

Analysis of Functionally Graded Material Plates using Sigmoidal Law

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ABSTRACT

Functionally Graded Material plate using First order Shear Deformation Theory is analyzed for strain displacement, stress-strain relations, stress resultants, and middle plane strain resultants. The direct stress, transverse shear stress, in-plane shear stress and the displacement of a simply supported rectangular functionally graded plate is analyzed. Transverse uniform compressive load is applied on the plate with constant Poisson's ratio and varying Young's modulus in sigmoidal law function along t the thickness direction with different materials on top layer and bottom layer of the plate. Result are compared for five degrees of freedom, (three displacements and two rotations).

Keywords : FGM, FSDT, Sigmoidal Law, Plates, Power law, Graded Material

INTRODUCTION

Composites are being used for its extensive benefits such as increased strength, stiffness, impact resistance, thermal conductivity, thermal insulation, acoustical insulation, and corrosion resistance. Composites are used as a replacement for steel, as it is being used for making lightweight mechanical parts without compromising on the desired strength and stiffness. They are made in layers to bond each other in stacks forms laminate.

In Functionally Graded Material (FGM) (Fig. 1) the mechanical properties of each element are different, while manufacturing the FGM it is necessary that the mechanical properties are gradually varying along the thickness direction. The mechanical properties mainly Young's modulus is made to vary using certain functions, to ensure smooth distribution of the stresses. The functions may be power-law function, sigmoidal law function and exponential law function. Most commonly used function is power-law function, but sigmoidal law is the combination of two power law function distribution suggested by Chung and Chi. This law is a dependent law, by using sigmoidal law, the abrupt changes in the Young's modulus at the top surface in the material and the stress concentration in the interfaces are eliminated and smooth transition of the material property (young's modulus) is guaranteed.

There are two types of forces's that are acting on a body, applied external force and the resisting internal force which is 'stress'. To understand the behaviour of any material, it is necessary to find the stresses and displacement. FGM are commonly used in aerospace industry, medical field, nuclear projects, energy sector, communication field, transportation field for antifriction coatings, sensors, solar cells, gas turbine engines.

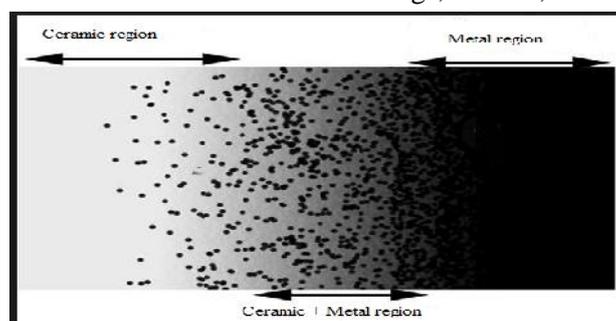


Fig.1 Ceramic-metal Functionally Graded Material Plate.

RELATED WORK

Ankit B and Khushbu C (2014) gave information about history, types, manufacturing, applications of FGM, then succeeds with the effects of stress concentration in FGM plate with cutout. Stress concentration factor is dimensionless was used to study the effects of stress concentration and magnitude of localized stress. The magnitude of stress concentration factor depends on the power-law index. Different value of forces were applied in different loading conditions. The response is measured mainly by experiment or analytical. It was learned that the stress concentration in case of biaxial loading is less than that of uniaxial loading. It was induced that as the radial Young's modulus increased, the stress is reduced, but it is not dependent on Poisson's ratio.

Ashraf M. Zenkour(2006) has considered the static response of FGM plate under transverse load. The author has presented a correct representation of the transverse shear strain, therefore the shear correction factor is neglected. The constituents of the plate is varied according to the power-law function in the thickness direction. They uses equilibrium equations of shear deformation theory for a functionally graded plate. The FGM plate is assumed to be simply supported therefore solution for the numerical model was obtained by Navier solutions. Plates with different mixtures were analyzed for deflections and it was found that they decrease smoothly as the volume fraction exponent decreases and as the ratio of metal-to-ceramic moduli increases. It was found that the response of the FGM plate that corresponds to properties intermediate to that of that metal and ceramic used lies in between that of ceramic and metal, irrespective of boundary condition.

Bhavani V.Sankar and Jerome T. Tzeng (2002) obtained longitudinal stress distribution using thermoelastic equilibrium equations and solved in closed-form for the functionally graded material plate. Euler Bernoulli-type beam theory was developed according to the assumptions made. Thermal stresses increases as the elastic constants and temperature increases through the thickness. FGM is assumed to vary in exponential function. The length of the FGM beam considered was 100mm and thickness as 10mm. The responses showed that the maximum tensile stress was near the neutral axis. It was found that when the Young's modulus increases the thermal stress increases rapidly. Thermal stress can be decreased by varying the thermoelastic properties. The results obtained was identical to the results obtained by elastic solution. Chih-Ping Wu and Hao-Yuan Li (2010) used the third order shear deformation theory. In the formulation of mid-plane displacement rotation and the transverse shear stress were taken as primary variables which were made to vary. The solution obtained was validated with the results obtained by PVD-based third order shear deformation theory. The change of through thickness-distribution of different modal field variables among FGM and homogenous plates is important when the material property gradient index grows big.

G N Praveen and J.N.Reddy(2010) taken functionally graded circular plate for the theoretical formulation. The objective was to relate the equations of first order shear deformation theory for deflection, forces, and moments to classic plate theory. Equations for radial and circumferential forces and moments were formulated with modulus of elasticity, Poisson's ratio and shear correction factor. Bending relationships for roller supported, hinged, clamped circular and clamped free angular plate was considered. Non-dimensional zed values of FGM for various boundary condition were presented for varying thickness radius ratio. The modulus E and the thermal coefficient of expansion are assumed to change throughout the plate thickness. The equation of motion was written in third order shear deformation theory. Third order shear deformation theory in the literature requires no shear correction factor and through the thickness the stress stresses are presented as quadratic. Non-linear part was also modeled using finite element model. Static linear, non-linear static and dynamic analysis were performed numerically. In static linear analysis it was learned that the difference between third order shear deformation theory and first order shear deformation theory is negligible. Third order shear deformation was presented in the paper. Among the two iterative methods Picard's iterative method is simple but the stiffness matrix is unsymmetrical unlike Newton-Raphson iterative method.

Trung Kien Nguyen, Karam Sab, and Guy Bonnet,(2008) has considered numerical examples on simply supported square plate and cylindrical bending sandwich FGM plates clamped at both end were analyzed. The objective of the paper was to identify a best suited shear correction factor. The material gradient properties are assumed to be changing continuously along the thickness according to power-law function. The predicted responses of deflections were validated with the available solutions from previous studies using finite element

model. The measurement of 'relative error was presented. It was found that the variation of shear correction factor does not affect the deflections of thin and medium thick plates. The difference between classic plate theory model and other models was remarked, the reason being the contribution of shear deformation energy for medium thick plate.

K Swaminathan and D T Naveenkumar(2013) has proposed theoretical formulation and analytical solutions, based on Hamilton's principle and Levi type solution for free vibration analysis of functionally graded material plate using first order shear deformation (FSDT).The objective of the paper was to find an accuracy of the results for free vibration response of simply supported functionally graded plates. The relationship between various material stiffness matrix with in-plane stress, bending stress and shear stress was formulated using displacement model, constitutive relations and governing equations. From two example problems it was found that the non-dimensional values of the natural frequency increased with the increase in aspect ratio and as the power law function value increases the value of natural frequency decreases. Manish Bhandari and Dr. Kamlesh Purohit (2014) carried out parametric studies by varying volume fraction distribution power-law, sigmoidal-law and exponential function and boundary conditions. Static analysis and verification for different volume fractions was presented in the literature by varying the mesh size and layer size. The FGM is modeled as metal on one side and ceramic on the other. In FEM modeling the FGM is made up various 'layers' in order to take care of the grading. FEM modeling was through ANSYS software. The result was presented in terms of non-dimensional parameters. The accuracy and proficiency of the study was compared with published result and the difference between the two results was below 3%, which proved very good solution accuracy.

Shyang-Ho Chi (2005) -FGM plate being elastic, rectangular, simply supported, subjected to transverse load is consider. In the beginning the investigators assumed that the Young's modulus and Poisson's ratio to be varying according to the thickness. Material properties of P-FGM, S-FGM, E-FGM was discussed in the literature. The stress fields, axial forces, shear forces, bending moment were obtained by classic plate theory, from research it was found that for a plate with thickness less than 0.1of its span the theory gives good results even though transverse shear deformations are neglected. Later the investigators found out that if both the Poisson's ratio and Young's modulus are considered to be varying the integration turns out to be complicated ,thereby they assumed the Poisson's ratio to be a constant. The closed-form solution with respect to Young's modulus of P-FGM, S-FGM, and E-FGM were obtained.

Victor Birman (2010):Fatigue response and fracture response of FGM were discussed. The investigator suggests that the coupling between micromechanics and heat transfer should be taken into consideration while formulating the solution. The equations of motion of FGM structures include stiffness and thermal expansion tensor coefficients. The strains are formulated using Lagrange's or Euler's equation. The basic step towards the foundation for the analysis of functionally graded material depends upon the gradient distribution, shape and orientation. It was found that for a 3D FGM thermoelastic analytical solution was preferred only when the effect of thermal load on the stiffness of the material is ignored. When the structure is subjected to temperature load only on one of it's the sides the distribution is nonuniform throughout the thickness.

Shyang-Ho Chi and Yen-Ling Chung (2006): MARC software which is based on finite element method was used for the analysis ,16 layers was used for gradual change of the material properties. Sigmoidal function was used for grading ,however power law and exponential law responses were also looked upon. The FGM plate was simply supported and perpendicular load was applied. Theoretical results calculated coincides well with the result from finite element analysis. It was remarked that the effect of varying Poisson's ratio of the FGM plate on its mechanical response is very small. From the analysis it was clear that maximum tensile stress of the FGM plate was along the bottom of the plate and maximum compressive stress was on the inner side of the plate. Zheng Zhon et al (2010) were considered a plate subjected to normal and shear fractions on top and bottom surfaces of the simply supported FGM plate. Cylindrical bending response based on two dimensional theory of elasticity of the functionally graded plate is obtained. Using Fourier series expansion and variable separation method a general solution procedure was developed. Numerical results were calculated for a simply supported FGM plate and the influence of different grade models on the stress and displacement fields were discussed. It was observed ,for a thick FGM plate the vertical displacement is uniformly distributed along the thickness direction and the horizontal displacement has a linear variation across the thickness, for a thick

FGM the vertical displacement is not uniform and horizontal displacement shows a deviation from linear distribution across the thickness.

SHEAR DERFORMATION LAMINATED PLATE THEORY (SDLPT)

Equivalent Single Layered Plate Theories - 3D problem is converted into a 2D problem making it easier to calculate the responses. The multi layered heterogeneous FGM plate is considered to behave as a single layer plate, but having complex material properties. There are two theories for the analysis of plates. First is the classical plate theory and the second is shear deformation theories. Classical laminated plate theory is derived from classical plate theory (Kirchhoff-Love theory) derived from Euler-Bernoulli beam theory which gives the relationship between deflection due to the load applied. Classical plate theory is useful for the prediction of deflection, stress and strains, while it does not account for transverse normal and shear stress. Shear responses to be found by post computation of the plate using 3D equilibrium equation which is complicated and unreliable, they are reliable only or homogenous and thin plates. Classical plate theory neglects the interlaminar forces between the layers.

Mindlin-Reissner plate theory, developed by Mindlin with the help of the theories developed earlier by Reissner in 1945. Mindlin-Reissner theory is also known as first order shear deformation theory as it takes into account the transverse shear stresses. This theory is used for thin plates. The displacement are function of thickness. Shear deformation theory uses constitutive equations which predicts the effect of transverse shear deformation on predicted deflections, frequencies and buckling loads. Constitutive equations are used as they are simple to calculate the interlaminar shear stress. The result obtained is a constant value and can be used for a thick plate.

First order Shear Deformation Theory (FSDT) - When the shear deformation theory is limited to first order (as first order shear deformation theory is easy to calculate), consider a rectangular functionally graded plate in the cartesian plane with a and b as the dimensions of the plate, h be the thickness of the plate, z be the thickness measured from the mid-plane surface. Load is assumed to be transverse and the plate is assumed to be microscopically graded according to the sigmoidal function. Sigmoidal function distribution ensures a smooth distribution of the stress in the plate and is obtained using the rule of mixture. According to first order shear deformation theory, the displacement model is used to find the displacement u,v,w along x,y and z coordinates.

$$u(x, y, z) = u_0(x, y, z) - z \theta_x(x, y) \quad (1)$$

$$v(x, y, z) = v_0(x, y, z) - z \theta_y(x, y) \quad (2)$$

$$w(x, y, z) = w_0(x, y) \quad (3)$$

where (u_0, v_0, w_0) are the displacement in the x, y, z plane and (θ_x, θ_y) are rotations of the transverse normal on the considered plate. The stress strain relationship for transverse shear deformation is given by $\tau = Q \gamma$

where

τ = Stress vector

Q = Transformed elastic stiffness matrix

γ = Strain vector

Strain Displacement relations : With the definitions of strains for linear theory of elasticity the strains of any point in the plate and its deformations are functions of the displacement field. The strain-displacement relations are as follows:

$$\epsilon_x = \frac{\partial u}{\partial x} \quad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

$$\epsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\epsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\epsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

Normal stress and shear stress relations

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{xz} \end{Bmatrix}$$

where,

$$Q_{11} = Q_{22} = \frac{E(z)}{(1-\nu^2)}$$

$$Q_{12} = Q_{21} = \frac{\nu E(z)}{(1-\nu^2)}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}$$

Power law distribution - The material properties are varied according to the power-law function.

$$G(z) = \left(\frac{z + h/2}{h} \right)^p \tag{4}$$

The Young's modulus of the FGM plate is calculated by rule of mixture

$$E(z) = (g(z) * E_1) + ((1 - g(z)) * E_2) \tag{5}$$

The general variation of young's modulus along the thickness for a Functionally Graded Materials is using Power law distributions as shown in Fig. 2

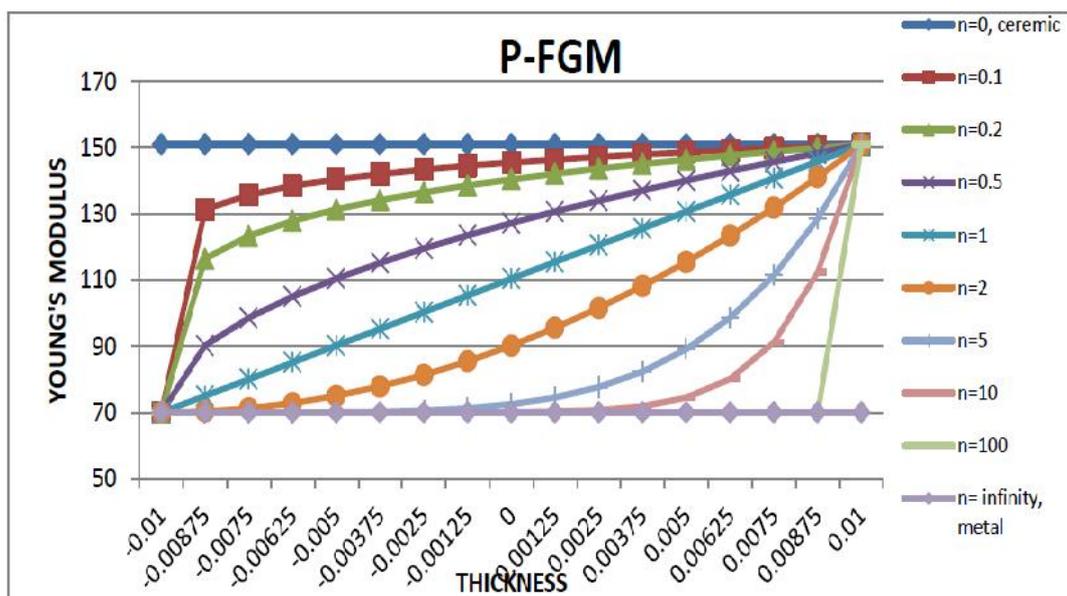


Fig.2 Variation of Young's modulus of a FGM plate along the thickness using Power law

Sigmoidal law distribution -

$$g_1(z) = 1 - \frac{1}{2} \left(\frac{\frac{h}{2} - z}{\frac{h}{2}} \right)^p \quad \text{for } 0 \leq z \leq \frac{h}{2} \quad (6)$$

$$g_2(z) = \frac{1}{2} \left(\frac{\frac{h}{2} + z}{\frac{h}{2}} \right)^p \quad \text{for } 0 \leq -\frac{h}{2} \leq z \leq 0 \quad (7)$$

The young's modulus is calculate using the rule of mixture

$$E(z) = (g_1(z) * E_1) + [1 - g_1(z)] E_2 \quad \text{for } 0 \leq z \leq \frac{h}{2} \quad (8)$$

$$E(z) = (g_2(z) * E_1) + [1 - g_2(z)] * E_2 \quad \text{for } -\frac{h}{2} \leq z \leq 0 \quad (9)$$

The general variation of young's modulus along the thickness for a Functionally Graded Materials is using Sigmoidal law distributions as shown in Fig. 3

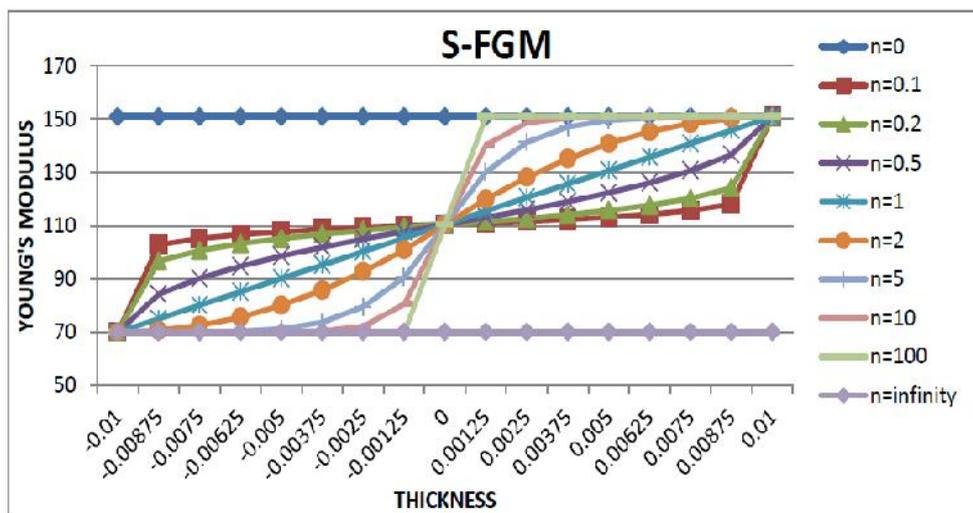


Fig. 3 Variation of Young's modulus of a FGM plate along the thickness using Sigmoidal law

ANALYTICAL MODELING

Stress-concentrations tend to settle on the interface layers of the FGM plate, while using a single power law distribution, since the material properties changes rapidly towards the top surface. FGM plates using volume fraction of two combined power-law distribution is considered to be beneficial than a single power law distribution. When two power laws (Sigmoidal law) are combined the transition of Young's modulus from the top surface to the bottom is gradual. Usage of sigmoidal function in FGM plate is upcoming and rare. Analysis of sigmoidal functionally graded material for stress and strains is found on its five degrees of freedom (displacements and rotations).

Generalized MATLAB program (Fig.4) for finding the response of a functionally graded plate being generalized with certain parameters as input by the user, related to the dimensions of the plate, length of the FGM plate under consideration, ratio of length to breadth of the FGM, ratio of length to thickness, mechanical

properties being Young's modulus of top layer and bottom layer, Sigmoidal parameter, Poisson's ratio, Load intensity, Position of the load acting.

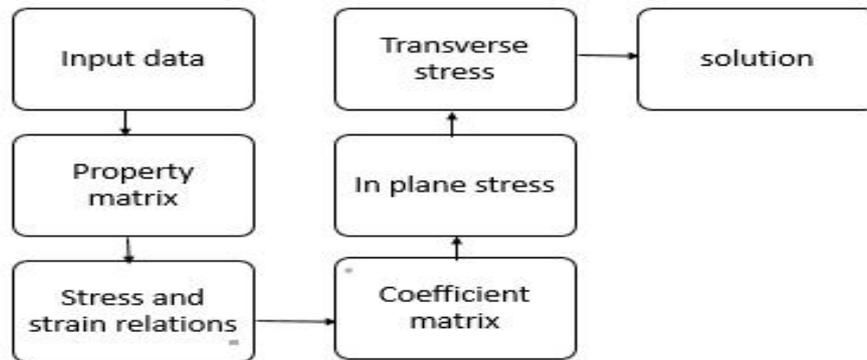


Fig. 4. Modules Used in MATLAB Program

Governing equations of rectangular FGM plates - An invariably elastic, uniformly medium thick rectangular FGM plate subjected to a transverse load is considered. It is assumed that

- Plane perpendicular to the middle surface of the plate before deformation remain normal after deformation.
- Compared to the thickness the deformations of the FGM plates are small.
- The Young's modulus and Poissons ratio for the non-homogeneous elastic FGM plate, are functions of the spatial coordinate z .
- The thickness is assumed to be small compared to the span of the FGM plate, therefore the normal stresses in the transverse direction is small and neglected

Theoretical formulations forms the basis for the analytical solutions. Theoretical formulation was found using displacement field by first order shear deformation theory, stress strain relations, strain displacement relations, stress resultant and middle plane strain resultants, equations of equilibrium and natural boundary conditions. The inplane and transverse stresses and the inplane and transverse shear stresses found are made to non-dimensionalized. Apply the boundary condition to the Naiver solution technique, the response of the FGM plate is simplified to be

$$\begin{aligned}
 U_{0mn} &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos r_x \sin s_y U_{0mn} \\
 V_{0mn} &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin r_x \cos s_y V_{0mn} \\
 W_{0mn} &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin r_x \sin s_y W_{0mn} \\
 \mu_x &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos r_x \sin s_y \mu_{xm} \\
 \mu_y &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin r_x \cos s_y \mu_{ym}
 \end{aligned} \tag{10}$$

Where, $\alpha = \frac{\pi m}{a}$ and $\beta = \frac{\pi n}{b}$

The simplified matrix is

$$\begin{Bmatrix} u \\ v \\ w \\ u_x \\ u_y \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} u_0 \cos r x \sin s y \\ v_0 \sin r x \cos s y \\ w_0 \sin r x \sin s y \\ x_0 \cos r x \sin s y \\ y_0 \sin r x \cos s y \end{Bmatrix} \quad (11)$$

For the analysis the FGM plate a square plate of length ‘a’ (assumed as 1unit) , ratio of length to breadth one and ratio of length to height 10 is considered. From the literature it is observed that the variation of material properties is varied using sigmoidal law by taking a value as 2.The Young’s modulus of the materials is taken as input by the user,. The Young’s modulus at the top is assumed as 380GPa (Zirconium) and the Young’s modulus at bottom is taken as70GPa (Aluminum).The Poisson’s ratio for ceramic is 0.3 and for metal is 0.2 is considered. .In this analysis the Poisson’s ratio is assumed to be constant and the value is taken as 0.3.The transverse load of 1000N is applied on the FGM plate. The response of the FGM from all the corners were taken into consideration. Corresponding inplane and transverse direct stresses, inplane and transverse shear stresses and displacement were estimated.

Analysis 1: The stresses and displacements are analyzed by changing the martial property at the top layer of a functionally graded plate with different Young’s modulus, sigmoidal values and length to thickness ratios (l/t). The data considered for the analysis are length 100mm, length to breadth ratio 1 and with a load of 1000KN, Poisson’s ratio 0.3. bottom metal aluminum (E 70Gp). The stresses and displacements with respect to thickness are carried out and the responses are recorded different Young’s modulus and shown in Fig 5 to Fig 9.

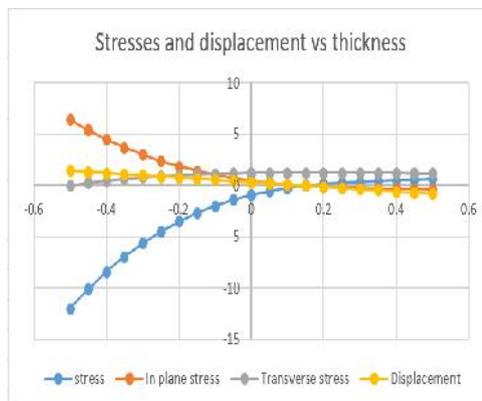


Fig.5 Plate with E 380 Gpa, Sigmoidal 2, & l/t=10

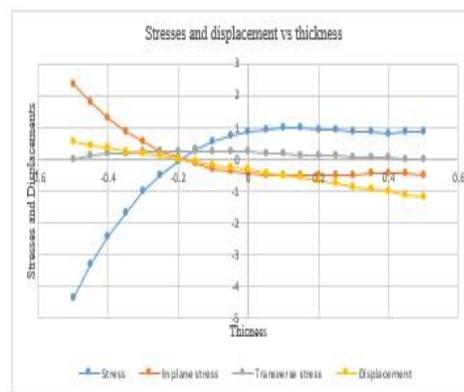


Fig. 6 Plate with E 420 Gpa, Sigmoidal 2, & l/t=10

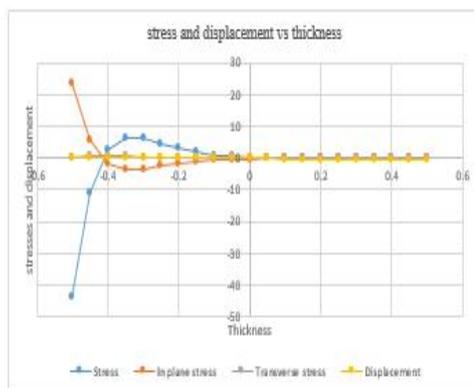


Fig.7 Plate with E 380Gpa, Sigmoidal 10 & l/t=10

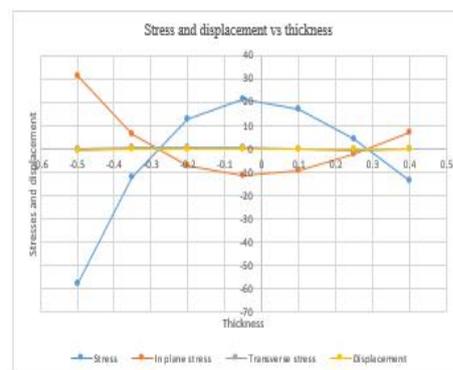


Fig.8 Plate with E 380Gpa Sigmoidal 10 & l/t=30

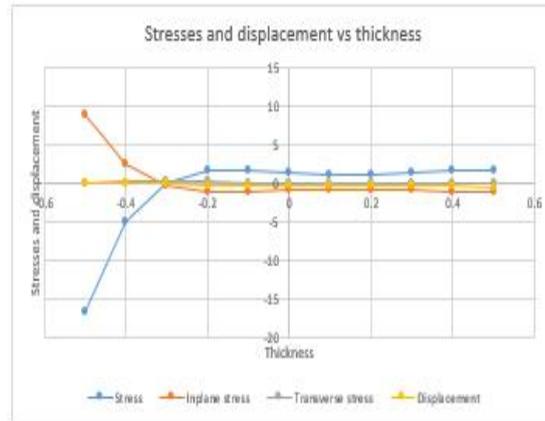


Fig.9 Plate with E 224Gpa, Sigmoidal 5 & l/t=20

Analysis 2: The bottom layer of the FGM plate is made using aluminum and the top layer of the FGM plate is made using a) Silicon Carbide, b) Steel alloy, c) Iron, and d) Titanium. The direct stress, transverse stress, and displacement of the FGM is recorded with respect to thickness and shown in Fig. 10 to Fig. 12 and Fig. 13 show the direct stresses with different Sigmoidal values.

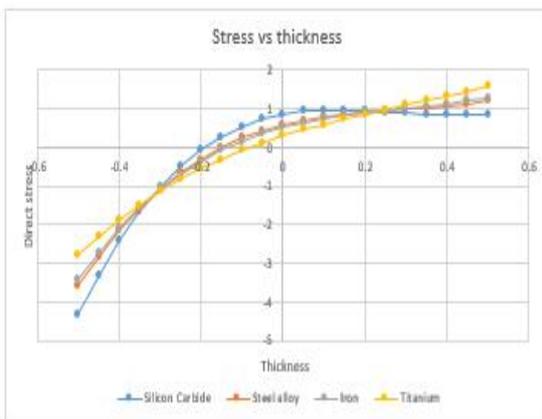


Fig.10. Variations of Direct stress

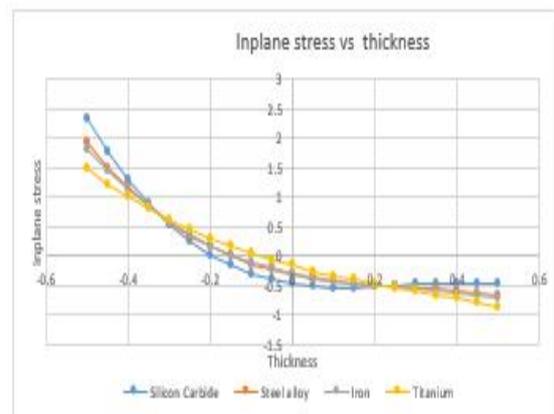


Fig.11. Variations of Inplane stress

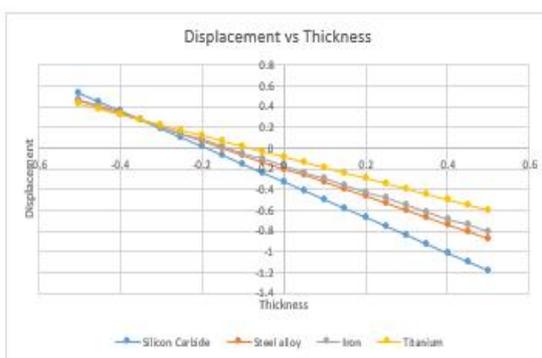


Fig.12 Variations of Displacements

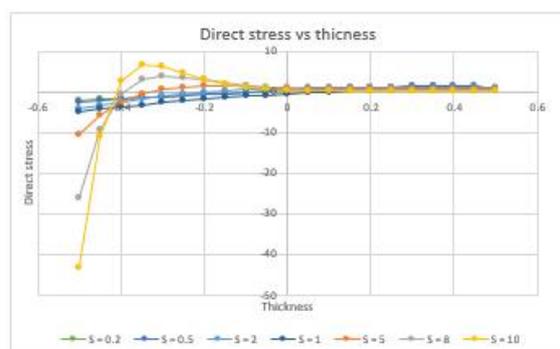


Fig.13. Direct Stresses with different Sigmoidal values.

Analysis 3: The top layer of the FGM plate is assumed as Zirconium and the bottom layer is Aluminum with different volume fractions of 0.2, 0.5, 1.0, 2.0, 5.0, 8.0 and 10.0. The variations of maximum direct stresses, inplane stresses along the thickness with different volume fractions are shown in Fig. 14 to Fig.15.

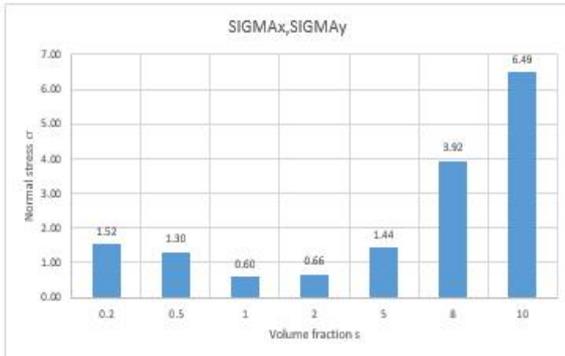


Fig.14 Max. Direct stresses with volume fractions

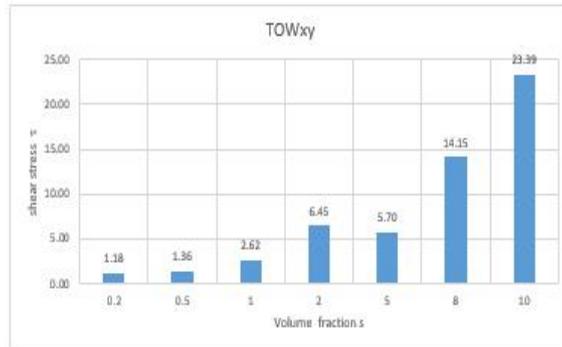


Fig.15 Max. inplane stresses with volume fractions

Analysis 4: The maximum direct stresses and transverse stresses were also compared with different materials (Young's modulus) are shown in Fig 16 and Fig 17 and the maximum direct stresses with different aspect ratios are shown in Fig. 18.

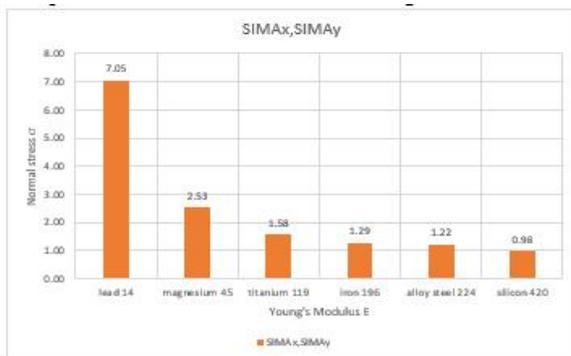


Fig. 16 Direct stresses with Young's modulus.

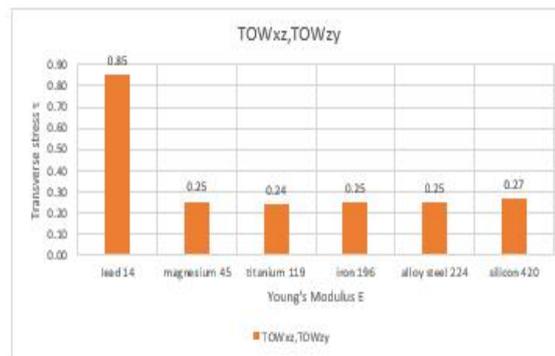


Fig 17. Transverse stresses with Young's modulus

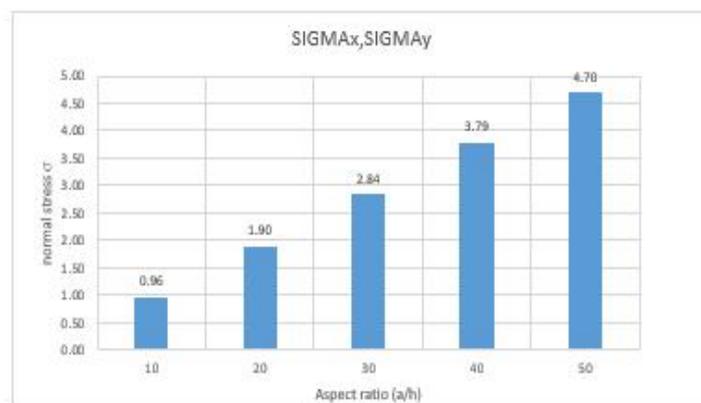


Fig. 18 Direct Stresses with Aspect ratios.

RESULTS AND DISCUSSIONS

An FGM long square plate made with Aluminum (E 70GPa) and Zirconium (E 380GPa) was analyzed using the First order Shear Deformation Theory formulation. The direct stresses coefficient and transverse shear coefficients were increased from 0.0 to 0.6 and 0 to 1.2 respectively and inplane stress coefficient value decrease from 1.4 to 0.1. When the FGM is made with Aluminum (E 70GPa) and Silicon carbide (E 420) the direct stress coefficient increases from 0.2 to 0.8, inplane stress coefficient decreases from 2.3 to 0.0, displacement coefficient decrease from 0.5 to 0.0 and there is no significant change in transverse stress.

As the load applied was transverse, the stress coefficient in z direction is 0. The stress coefficients in x and y direction was found to have the same magnitude. Similarly shear stress in xz and yz plane were found to be the same magnitude. If the sigmoidal law value is increased to 10 the direct stress coefficient increases from 2.6 to 0.2. The inplane and transverse stress coefficient decrease from 5.8 to 0 and 0.6 to 0.01. If the FGM plate has less thickness, the in plane stresses coefficient decreases from 30 to 7 and direct stress coefficient increases from 12 to 3. when the sigmoidal law value 5 and the metal used was steel (E 224) there was only significant change in the in plane stress coefficient from 8.8 to 2.5.

From the results Silicon carbide is found to have less stress compared to other considered metals. When the direct stress coefficient was compared with the sigmoidal parameter, the sigmoidal parameter with 2 show the less value of direct stress coefficient, as it enhances the smooth distribution of the same. When the sigmoidal law is 10, the maximum direct stress coefficient of the particle in the FGM reaches up to 6.49, when the sigmoidal law is 0.2, the direct stress is 1.5. The acceptable value of sigmoidal law parameter was found to be 1 as it had the less value for max direct stress for the particle in the FGM. While comparing the sigmoidal law parameter with the stress, sigmoidal law 2 show a gradual change in the stress as the thickness increases and therefore smooth stress distribution is obtained. When the aspect ratio (a/h) is made as 10, the response in 21 points of varying height of the plate was considered whereas when the aspect ratio (a/h) is 30, the response in 7 points of the FGM plate was considered. It was found that as the height to thickness ratio increases the accuracy of the result increases.

CONCLUSION

When a transverse load is applied to an FGM, the bottom portion of the FGM plate experiences the high value of stress compared to the portion in the top, therefore it is necessary to manufacture the FGM with the high value of Young's modulus at the bottom to prevent the fracture of the FGM. In case of the inplane stress, the portion in the top experiences the maximum inplane stress. The topmost particle in the FGM experiences the displacement the maximum. For the smooth distribution of the stresses sigmoidal law with 2 shows the significant results. It is concluded that the stress in x direction and in y direction x and y has the same magnitude and the z has no magnitude as the load applied was transverse load. Comparison of laminated composites with functionally graded composites with respect to temperature shall be considered. The effect of thermal stresses in FGM plates and the comparison with different metals can be found. The energy absorption capacity of the FGM can be regarded. Percentage of the error for the result to be found.

REFERENCES

- [1] A. M. Zenkour, (2006), "Generalized shear deformation theory for bending analysis of functionally graded plates," Applied Mathematical Modelling, 30 (2006) 67-84.
- [2] Ankit B and Khushbu C., "A Review of Stress Analysis of Functionally Material Plate with Cut-out" International Journal of Engineering Research and Technology, 3 (2014) 2020-20125.
- [3] B. R. Cheng ZQ, "Three-dimensional thermoelastic deformations of functionally graded elliptic plates.," Journal Sound Vibration, (2000) 97-106,.
- [4] Bhavani V. Sankar and Jerome T. Tzeng, (2002), "Thermal stresses in functionally graded beams," AIAA journal., 40 (2002) 1228-1232.
- [5] C. Y. Chi S, "Mechanical behavior of functionally graded material plates under transverse load – Part I: Analysis," International Journal Solids Structures, (2010) 3657-3674.

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- [6] Chih-Ping Wu and Hao-Yuan Li, “An RMVT-based third-order shear deformation theory of multilayered functionally graded material plates,” *Composite Structures*, 92(2010), 2596-2605.
- [7] E. Pan and F. Han, “Exact solution for functionally graded and layered magneto-electro-elastic plates,” *International Journal of Engineering Science*, 43(2015),321-339.
- [8] Fares, M. E and Elmarghany, “An efficient and simple refined theory for bending and vibration of functionally graded plates,” *Composite Structures*,91(2013) 296-305.
- [9] G. N. Praveen and J. N. Reddy, “Nonlinear transient thermoelastic analysis of functionally graded ceramic-metal plates,” *International Journal Solids Structure*, 20(2010),. 4457-4474.
- [10] Hiroyuki Matsunaga, “Stress analysis of functionally graded plates subjected to thermal and mechanical loading,” *Composite Structures*, 12(2009)344-357.
- [11] K. Swaminathan and Naveen Kumar, “ Three dimensional modeling of the plate,” *International journal of solids and structures*, 42(2014) 3341-3362.
- [12] Kashtalyan M, “Three-dimensional elasticity solution for bending of functionally graded rectangular plates,” *European Journal of Mechanics and Solids*, 15(2012)135-157.
- [13] Manish Bhandari and Kamlesh Purohit, "Analysis of Functionally Graded Material Plate under Transverse Load for Various Boundary Conditions," *IOSR Journal of Mechanical and Civil Engineering*, 10 (2014), 46-55.
- [14] Mohammad Talha and B N Singh, "Thermo-mechanical deformation behavior of functionally graded rectangular plates subjected to various boundary conditions and loadings," *International Journal of Aerospace and Mechanical Engineering*, 43(2012) 14-25.
- [15] Shyang-Ho Chi and Yen-Ling Chung, "Mechanical behavior of functionally graded material plates under transverse load—Part II: Numerical results," *International Journal of Solids and Structures*, 43(2006) 3675-3674.
- [16] Trung Kien Nguyen, Karam Sab, and Guy Bonnet, "First-order shear deformation plate models for functionally graded materials," *Composite structures*, 83 (2008) 25-36.
- [17] Victor Birman and Larry W. Byrd, “Modelling and Analysis of Functionally Graded materials and Structures,” *Applied mechanics Reviews*, 60(2007)195-209.
- [18] Zheng Zhong and Ertao Shang, “Closed-Form Solutions of Three-Dimensional Functionally Graded Plates,” *Mechanics of Advanced Materials and Structures*, 15(2008) 355-363.