
Fault Prediction of a Cantilever Beam Using Fuzzy Controller

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ABSTRACT

Crack is most dangerous structural defect for a beam that may be cantilever, simply supported beam. Once the crack is developed in a structure that will affect the whole structure. So detection of crack is main major issues now-days. This paper focus on crack detection by finite element modeling and finite element formulation by making proper relation between natural frequency with crack depth and natural frequency with crack location. If a crack is present in a structure that not only effect strength but also affect the stiffness and natural frequency of a beam. This paper described for keeping crack depth constant by varying the crack length how the natural frequency will be varied and vise versa. From that we can easily identify effect of modal natural frequency at different location and at different position. This paper is based on measuring natural frequency and mode shape for beam by using single transverse crack.. This analysis can be carried out by taking free vibration analysis for a cracked cantilever beam. Numerical analysis obtained from MATLAB is validated with the help of finite element modeling ANSYS Software. The result obtained from ANSYS is taken in fuzzy controller. . Modal natural frequency was found to be lessened with increase in crack depth. But modal natural frequency is going to be increased with increase in position from the fixed point.

KEYWORDS:Free Vibration; Cantilever beam; Transverse Crack; ANSYS; MATLAB; Natural frequency; fuzzy logic.

INTRODUCTION

Beams are primarily used in building construction, steel construction and structural frames of industrial sheds. So Detection of crack in structural beam is play very important role in structural health monitoring application due to static and dynamic loading. This Paper generally focused on crack detection deals with change in natural frequency and mode shape of a beam by changing of crack depth and crack location. Structural health monitoring techniques are generally based on linear and nonlinear approach. This paper is generally based on linear approach technique which is used to detect the crack by taking free vibration analysis for both single and double crack. But nonlinear approach is only for single crack. When a crack induced in a structure that not only affect dynamic response but also affect several other properties like stiffness, natural frequency, mode shape, damping etc and also leads sudden failure of the structure that is call catastrophic failure of a structure. Different methods are there to find out the crack in beam like impact echo method, Visual inspection method, slope and deflection method and natural frequency and mode shape method etc. This paper addresses single crack detection in structural elements like cantilever beam by monitoring natural frequency and mode shape of a beam by predicting crack depth and crack location using finite element modeling based ANSYS and fuzzy logic.

Sadettin Orhan[1]: This paper described the free and forced vibration analysis of single crack. The result obtained by this paper that free vibration analysis provides the information for detection of single and double crack whereas forced vibration can detect the crack only for single crack. Nitesh A. Meshram [2]: This paper has taken finite element method to detect the single crack in cantilever beam. Only single crack at different depth and at different location are evaluated and analysis reveals the relationship between crack depth and modal natural frequency. Analysis was performed by using ANSYS software. Zheng et al. [3] has studied the free vibration analysis of cracked beam by using finite element method. In this paper natural frequency and mode shape of a cracked cantilever beam are obtained by using finite element method. They have taken shape

function that satisfy the local flexibility conditions at the crack location which can give more accurate result in vibration modes. Chondros et al. [4] has developed a continuous cracked bar vibration model for the lateral vibration of a cracked Euler Bernoulli cantilever beam with an edge crack. The crack was modeled as a continuous flexibility using the displacement field in the vicinity of the crack found with fracture mechanics methods. The results of three independent evaluations of the lowest natural frequency of lateral vibrations of an aluminum cantilever beam with a single-edge crack are presented: the continuous cracked beam vibration model, the lumped crack model vibration analysis, and experimental results. Sutar Mihir [5] has described finite element analysis of a cracked cantilever beam and analyses the relation between the modal natural frequency with crack depth and modal natural frequency with crack location. This paper evaluated modal natural frequency for single crack. Bhinge et al. [6] described crack detection in a cantilever beam by using vibration techniques. In theoretical analysis, Euler beam equation here taken as characteristic equation which help in finding the relationship between the stiffness and location of crack. Here natural frequency for cracked beam can be determined by using finite element method. H Nahvi et al. [7] proposed crack detection in cantilever beam by using experimental model data and finite element method. This paper described both analytical and experimental approach to the crack detection in a cantilever beam by vibration analysis method by relating the crack location and depth. Tsai and Wang [8] have investigated diagnostic method of determining the position and size of a transverse open crack on a stationary shaft without disengaging it from the machine system. To obtain the dynamic characteristics of a stepped shaft and a multidisc shaft the transfer matrix method is employed on the basis of Timoshenko beam theory. A.S. Sekhar et al. [9] proposed crack detection and vibration characteristics of a cracked shaft. This paper represented analytical expressions and derived curve relating the crack depth and crack location on the shaft to change in the first three natural frequencies of the shaft. The element stiffness matrix for a beam with crack is derived from an integration of stress intensity factors.

1. FINITE ELEMENT FORMULATION:

1.1. Governing equation of free vibration

The beam with single transverse crack is fixed at one end and other end is free is taken into account for detection of an open crack. This study has to be taken without considering the damping. The free vibration analysis of a cantilever beam is carried by taking Euler Bernoulli beam having rectangular cross section is given by a differential equation:

$$EI \frac{d^4 y}{dx^4} - m \omega_i^2 y = 0 \quad \dots 1$$

Where m is the mass of the beam per unit length (kg/m), ω_i is the natural frequency of the i th mode (rad/sec), E is the modulus of elasticity (N/m²) and I is the moment of inertia (m⁴). By defining $\omega_i^4 = \omega_i^2 m/EI$ equation (1) is rearranged as a fourth-order differential equation as follows:

$$\frac{d^4 y}{dx^4} - \omega_i^4 y = 0 \quad \dots \dots \dots 2$$

The general solution to equation (2) is

$$y = A \cos \omega_i x + B \sin \omega_i x + C \cosh \omega_i x + D \sinh \omega_i x$$

Where A, B, C, D are constants and ω_i is a natural frequency parameter. The crack assumed to be opened. Taking Hermitian shape functions, the stiffness matrix of the two-noded beam element without a crack is obtained using the standard integration based on the variation in flexural rigidity.

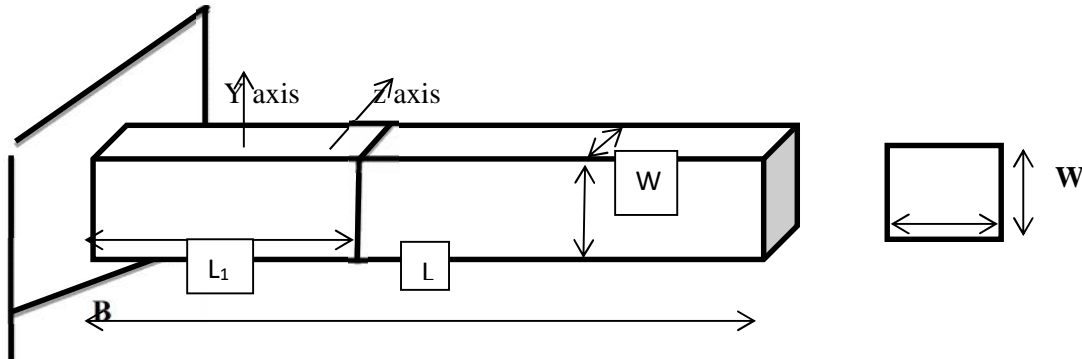


Fig:1.1(a) geometry of cantilever beam. (b) Cross-sectional view of the beam

The stiffness matrix of the un cracked beam is obtained from

$$[k^e] = \int [B(X)]^T EI [B(X)] dx$$

Where $[B(X)] = \{H_1''(X) H_2''(X) H_3''(X) H_4''(X)\}$

And $H_1(x), H_2(x), H_3(x), H_4(x)$ are the Hermitian shape functions defined as

$$H_1(x) = \frac{1}{4}(2 - 3x + x^3) \quad \dots\dots 6$$

$$H_2(x) = \frac{1}{4}(1 - x - x^2 + x^3) \quad \dots\dots 7$$

$$H_3(x) = \frac{1}{4}(2 + 3x - x^3) \quad \dots\dots 8$$

$$H_4(x) = \frac{1}{4}(-1 - x + x^2 + x^3) \quad \dots\dots 9$$

Assuming the beam rigidity EI is constant and given by EI_0 within the element, and then the element stiffness is

$$[K^e] = \frac{EI_0}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

Assuming the stiffness reduction caused by as open crack falls within a single element, and then the stiffness

matrix $[K_c^e]$ of the cracked element can be written as $[K_c^e] = [K^e] - [K_c]$

Where $[K_c]$ is the reduction in the stiffness matrix due to the crack. According to peng et al[2],the matrix $[K_c]$ is

$$[K_c] = \begin{bmatrix} k_{11} & k_{12} & -k_{11} & k_{14} \\ k_{12} & k_{22} & -k_{12} & k_{24} \\ -k_{11} & -k_{12} & k_{11} & -k_{14} \\ k_{14} & k_{24} & -k_{14} & k_{44} \end{bmatrix}$$

$$k_{11} = \frac{12E(I_0 - I_c)}{L^4} \left[\frac{2l_c^3}{L^2} + 3l_c \left(\frac{2L_1}{L^2} - 1 \right)^2 \right] \dots\dots\dots 13$$

$$k_{12} = \frac{12E(I_0 - I_c)}{L^3} \left[\frac{l_c^3}{L^2} + l_c \left(2 - \frac{7L_1}{L} + \frac{6L_1^2}{L^2} \right)^2 \right] \dots\dots\dots 14$$

$$k_{14} = \frac{12E(I_0 - I_c)}{L^3} \left[\frac{l_c^3}{L^2} + l_c \left(2 - \frac{5L_1}{L} + \frac{6L_1^2}{L^2} \right)^2 \right] \dots\dots\dots 15$$

$$k_{22} = \frac{12E(I_0 - I_c)}{L^3} \left[\frac{3l_c^2}{L^2} + 2l_c \left(\frac{3L_1}{L} - 2 \right)^2 \right] \dots\dots\dots 16$$

$$k_{24} = \frac{12E(I_0 - I_c)}{L^2} \left[\frac{3l_c^3}{L^2} + 2l_c \left(2 - \frac{9L_1}{L} + \frac{9L_1^2}{L^2} \right)^2 \right] \dots\dots\dots 17$$

$$k_{44} = \frac{12E(I_0 - I_c)}{L^2} \left[\frac{3l_c^2}{L^2} + 2l_c \left(\frac{3L_1}{L} - 1 \right)^2 \right] \dots\dots\dots 18$$

$l_c=1.5d$, L =total length of the beam, L_1 =distance between left node and crack, where $I_0 = \frac{B W^3}{12}$ =moment of inertia of the beam cross section

$$I_c = \frac{B(W - a_1)^3}{12}$$
 =moment of inertia of the beam with crack

B =width of the beam, W =depth of the beam

It is considered that crack doesn't affect the mass distribution of the beam. Therefore, consistence mass matrix of the beam element can be formulated directly as:

$$[M^e] = \int_0^l \frac{\rho A l}{2} [H(x)]^T [H(x)] dx \dots\dots\dots 19$$

And we have

$$[M^e] = \frac{\rho A l}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \dots\dots\dots 20$$

Where $[H(x)] = \{ H_1(x) H_2(x) H_3(x) H_4(x) \}$

Then the natural frequency can be determined from the following equation :

$$- \omega^2 [M] + [K] \{ \bar{q} \} = 0 \dots\dots\dots 22$$

Where q =displacement vector of the beam.

2. CONFIGURATION OF SIMULATED CRACK

At different crack location and crack depth the numerical analysis is performed to find out natural frequency for cracked cantilever beam having single open transverse crack. The results which have obtained analytically are validated with the results obtained by simulation analysis. Structural steel has taken the beam. The cracked cantilever beam of the current research has the following dimensions.

Properties:

Length of the beam=700mm

Width of the beam=50mm

Depth of the beam=6mm

Density= $7850 \times 10^9 \text{kg/mm}^3$

Young's modulus of the beam= $2 \times 10^5 \text{N/mm}^2$

Poisson's ratio=0.3

End condition of the beam= one end is fixed and other end is free.

3. FINITE ELEMENT MODELING OF CRACK USING ANSYS

The ANSYS 15.0 finite element program was used for free vibration of the uncracked and cracked beams. For this purpose, the key points were first created and then line segments were formed. The lines were combined to create an area. Finally, this area was extruded and a three-dimensional V-shaped edge cracked beam model was obtained. . The beam was consisting of 345 elements with 2788 nodes. Cantilever boundary conditions can also be modeled by constraining all degrees of freedoms of the nodes located on the left end of the beam. Modal analysis system is required to calculate natural frequency at different mode shape of the beam. For modeling of beam SOLIDWORKS was selected.

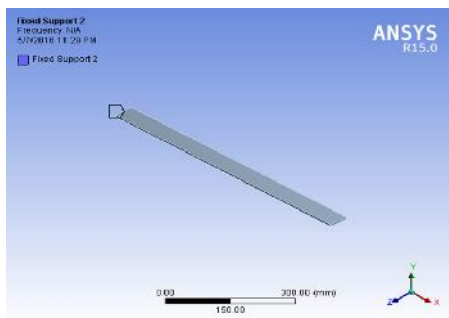


Fig: 3.1 (a). Cantilever Beam in ANSYS

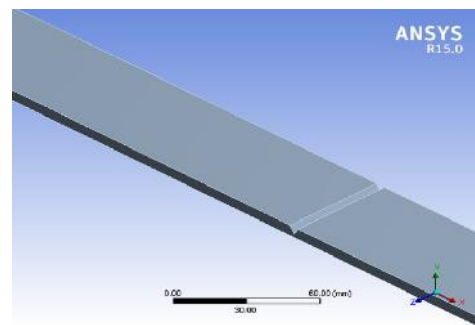


Fig: 3.2 (b). Cantilever Beam in ANSYS nature

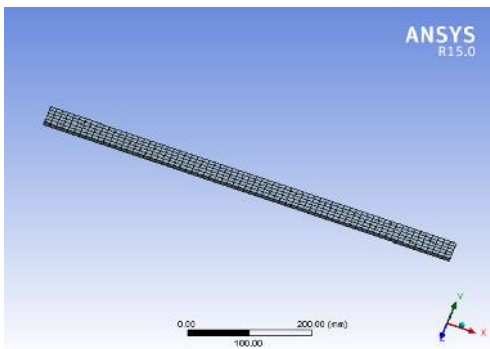


Fig: 3.3(c).Uncrack Meshing

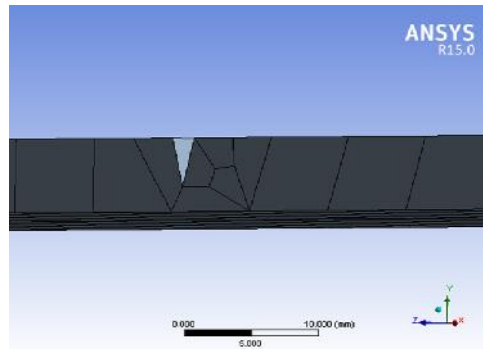


Fig: 3.4(d).Crack Meshing

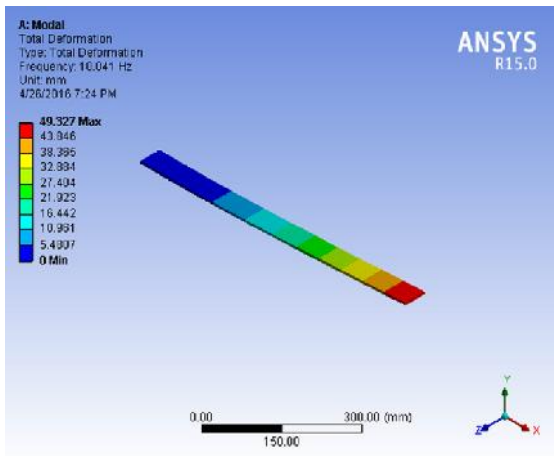


Fig: 3.5.(a): generated first mode shape for beam without crack.

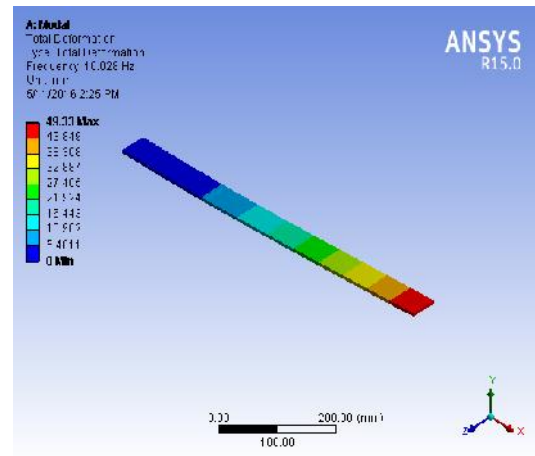


Fig: 3.6.(b): generated first mode shape For beam with crack.

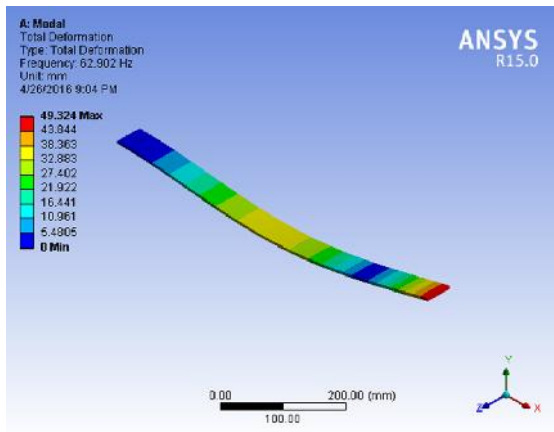


Fig:3.7.(c): Generated second mode shape for beam without crack

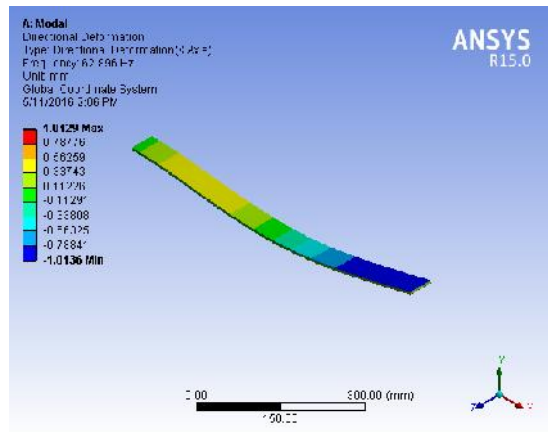


Fig: 3.8.(d): Generated second mode shape for beam with crack

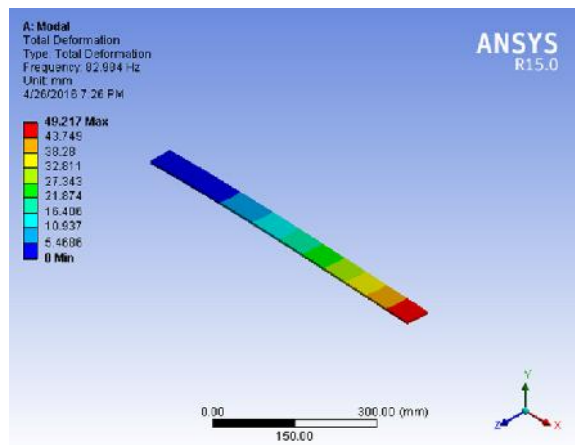


Fig: 3.9.(e): Generated third mode shape for beam without crack.

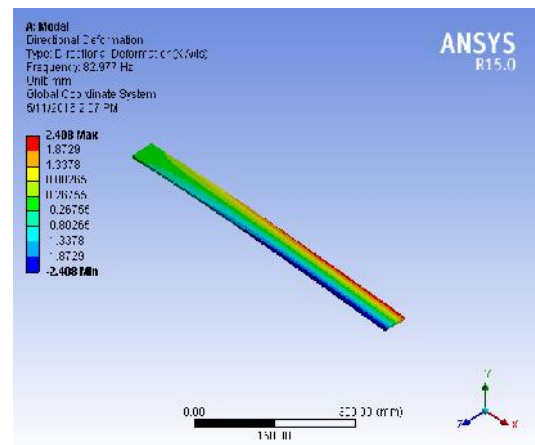


Fig: 4.(f): Generated third mode shape for beam with crack

4. FUZZY INFERENCE SYSTEM

A fuzzy inference system is the process of formulating the mapping from a given input to an output using fuzzy logic. Fuzzification is the first step in the fuzzy inferencing process. This involves a domain transformation where crisp inputs are transformed into fuzzy inputs. Crisp inputs are exact inputs measured by sensors and passed into the control system for processing. The purpose of fuzzification is to map the inputs from a set of devices to values from 0 to 1 using a set of input membership functions. Degree of membership is the degree to which a crisp value is compatible to a membership function, value from 0 to 1, also known as truth value or fuzzy input. The membership function is the crucial component of a fuzzy set. Therefore all the operations on fuzzy sets are defined based on their membership functions. The membership function for a set maps each element of the set to a membership value between 0 and 1 and uniquely describes that set. The values 0 and 1 describe “not belonging to” and “belonging to” a conventional set respectively; values in between represent “fuzziness.” Systems are controlled by fuzzy logic controllers based on rules instead of equations. This collection of rules is known as the rule base usually in the form of IF-THEN-ELSE statements. Here the IF part is known as the antecedent and the THEN part is the consequent. Fuzzy rules are a collection of linguistic statements that describe how the FIS should make a decision regarding classifying an input or controlling an output. The conversion of a fuzzy set to single crisp value is called defuzzification and is the reverse process of fuzzification.

$$\text{Relative crack location} = \text{rcl} = \frac{\text{d o c g f i e}}{\text{l e o t l b}}$$

$$\text{Relative crack depth} = \text{rcd} = \frac{\text{d h o t h e c}}{\text{d h o t h e b}}$$

$$\text{Relative Natural Frequency} = \frac{\text{b u o f i N t i}}{\text{b c o f i N}}$$

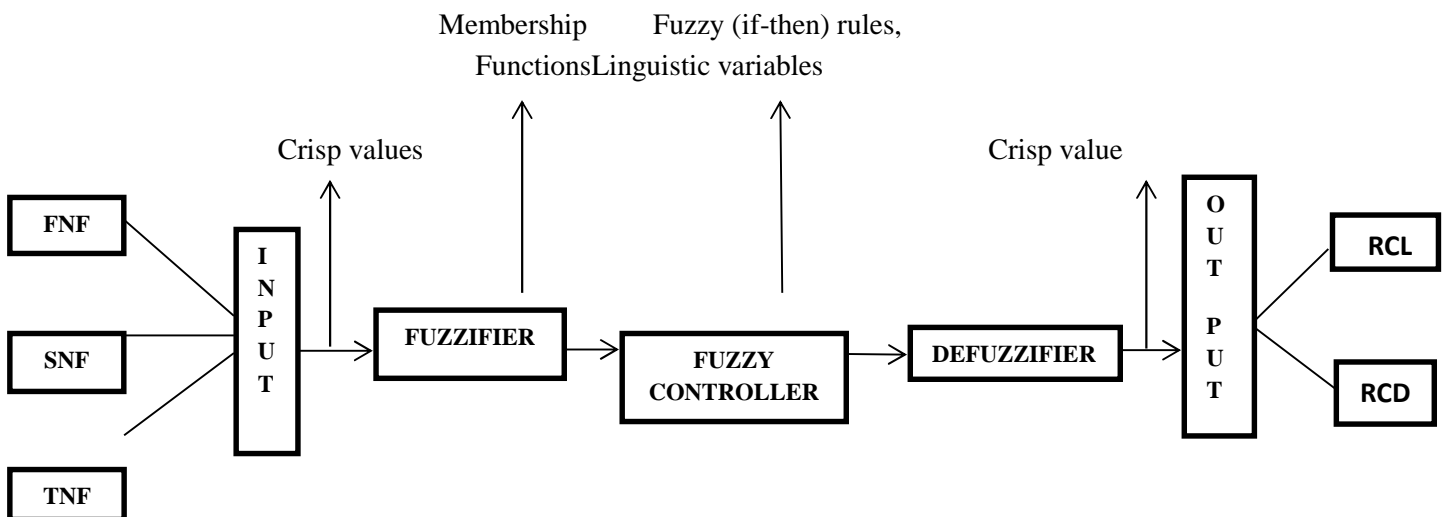


Figure: 4.1 Schematic diagram of Fuzzy Inference System

5. COMPARISONS OF THE RESULTS

Table: 5.1. Comparison of Results obtained from Theoretical Analysis, ANSYS and Fuzzy Controller Analysis.

Relative First natural frequency fnf	Relative Second natural frequency snf	Relative Third natural frequency tnf	Theoretical results		ANSYS results		Fuzzy Controller	
			Rcd	Rcl	Rcd	Rcl	Rcd	Rcl
0.925	0.958	0.964	0.635	0.140	0.632	0.138	0.626	0.128
0.927	0.96	0.965	0.62	0.145	0.616	0.142	0.597	0.128
0.939	0.966	0.970	0.585	0.209	0.583	0.207	0.583	0.134
0.948	0.968	0.973	0.583	0.313	0.580	0.307	0.577	0.195
0.954	0.972	0.975	0.563	0.396	0.561	0.392	0.547	0.248
0.959	0.976	0.983	0.515	0.470	0.512	0.469	0.49	0.331
0.96	0.982	0.985	0.453	0.483	0.452	0.481	0.444	0.465
0.973	0.985	0.987	0.363	0.539	0.361	0.535	0.352	0.534
0.984	0.99	0.995	0.332	0.718	0.33	0.714	0.323	0.704
0.9925	0.997	0.999	0.168	0.861	0.166	0.859	0.132	0.856

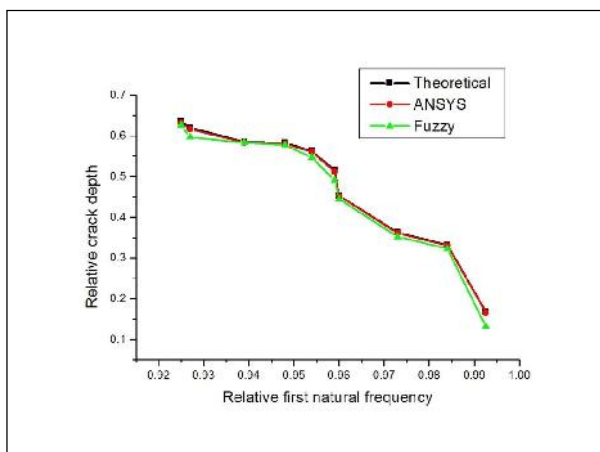


Figure: 5.1. Relative First Natural Frequencies versus Relative Crack Depth

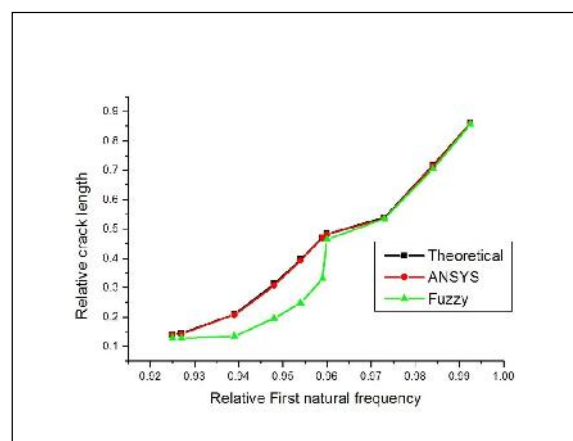


Figure: 5.2. Relative First Natural Frequencies versus Relative Crack Location

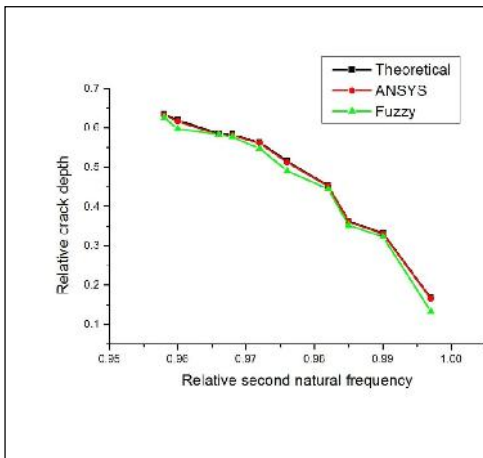


Figure: 5.3. Relative Second Natural Frequencies versus Relative Crack depth

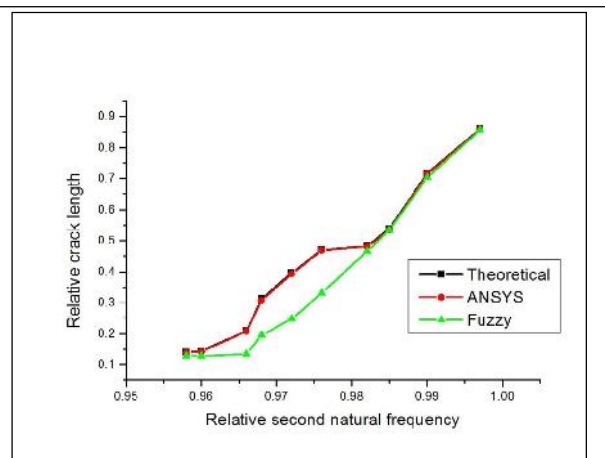


Figure: 5.4. Relative Second Natural Frequencies versus Relative Crack location

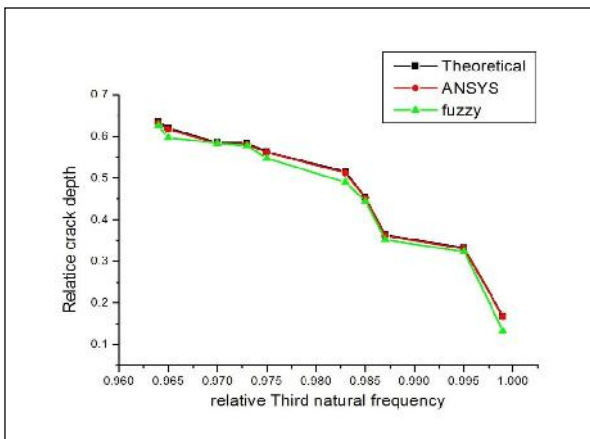


Figure: 5.5. Relative Third Natural Frequencies versus Relative Crack depth

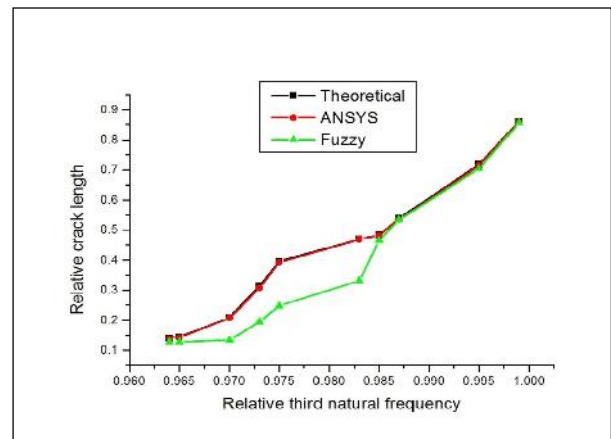


Figure: 5.6. Relative Third Natural Frequencies versus Relative Crack location

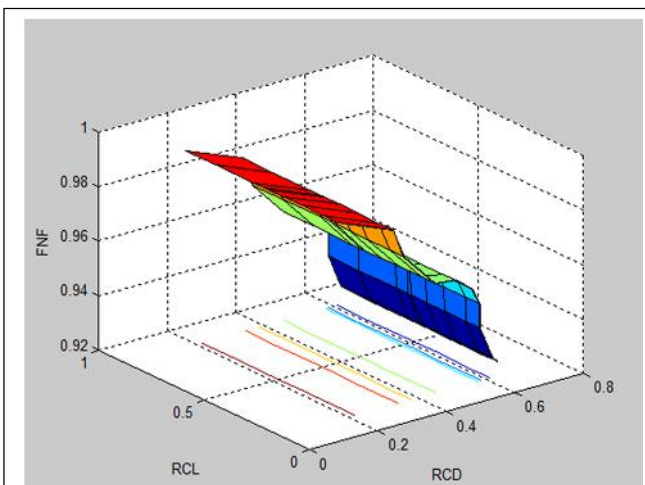


Fig: 5.7. Three dimensional cum surface plot for relative 1st mode natural frequency

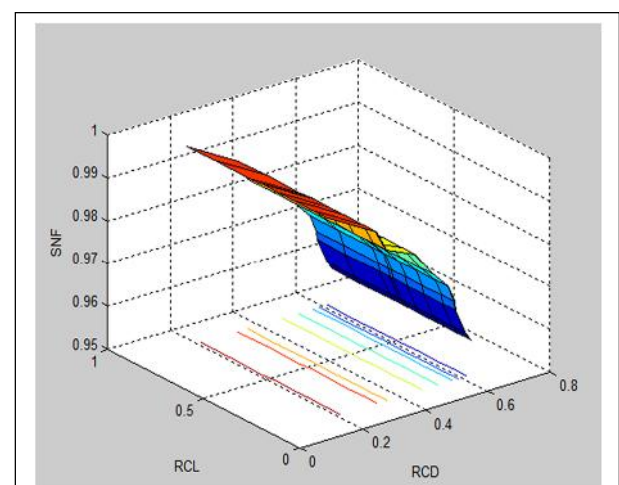


Fig: 5.8. Three dimensional cum surface plot for relative 2nd mode natural frequency

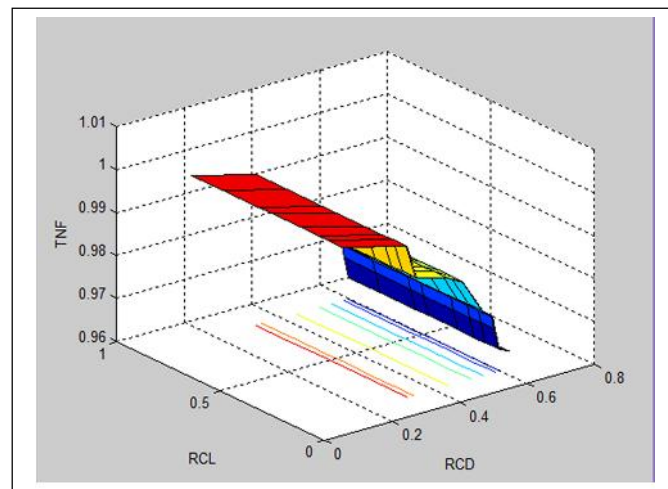


Fig: 5.9. Three dimensional cum surface plot for relative 3rd mode natural frequency

6. CONCLUSIONS:

The analysis has been done on the presence of a transverse crack and it is observed that the presence of crack affects the natural frequency, as a result the natural frequency decreases with the increase in crack depth and it increases with the increase in crack location at particular crack location and crack depth. So it is concluded that the analysis of change of natural frequencies is effective for prediction of crack in beam like structures. The results of the crack depth and crack location have been obtained from the comparison of the results of the uncracked and cracked beam during the free vibration analysis.

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