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# Application of Response Spectra in the Analysis of Sea Structures Subjected to Sea Wave Forces

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## ABSTRACT

*One of the primary aims of engineering is to achieve the safety and cost efficiency of structures with respect to natural phenomenon. The sea waves are one among the most destructive natural phenomenon for near shore structures, their evaluation is thus a necessity for structural engineering and for the safety and economy of the structures in near-shore. In near-shore structural design, hydrodynamic interaction effects and dynamic response become major considerations. There are few design standards available for the design and construction of offshore structures. American Petroleum Institute (API 2A WSD) code recommends use of wave time history method for dynamic analysis. These standards do not talk about the concept of application of response spectrum method to simulate wave forces for the analysis. The idea of response spectrum due to sea wave forces is not new and many attempts are made by researchers in this field. This paper is aimed to develop a new approach to develop response spectra due to sea wave forces and to verify the applicability of response spectrum method for calculating sea wave forces, and verifying its accuracy by comparing with time history method.*

## KEYWORDS

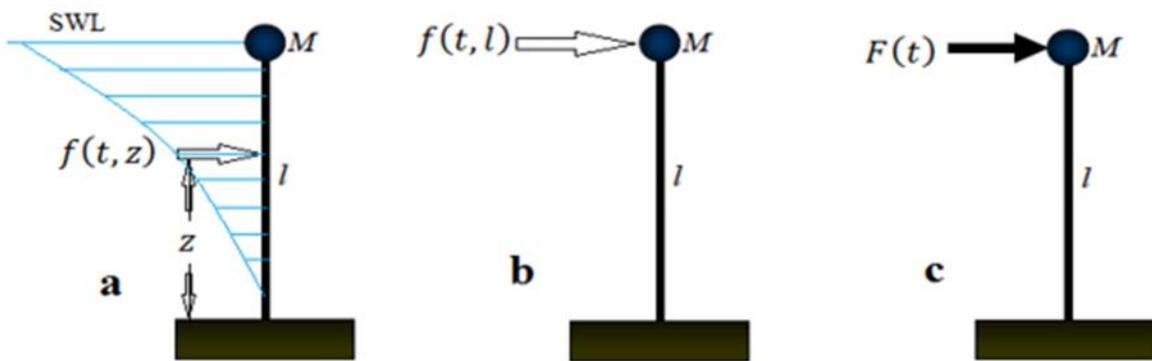
*Response spectra, Near-shore structures, Time history, Morison equation*

## INTRODUCTION

Several researchers had published their papers regarding developing response spectra due to sea wave forces. Response spectra is a plot between maximum response of single degree of freedom systems of different time period and its time period subjected to specified wave force. In his paper, a new method to develop response spectra due to sea wave forces for near-shore structures is discussed. The effect of wind, current and buoyancy are neglected and also depth of sea water is limited to 200m. The structure is modeled by lumped mass system and the wave forces are calculated according to Morison equation as per the procedure given in [1]. The response of a structure to wave loads depends on its mass, height, cross section, type of material, wave characteristics etc. The wave characteristics of the prominent seashores across the globe is known, using this the response spectra of a structure due to sea wave forces can be developed. It is difficult to generalize the response spectra applicable for all structures because it depends on various factors as mentioned. In this paper emphasis is given in developing the response spectra of a structure rather than generalizing them. Its applicability is verified by finding out the base shear and base bending moment of a structure using response spectra and comparing the results with time history method.

## DETERMINATION OF RESPONSE

For this study consider a single column structure standing in sea water of depth less than 200m. The mass of column and the wave pressure are distributed along the depth of sea water, for simplicity mass of the structure is lumped at the mean sea water level and bottom support is assumed to be fixed as shown in Figure 1.



**Fig 1: Force acting at a random height z (a) Equivalent force acting on lumped mass (b) Total equivalent force acting on lumped mass (c)**

The time history of equivalent wave force acting on the structure at sea water level is determined based on the condition that the displacement produced by wave force  $f(t, z)$  acting at a height  $z$  from sea bed and the equivalent force  $f(t, l)$  acting at lumped mass location are the same at any time  $t$ .

Deflection of cantilever at lumped mass location due to  $f(t, z)$  is equal to

$$\delta = \frac{f(t, z)z^3}{3E} + \frac{f(t, z)z^2}{2E}(l - z) \quad (1)$$

Equivalent force acting at lumped mass location is equal to

$$f(t, l) = f(t, z) \left[ \frac{z^3}{3} + \frac{z^2}{2}(l - z) \right] \quad (2)$$

The total equivalent force acting on the lumped mass is obtained by finding the summation of equivalent force of  $f(t, z)$  from  $z = 0$  to  $z = l$ .

$$F(t) = \sum_{z=0}^{z=l} f(t, z) \left[ \frac{z^3}{3} + \frac{z^2}{2}(l - z) \right] \quad (3)$$

The dynamic equation of equilibrium of the lumped mass system having mass  $m$ , length  $l$ , stiffness  $k$ , and damping  $c$  can be written as below

$$m\ddot{x} + c\dot{x} + kx = F(t) \quad (4)$$

Response of the structure at different time steps can be determined by any suitable numerical integration methods. In this study Newmark-beta method is used. A Computer program is written in MATLAB in which the response of the structure is calculated for various time periods of the structure and thus the program generates response spectra. Analytically it can be shown that the maximum value of bending moment developed at the bottom support due to equivalent force  $F(t)$  is always less than that due to original wave forces ( $f(t, z)$ ) which are acting throughout the depth of water. Therefore a correction factor should be applied to the force value obtained from response spectra. The correction factor is equal to the ratio of maximum bending moment developed at the bottom support due to original wave force to that due to the equivalent force acting at the top. A program is written in MATLAB to draw the plot for force correction factor as per following steps.

Step 1: Start

Step 2: Input depth of water, drag coefficient, inertia coefficient, damping coefficient, diameter of pier, time period of wave, wave height, and density of sea water.

Step 3: Calculate the time history of wave force acting on the pier throughout the depth. Morison's equation is used for this.

$$F = \frac{1}{2} C_D \rho_w D |V| + \frac{\pi D^2}{4} C_m \rho_w a$$

Where  $F$  is wave force per unit height is,  $C_D$  is drag coefficient,  $C_m$  is inertia coefficient,  $\rho_w$  is density of water,  $V$  is velocity of water particle,  $D$  is diameter of pier and  $a$  is acceleration of water particle.

Step 4: Find out the total bending moment at the base of pier due to wave forces at different timesteps and its maximum value.

Step 5: Find out the time history of equivalent force acting at the location of lumped mass.

Step 6: Calculate the bending moment at the base of pier due to maximum of equivalent force.

Step 7: Calculate the correction factor as the ratio of maximum bending moment at the bottom support due to original wave force obtained as per step 4 to the maximum bending moment due to the equivalent force obtained as per step 6.

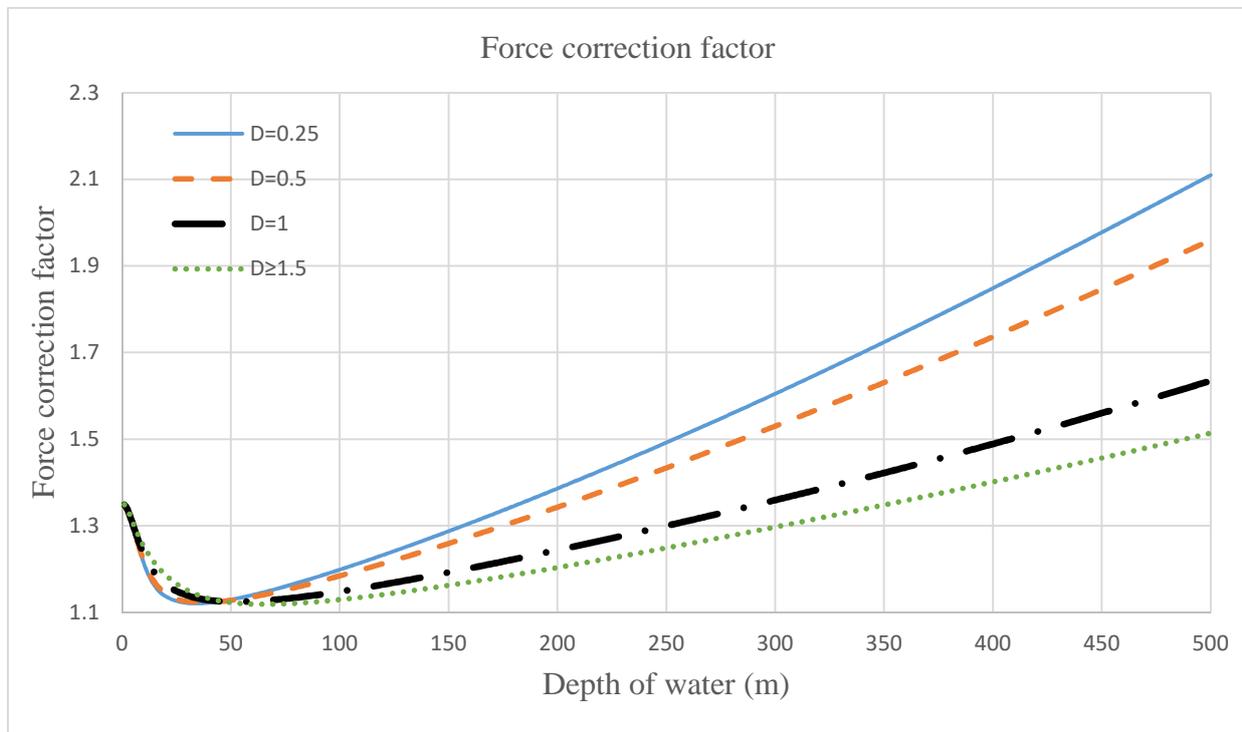
Step 8: Change the value of depth of water and repeat the steps from 3 to 7.

Step 9: Change the diameter of pier and repeat the steps from 3 to 8.

Step 10: The variation of force coefficient can be plotted against depth of water for different values of diameter of pier.

Step 11: Stop.

The force correction factor is evaluated for a wave period ( $T_w$ ) = 5 s and wave height ( $H$ ) = 10 m and shown in Figure 2.



**Fig2: Force correction factor**

This plot drawn for calculating force correction factor changes with wave period and wave height. But by knowing the maximum wave height and wave period at important sea shores it is possible to construct the graph for calculating force correction factor.

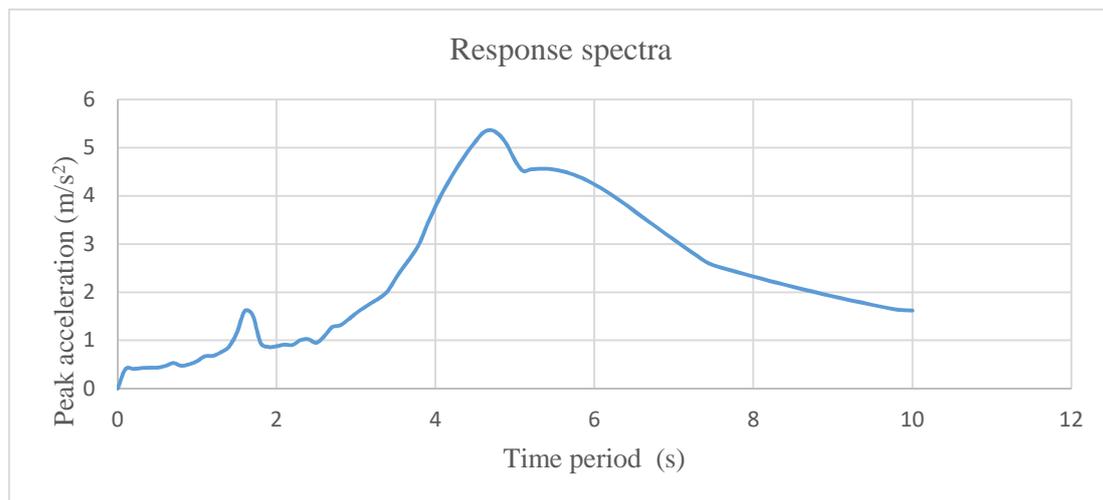
## NUMERICAL EXAMPLE

For verification a concrete pier constructed in sea water is considered. The diameter of pier (D) is 1.5 m, depth of water (h) is 40 m, wave height (H) is 10 m, wave period ( $T_w$ ) is 5 s and wave length (  $\lambda$  ) 39 m. M 35 concrete is used for construction of pier. Assume density of concrete ( $\rho_c$ ) and sea water ( $\rho_w$ ) to be 2500  $\text{kg/m}^3$  and 1024  $\text{kg/m}^3$  respectively. Assume 5% damping. V represents volume of pier.

Lumped mass, stiffness and natural period of vibration are calculated and shown in Table 1.

**Table 1. Calculated values of lumped mass, stiffness and natural period of vibration**

Mass of pier (a)	Added mass (b)	Lumped mass (m)	Stiffness (k)	Natural period (T)
kg	kg	kg	N/m	s
$V \times \rho_c$	$V \times \rho_w$	$(a+b)/2$	$3EI/l^3$	$2\pi \sqrt{\frac{m}{k}}$
176714	72382	124548	344572	3.77



**Fig 3: Response spectra**

The response spectra as shown in Figure 4.5 is obtained by solving equation (4) using Newmark- beta method in MATLAB software. From Figure 4.5 the peak acceleration corresponding to time period of 3.77 s is obtained as 2.93  $\text{m/s}^2$  and from Figure 4.2 force correction is obtained as 1.13 for a height of 40 m.

Therefore force acting on the pier = 1.13 x 124548 x 2.93 = 412kN.

Bending moment at the base = 412.36 x 40 =16494kN m.

## COMPARISON OF RESULTS

The pier is modelled in SAP 2000 as vertical column and wave time history analysis is done. The results obtained from different methods of analysis are tabulated in Table 2.

**Table 2. Comparison of results obtained from different methods**

Method	Base shear (kN)	Bending moment (kN m)
Response spectra method (SDOF)	412	16494
Time history method	266	10257

The ratio of base shear obtained from equivalent static method and time history method is 1.55.

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## CONCLUSIONS

Comparing the results obtained from time history method and response spectrum method it is clear to conclude that, response spectrum method can be used for calculating equivalent wave force acting on the structure very easily which will be helpful in the preliminary design of near-shore structures.

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