$

$$\text{Max}\{c\,P_{i-1}+p_{i+1}+P_{i+1}+p_i+P_i+a_i+q_i+Q_i+b_i+r_i+R_i+c_i+s_i+S_i+d_i+v_i+V_i,\ c\,P_{i-1}+p_{i+1}+P_{i+1}+a_{i+1}+q_{i+1}+Q_{i+1}+b_{i+1}+r_{i+1}+R_{i+1}+c_{i+1}+s_{i+1}+S_{i+1}+d_{i+1}+v_{i+1}+V_{i+1}+v_i+V_i,\ c\,V_{i-1}+v_{i+1}+V_{i+1}+v_i+V_i\} \quad \text{-------- (8)}$$

Subtracting last term from both sides, we get

$$Max\{c\,P_{i-1}+p_i+P_i+p_{i+1}+P_{i+1}+a_{i+1}+q_{i+1}+Q_{i+1}+b_{i+1}+r_{i+1}+R_{i+1}+c_{i+1}+s_{i+1}+S_{i+1}+d_{i+1}+v_{i+1}+V_{i+1},\ c\,P_{i-1}+p_i+P_i+a_i+q_i+Q_i+b_i+r_i+R_i+c_i+s_i+S_i+d_i+v_i+V_i+v_{i+1}+V_{i+1}\}<$$

$$\text{Max}\{c\,P_{i-1}+p_{i+1}+P_{i+1}+p_i+P_i+a_i+q_i+Q_i+b_i+r_i+R_i+c_i+s_i+S_i+d_i+v_i+V_i,\ c\,P_{i-1}+p_{i+1}+P_{i+1}+a_{i+1}+q_{i+1}+Q_{i+1}+b_{i+1}+r_{i+1}+R_{i+1}+c_{i+1}+s_{i+1}+S_{i+1}+d_{i+1}+v_{i+1}+V_{i+1}+v_i+V_{i,}\}$$

Subtracting $c\,P_{i-1}+p_i+P_i+p_{i+1}+P_{i+1}+a_i+a_{i+1}+b_i+b_{i+1}+c_i+c_{i+1}+d_i+$ di+1 +qi+Qi +qi+1+Qi+1 +ri +Ri+ri+1+ Ri+1+si +Si+ si+1+Si+1+vi +Vi+vi+1+ Vi+1from both sides we get

$$\text{Max}\ \{-a_i-b_i-c_i-d_i-q_i-Q_i-r_i-R_i-s_i-S_{i,}-v_i-V_{i,}\ -p_{i+1}-P_{i+1}-a_{i+1}-b_{i+1}-c_{i+1}-d_{i+1}-q_{i+1}-Q_{i+1}-r_{i+1}-R_{i+1}-s_{i+1}-S_{i+1}\}<$$

$$\text{Max}\ \{-a_{i+1}-b_{i+1}-c_{i+1}-d_{i+1}-q_{i+1}-Q_{i+1}-r_{i+1}-R_{i+1}-s_{i+1}-S_{i+1}-v_{i+1}-V_{i+1,}\ -p_i-P_i-a_i-b_i-c_i-d_i-q_i-Q_i-r_i-R_i-s_i-S_i\}$$

Therefore,

$$\text{Min}\ (p_i+P_i+a_i+q_i+Q_i+b_i+r_i+R_i+c_i+s_i+S_i+d_i\ a_{i+1}+q_{i+1}+Q_{i+1}+b_{i+1}+r_{i+1}+R_{i+1}+c_{i+1}+s_{i+1}+S_{i+1}+d_{i+1}+v_{i+1}+V_{i+1})$$

$$< Min(p_{i+1}+P_{i+1}+a_{i+1}+q_{i+1}+Q_{i+1}+b_{i+1}+r_{i+1}+R_{i+1}+c_{i+1}+s_{i+1}+S_{i+1}+d_{i+1},\ a_i+q_i+Q_i+b_i+r_i+R_i+c_i+s_i+S_i+d+v_i+V_i)$$

Hence proved.

1.4 Five machines algorithm

This theorem can be used in solving five machines problem of optimization for getting total elapsed time. Solution of numerical problem can be obtained by applying above proved theorem. In this problem five machines with their setup time, processing time, transporting time, as well as waiting time are considered.

The problem can be given in tabular form as follows:

Article	p_i	P_i	a_i	q_i	Q_i	b_i	r_i	R_i	c_i	s_i	S_i	d_i	v_i	V_i	wt_i

A_1	P_1	P_1	a_1	q_1	Q_1	b_1	r_1	R_1	c_1	s_1	S_1	d_1	v_1	V_1	wt_1
A_2	P_2	P_2	a_2	q_2	Q_2	b_2	r_2	R_2	c_2	s_2	S_2	d_2	v_2	V_2	wt_2
A_3	P_3	P_3	a_3	q_3	Q_3	b_3	r_3	R_3	c_3	s_3	S_3	d_3	v_3	V_3	wt_3
A_4	P_4	P_4	a_4	q_4	Q_4	b_4	r_4	R_4	c_4	s_4	S_4	d_4	v_4	V_4	wt_4
A_5	P_5	P_5	a_5	q_5	Q_5	b_5	r_5	R_5	c_5	s_5	S_5	d_5	v_5	V_5	wt_5

In the givenproblem five machines namely P, Q,R,S,Varearranged in series such that the articles A_1, A_2, A_3, ------ A_n are transported by transporting agent from machine Pto machine Q,machine Qto machine R,machine Rto machine S and machine S to V in such a way that after delivering the articles to machine V without delay come back to machine Pfor transferring the next item. The problem is solved for finding the solution which is optimum for the given production system in minimum time period.

Let

a_i denotes transporting time that is the time required for transportation of the articles from P to Q,

b_i denotes transporting time required for transportation of the articles from Q to R,

c_i denotes transporting time required for transportation of the articles from R to S.

d_i denotes transporting time required for transportation of the articles from S to V.

For producing the articles machines required the set up time which is denoted by

p_i :- set up time of machine P, **q_i**, :- set up time of machine Q,**r_i**, :- set up time of machine R, **s_i**:- set up time of machine S, **v_i** :- set up time of machine V,

Let the time required for the transporting agent from Vand Pwhich is called as the returning time. When the machine P has completed the production of A_{i-1} article and the transporting agent delivering this article to machine V come back to machine P but if the processing of last article by machine Vis completed as it starts this processing instantly after giving out of previous article then machine Phas to wait for the transporting agent if it didn't come back by that time.

Let**P_i**:- Processing time of machine P, **Q_i** :- Processing time of machine Q,

R_i :- Processing time of machine R, **S_i**:- Processing time of machine S,

V_i:- Processing time of machine V,

The problem can be solved in 5 steps in the following way:

Step I:

Five machine problem reduced into four machine problem by introducing four assumed machines E, F, G and O with the service time Ei, $F_i G_i$and $O_i$resp.

Where, $E_i = p_i + P_i + a_i + q_i + Q_i + b_i$, $F_i = a_i + q_i + Q_i + bi + r_i + R_i$

$G_i = bi + r_i + Q_i + c_i + s_i + S_i$, $O_i = c_i + s_i + S_i + di + v_i + V_i$

Article	E_i	F_i	G_i	O_i	wt_i
A_1	E_1	F_1	G_1	O_1	wt_1
A_2	E_2	F_2	G_2	O_2	wt_2
A_3	E_3	F_3	G_3	O_3	wt_3
A_4	E_4	F_4	G_4	O_4	wt_4
A_5	E_5	F_5	G_5	O_5	wt_5

Also

1) Min $(p_i + P_i + a_i) \geq$ Max $(a_i + q_i + Q_i)$2) Min $(bi + r_i + R_i) \geq$ Max $(a_i + q_i + Q_i)$

3) Min (c_i +s_i +S_i)≥ Max (c_i +r_i+ R_i)4) Min (d_i + v_i +V_i)≥ Max (d_i + s_i +S_i)

Step II:

Now this problem will be reduced into two machine problem.

But first we have to convert it into three machines H,K,L and then by considering two fictitious machines M and Nthe problem will be converted into two machines problem.

Let Hi = $E_i + F_i$, Ki = $F_i + G_i$, Li = $G_i + O_i$

Therefore M and N are represented as follows:

Mi = $H_i + K_i$,Ni = $K_i + L_i$

Also1) If min (M,N) = M_i then $M'_i = M_i - wt_i$and $N'_i = N_i$

2) If min (M,N) = N_i then $M'_i = M_i$ and $N'_I = N_i + wt_i$

Step III:

For getting the proper sequence the new problem is defined as follows and represented in the table form: $M'i = M'i/wt_i$and $N'i = N'i/wt_i$

Article	M'_i/wt_i	N'_i/wt_i	wt_i
A_1	M'_1 / wt_1	N'_1/ wt_1	wt_1
A_2	M'_2/ wt_2	N'_2 / wt_2	wt_2
A_3	M'_3 / wt_3	N'_3 / wt_3	wt_3
A_4	M'_4 / wt_4	N'_4 / wt_4	wt_4
A_5	M'_5 / wt_5	N'_5 / wt_5	Wt_5

Step IV:

Considering the breakdown interval which is already decided or known and the effect of this breakdown interval should be observed on all jobs. With the effect of this breakdown interval, the problem will be redefined as follows:

1)If the job is affected by the breakdown interval (u_1-u_2) then

$P'_i = P_i +(u_1-u_2)$, $Q'_i = Q_i + (u_1-u_2)$, $R'_i = R_i + (u_1-u_2)$,$S'_i = S_i + (u_1-u_2)$,$V'_i = V_i + (u_1-u_2)$

2) If the job is not affected by the breakdown interval (u_1-u_2)then

$P'_i = P_i$, $Q'_i = Q_i$, $R'_i = R_i S'_i = S_i$ $V'_i = V_i$

Step V: Applying steps I,II,III, IV the problem has been solved for getting the optimal sequence.

Every job has not equal importance hence all the jobs are assigned with weights according to their importance in the sequence of productionand it is considered as one of the measure in the computation of total make-span. So the entirety weighted flow time is summation of product value of weighted value of the job and its flowtime. Also the mean weighted flow time is the ratio of weighted flow time and sum of the values of weight of all the jobs.

Hence, **Mean weighted flow time = weighted flow time/ sum of weights**

The scheduling of all the course of action is in such a way that the minimum time should be required for getting the optimum solution or whole production For finding the algorithm the above information can be symbolized and represented in table for finding the sequence by using Johnson's rule of sequencing.

1.5 Numerical problem- Description and Solution

Let us consider the problem of five machinesarranged in series with set up time, processing time, a transporting time, weight of the jobs,:

Table 1.5.1

Article	p_i	P_i	a_i	q_i	Q_i	b_i	r_i	R_i	c_i	s_i	S_i	d_i	v_i	V_i	wt_i
A_1	2	6	4	3	5	3	4	6	2	5	6	3	6	6	2
A_2	3	5	6	3	2	5	2	5	3	6	5	2	5	7	4
A_3	3	4	5	2	4	2	4	6	3	4	6	3	5	6	3
A_4	4	5	3	2	4	4	2	7	2	3	8	3	4	8	5
A_5	2	8	3	1	3	6	1	8	2	4	8	2	5	8	2

Solution: **Step I:**

Min $(p_i + P_i + a_i)$=12 and Max $(a_i + q_i + Q_i)$= 12

Similarly other inequalities are also satisfied as mentioned in algorithm.

Letfour fictitious machines E, F, G and O with their service times $E_{i,}$ $F_{i,}$,G_i andO_i respectively.The problem is reduced and it is given in the following table.

Table 1.5.2

Article	$E_i = p_i + P_i + a_i + q_i + Q_i + b_i$	$F_i = a_i + q_i + Q_i + bi + r_i + R_i$	$G_i = bi + r_i + Q_i + c_i + s_i + S_i$	$O_i = c_i + s_i + S_i + di + v_i + V_i$	wt_i
A_1	23	25	26	28	2
A_2	24	23	26	28	4
A_3	20	23	25	27	3
A_4	22	22	26	28	5
A_5	23	22	29	29	2

Step II:

Now this problem will be reduced into two machine problem by considering three fictitious machines H, K, L first and then two fictitious machines M and N where

$Hi = E_i + F_i$, $Ki = F_i + G_i$, $Li = G_i + O_i$

Table 1.5.3

Article	H	K	L	wt_i
A_1	48	51	54	2
A_2	47	49	54	4
A_3	43	48	52	3
A_4	44	48	54	5
A_5	45	51	58	2

Now, $Mi = H_i + K_i$, $Ni = K_i + L_i$

Table 1.5.4

Article	$Mi = H_i + K_i$	$Ni = K_i + L_i$	wt_i
A_1	99	105	2
A_2	96	103	4
A_3	91	100	3
A_4	92	102	5
A_5	96	109	2

Also

1) If min (M,N) = M_i then $M'_i = M_i - wt_i$ and $N'_i = N_i$
2) If min (M,N) = N_i then $M'_i = M_i$ and $N'_I = N_i + wt_i$

Table 1.5.5

Article	M'_i	N'_i	wt_i
A_1	97	105	2
A_2	92	103	4
A_3	88	100	3
A_4	87	102	5
A_5	94	109	2

Step III;**Table 1.5.6**

Article	M'_i / wt_i	N'_i /wt_i	wt_i
A_1	48.5	52.5	2
A_2	23	25.75	4
A_3	29.33	33.33	3
A_4	17.4	20.4	5
A_5	47	54.5	2

By Johnson's rule the optimal sequence obtained is 4, 2, 3, 5, 1.

Table 1.5.7

Ai	p_i	P_i		a_i	q_i	Q_i		b_i	r_i	R_i		c_i	s_i	S_i		d_i	v_i	V_i		wt_i
		I	O			I	O			I	O			I	O			I	O	
A_4	4	4	9	3	2	14	18	4	2	24	31	2	3	**36**	**44**	3	4	51	59	5
A_2	3	12	17	6	3	26	28	5	2	**35**	**40**	3	6	50	55	2	5	62	69	4
A_3	3	20	24	5	2	31	35	2	4	44	50	3	4	59	65	3	5	74	80	3
A_5	2	26	34	3	1	**38**	**41**	6	1	51	59	2	4	69	77	2	5	85	93	2
A_1	2	**36**	**42**	4	3	49	54	3	4	63	69	2	5	82	88	3	6	99	105	2

Step IV :

Effect of breakdown interval (36, 44):

The effect of breakdown interval is on jobs P_i1 Q_i5, R_i2, and on S_i4, hence the original problem is converted into new problem.

If the job is affected by the breakdown interval then

$P'_i = P_i + (u_2\text{-}u_1)$, $Q'_i = Q_i + (u_2\text{-}u_1)$, $R'_i = R_i + (u_2\text{-}u_1)$, $S'_i = S_i + (u_2\text{-}u_1)$,

$V'_i = V_i + (u_2\text{-}u_1)$

Therefore repeating the same procedure of step 1,2,3 for finding the sequence and getting the optimal solution.

Table 1.5.8

Article	p_i	P_i	a_i	q_i	Q_i	b_i	r_i	R_i	c_i	s_i	S_i	d_i	v_i	V_i	wt_i
A_1	2	**14**	4	3	5	3	4	6	2	5	6	3	6	6	2
A_2	3	5	6	3	2	5	2	**13**	3	6	5	2	5	7	4
A_3	3	4	5	2	4	2	4	6	3	4	6	3	5	6	3
A_4	4	5	3	2	4	4	2	7	2	3	16	3	4	8	5
A_5	2	8	3	1	11	6	1	8	2	4	8	2	5	8	2

Repeating the same procedure as above, we get

Table 1.5.9

Article	E_i	F_i	G_i	O_i	wt_i
A_1	31	25	26	28	2
A_2	24	31	34	28	4
A_3	20	23	25	27	3
A_4	22	22	34	36	5
A_5	31	30	29	29	2

By using step II, we get

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Table 1.5.10

Article	$Hi = E_i + F_i$	$Ki = F_i + G_i$	$Li = G_i + O_i$	wt_i
A_1	56	51	54	2
A_2	55	65	62	4
A_3	43	48	52	3
A_4	44	56	70	5
A_5	61	59	58	2

Table 1.5.11

Article	$Mi = H_i + K_i$	$Ni = K_i + L_i$	wt_i
A_1	107	105	2
A_2	120	127	4
A_3	91	100	3
A_4	100	126	5
A_5	120	117	2

1) If min (M,N) = M_i then $M'_i = M_i - wt_i$ and $N'_i = N_i$
2) If min (M,N) = N_i then $M'_i = M_i$ and $N'_I = N_i + wt_i$

Table 1.5.12

Article	M'_i	N'_i	wt_i
A_1	107	107	2
A_2	116	127	4
A_3	88	100	3
A_4	95	126	5
A_5	120	119	2

Step III;**Table 1.5.13**

Article	M'_i / wt_i	N'_i /wt_i	wt_i
A_1	53.5	53.5	2
A_2	29	31.75	4
A_3	29.33	33.33	3
A_4	19	25.2	5
A_5	60	59.5	2

By Johnson's rule the optimal sequence obtained for above reduced problem is 4,2,3, 1, 5

Table 1.5.14

Result:

Minimum weighted flow time (MWFT)

Article	p_i	P_i		a_i	q_i	Q_i		b_i	r_i	R_i		c_i	s_i	S_i		d_i	v_i	V_i		wt_i
		I	O			I	O			I	O			I	O			I	O	
A_4	4	4	9	3	2	14	18	4	2	24	31	2	3	**36**	**52**	3	4	59	67	5
A_2	3	12	17	6	3	26	28	5	2	**35**	**48**	3	6	58	63	2	5	72	79	4
A_3	3	20	24	5	2	31	35	2	4	52	58	3	4	67	73	3	5	84	90	3
A_1	2	26	40	4	3	**47**	**52**	3	4	62	68	2	5	78	84	3	6	96	102	2
A_5	2	**42**	**50**	3	1	54	65	6	1	72	80	2	4	88	96	2	5	107	115	2

= (67*5) + (79- 9) *4 + (90 -17)*3+(102-24)*2 + (115- 40)*2

= 335+280+219+156+150/5+4+3+2+2

= 1140/16

= 71.25 hours

Conclusion: From above table it is shown that the time gets reduced for total production by using the sequence obtained with the help of Johnson's rule. The total elapsed time for the complete process is 115 hrs and minimum weighted flow time is 71.25hrs. If there is a change in the breakdown interval then there is not a major change in the total make-span of the production. Johnson's rule is effective in finding the sequence of the jobs. As pre-emption of jobs is not allowed therefore the machines are working properly with decided job sequence and there is no extra waiting time for the machines which are in working.

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