International Journal of Engineering Technology Science and Research
IJETSR
www.ijetsr.com
ISSN 2394 – 3386
Volume 4, Issue 8
August 2017

Effect of Different Types of Boundary Conditions on Conjugate Natural Convection in a One Side Wall

Balesh Babali
Mechanical Department PES University Bangalore, Karnataka, India
Aswatha
Mechanical Department Bangalore Institute of Technology Bangalore, Karnataka, India
K. N. Seetharamu
Mechanical Department PES University Bangalore, Karnataka, India

ABSTRACT

In the present study a finite volume based computational procedure is used to investigate conjugate natural convection in a square cavity. The enclosure used for flow and heat transfer analysis has been bounded by adiabatic top and bottom walls, constant temperature / constant heat flux / convective boundary conditions at right solid wall of different thickness and left cold wall. The Grashof Number (Gr) is varied from 10^3 - 10^7 while Prandtl number (Pr) ranges from 0.7 to 17. When the Grashof number is increased, rate of heat transfer also increases. Average Nusselt number increases with Pr and conductivity ratio. The effect of wall thickness to length ratio (t/l) of cavity on average Nusselt number is negligible.

Key words: Conjugate heat transfer; square enclosure; natural convection;

1. INTRODUCTION

Natural convection in cavities has gained importance in many engineering and domestic applications. Applications include design of energy efficient buildings, cooling of electronic cabinets, operation and safety of nuclear reactors, thermal comfort of an automobile passenger cabin and others. Natural convection cooling is desirable because it doesn't require energy source for cooling and hence more reliable. In most of the cases the wall thickness of the cavity is neglected. But in practical applications the conduction of walls of cavity will play a crucial role in heat transfer and hence study of conjugate heat transfer is important.

The literature on conjugate heat transfer is sparse and the problem is limited to boundary conditions such as uniform temperature and heat flux boundary conditions. Kaminski and Prakash (1986) numerically analyzed the effect of conduction on one of the vertical walls having thickness (t) on natural convection in a square cavity. Numerical analysis of laminar natural convection flow in a square enclosure having thickness on all sides of enclosure and containing volumetric sources is carried out by Liaqat and Baytas (2001).

NOMENCLATURE

G	acceleration due to gravity (ms^{-2})	T_h	temperature of right vertical wall (K)
Gr	Grashof number ($g\,\beta\Delta TL^3)/\gamma^2$	u	x- component of velocity ($m\,s^{-1}$)
H	Heat transfer coefficient, W/m^2 K	v	y- component of velocity ($m\,s^{-1}$)
K	thermal conductivity ($W\,m^{-1}\,K^{-1}$)	***Greek symbols***	
L	length of the square cavity (m)		
Nu	local Nusselt number	α	thermal diffusivity (m^2s^{-1})
$\overline{Nu}$	Average Nusselt number	β	volume expansion coefficient (K^{-1})
Pr	Prandtl number	θ	dimension temperature
q''	heat flux ($W\,m^{-2}$)	γ	kinematic viscosity (m^2s^{-1})
t	thickness of solid wall	ρ	density ($kg\,m^{-3}$)
T	temperature (K)	ψ	stream function ($m^2\,s^{-1}$)
T_c	temperature of left vertical wall (K)		

The objective of the present investigation is to study the conjugate natural convection in a square cavity. The cavity used has been bounded by adiabatic top and bottom walls, constant temperature / constant heat flux / convective boundary conditions at right solid wall of different thickness and left cold wall.

2. MATHEMATICAL FORMULATION

Fig. 1 shows the details of the physical situation to be analyzed.

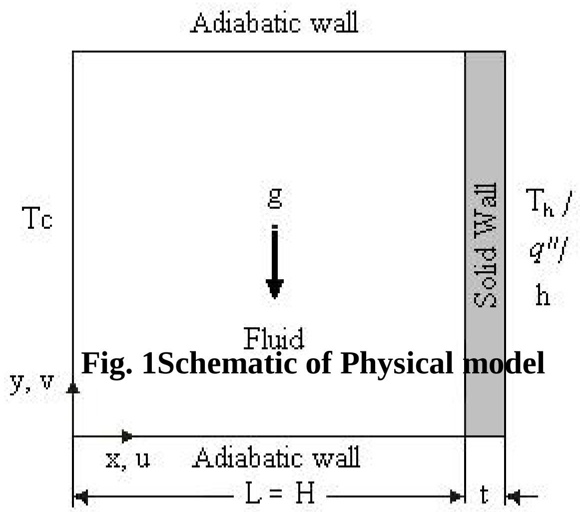


Fig. 1Schematic of Physical model

The governing equations for natural convection flow are conservation of mass, momentum and Energy equations in Kaminski et al. (1986):

No-slip boundary conditions are specified at all walls.

Right wall: $T(L+t, y) = T_h$, q″, h

Bottom wall: $\frac{\partial T}{\partial y}(x,0) = 0$ and

Top wall: $\frac{\partial T}{\partial y}(x,L) = 0$ (1)

Left wall: $T(0, y) = T_c$

The fluid is assumed to be Newtonian and its properties are constant. Only the Boussinesq approximation is invoked for the buoyancy term.

The changes of variables are as follows:

$$– = \frac{T - T_c}{T_h - T_C}, \quad \Pr = \frac{¥}{\sim}, \quad Gr = \frac{g \; (T_h - T_C) L^3}{¥^2} \qquad (2)$$

3. NUMERICAL PROCEDURE

In the present investigation, the set of governing equations are integrated over the control volumes, which produces a set of algebraic equations. The PISO (Pressure Implicit with Splitting of Operators) algorithm developed by Issa (1985) is used to solve the coupled system of governing equations. The set of algebraic equations are solved sequentially by ADI. A second-order upwind differencing scheme is used for the formulation of the convection contribution to the coefficients in the finite-volume equations. Central differencing is used to discretize the diffusion terms.

International Journal of Engineering Technology Science and Research
IJETSR
www.ijetsr.com
ISSN 2394 – 3386
Volume 4, Issue 8
August 2017

4. STREAM FUNCTIONS AND NUSSELT NUMBERS

4.1 Stream Function

The fluid motion is displayed using the stream function obtained from velocity components u and v. The relationship between stream function and velocity components for two dimensional flows is given by Batchelor (1993):

$$u = \frac{\partial \cdot}{\partial y} \text{ and } v = -\frac{\partial \cdot}{\partial x}, \quad \text{leads to a single equation: } \frac{\partial^2 \cdot}{\partial x^2} + \frac{\partial^2 \cdot}{\partial y^2} = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \qquad (3)$$

4.2 Nusselt Number

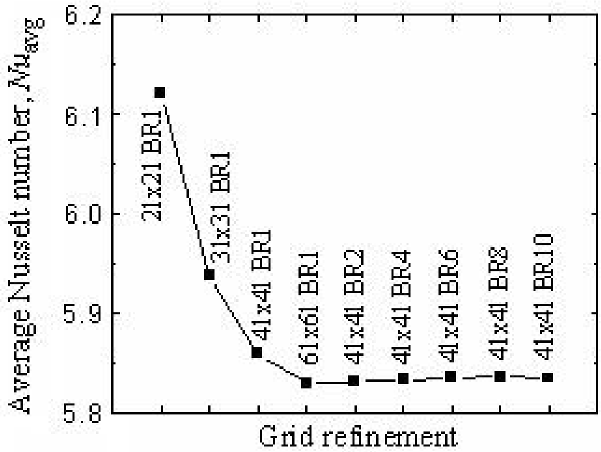
In order to determine the local Nusselt numbers on the cold wall and hot wall, if the temperature boundary condition is specified. The temperature profiles are fit with quadratic, cubic and bi-quadratic polynomials and their gradients at the walls are determined. It has been observed that the temperature gradients at the surface are almost the same for all the polynomials considered. Hence only a quadratic fit is made for the temperature profiles to extract the local gradients at the walls to calculate the local heat transfer coefficients from which the local Nusselt numbers are obtained. Integrating the local Nusselt number over each side, the average Nusselt number for each side wall is obtained as

$$\overline{\mathrm{Nu}}_s = \int_0^H \mathrm{Nu}_s \, dy \qquad (4)$$

5. VERIFICATION OF THE PRESENT METHODOLOGY

The grid independent study has been made with different grids and biasing to yield the consistent values withSharif and Taquiur (2005) paper. The physical systems considered for analysis are bounded by uniform temperature vertical side walls and adiabatic top wall. The bottom wall is subjected to a uniform heat flux distributed 20 % to 80 % of the length from the centre and the remaining length is considered as adiabatic. The Grashof number (Gr) is varied from $10^3$to 10^6. A grid refinement study is performed for a square cavity (AR = 1) with heating 80 % of the length of the bottom wall. Fig. 2(a) shows the convergence of the average Nu at the heated surface with grid refinement for Gr = 10^5 of Sharif and Taquiur (2005).Different grid sizes of 31 x 31, 41 x 41, 51 x 51 and 61 x 61 with uniform mesh as well as biasing have been studied.

The grid 41 x 41 biasing ratio (BR) of 2(The ratio of maximum cell to the minimum cell is 2, thus making cells finer near the wall) gave results identical to that of 61 x 61 uniform mesh. In view of this, 41 x 41 grid with biasing ratio 2 is used in all further computations. Fig. 2(b) shows variation of the average Nusselt number with the journal values. The percentage of error was within 2.8 %.



(a)

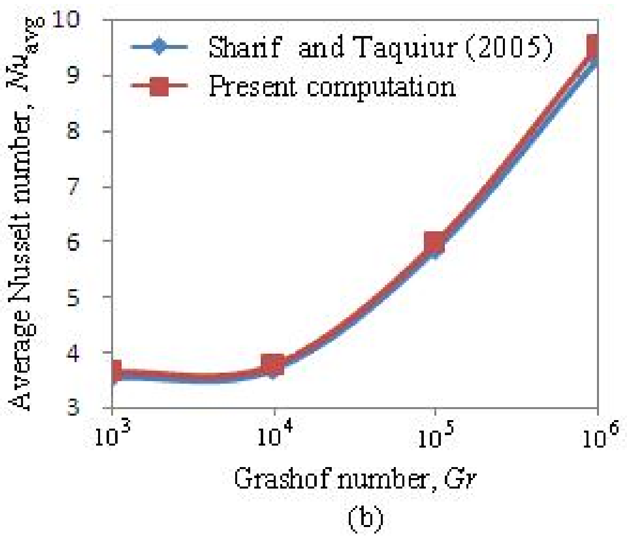


Fig.2 Convergence of $\overline{Nu}$ with (a) grid refinement and (b) Comparison with Sharif and Taquiur (2005)

Gr	$\frac{k_w L}{k_f t}$	$\overline{Nu}$ **for t/L = 0.2**		$\overline{Nu}$ **for t/L = 0.4**	
		Kaminski (1986)	**Present Study**	**Kaminski (1986)**	**Present Study**
10^3	5	0.87	0.8451	0.87	0.845
	25	1.02	0.9829	1.02	0.982
	50	1.04	1.0113	1.04	1.021
	∞	1.06	1.042	1.06	1.042
10^5	5	2.08	2.018	2.08	2.04
	25	3.42	3.20665	3.41	3.297
	50	3.72	3.4341	3.71	3.427
	∞	4.08	3.9606	4.08	4.058
10^6	5	2.87	2.7488	2.87	2.740
	25	5.89	5.6581	5.88	5.626
	50	6.81	6.5314	6.8	6.5017
	∞	7.99	7.4493	7.99	7.545
10^7	5	3.53	3.52554	3.53	3.518
	25	9.08	8.98223	9.06	8.928
	50	11.39	11.1956	11.38	11.131
	∞	15.09	14.5765	15.09	14.567

Table 1 Comparison of the present numerical method with the benchmark solution for conjugate heat transfer Kaminski (1986).

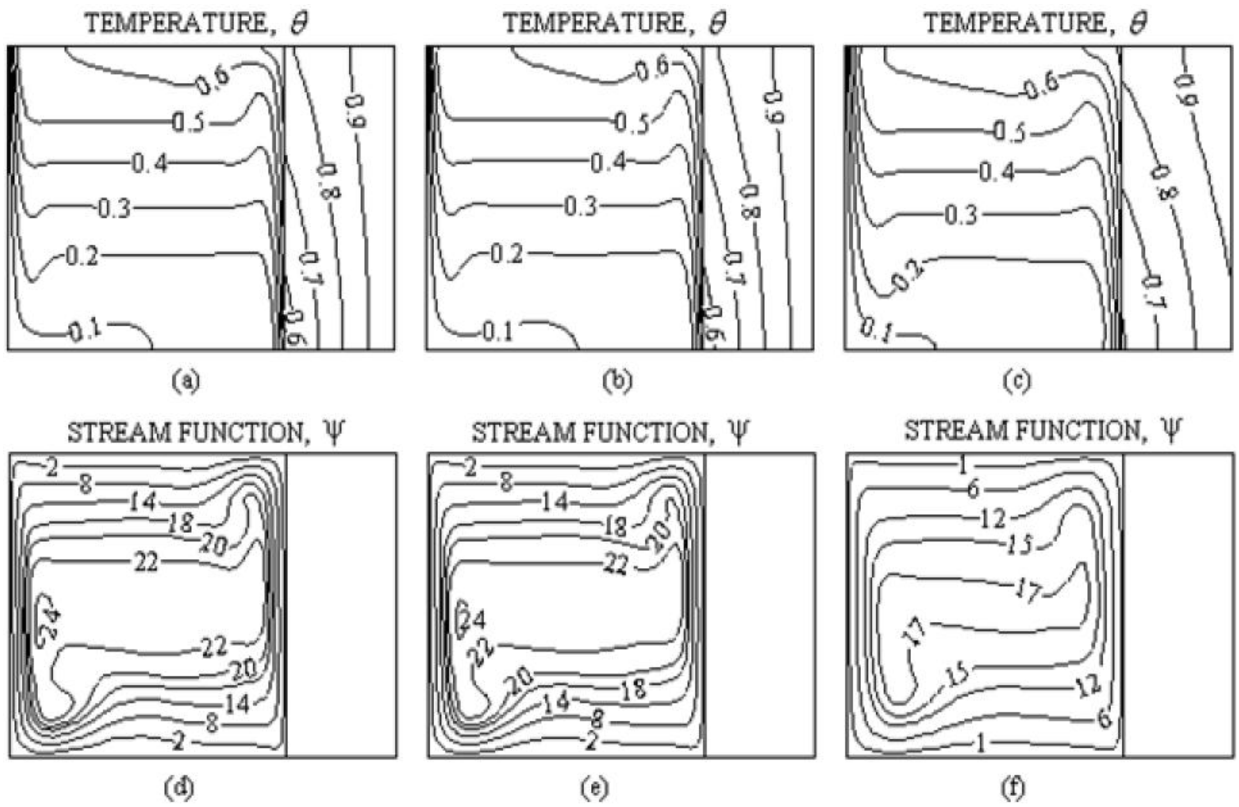


Fig. 3(a), (b), (c) represents the temperature contours and (d), (e), (f) stream function contours for *Gr* = 10^7, t/L = 0.4and $k_wL/k_f t$ = 25, when solid wall is subjected to the uniform temperature, convection boundary condition and uniform heat flux cases respectively.

This is found to be in good agreement with Sharif and Taquiur (2005).However, in order to predict the computational capabilities of the present code, conjugate natural convection in a square enclosure with the effect of conduction in one of the vertical walls studied by Kaminski and Prakash (1986) has been selected. The convergence a criterion is carried out, the 41 x 41 with biasing ratio 4 is much suitable for this case. The comparison of the present code results with that of Kaminski and Prakash (1986) is tabulated in table 1. The maximum error obtained between these two is well within 3%. It has been observed that thickness to length ratio of cavity (t/l) has no effect on $\overline{Nu}$.

6. RESULTS AND DISCUSSION

Fig. 3(a), (b), and (c) represent the isotherms and (d), (e) and (f) shows the stream function for uniform temperature case, convection boundary conditions and uniform heat flux case respectively. It is observed that the contour plots for isotherms and stream functions of first two types of boundary conditions are similar as expected, whereas the third boundary condition is quite different from first two cases. Since the value of non-dimensional temperature θ= 0.8 and 0.9span for the entire vertical hot solid wall and θ= 0.6 also exists in the first two cases. However the θ= 0.7, 0.8 and 0.9 are existing partially in the solid wall for heat flux cases. Hence the solid wall is hotter than the first two cases. Fig. 3(d), (e) and (f) shows the stream function for all the three cases considered. But it is evident from the stream function contours that the magnitude of the stream function is less for heat flux cases as compared to temperature and convective boundary conditions. Hence, heat transfer rate is less in constant heat flux boundary conditions.

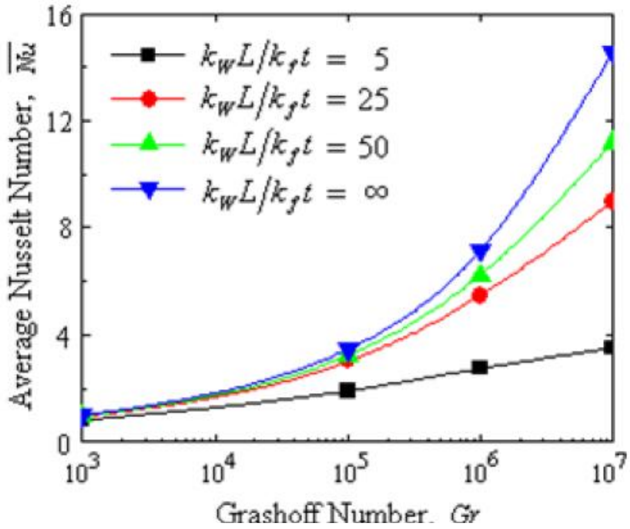


Fig. 4 shows the variation of $\overline{Nu}$ with conductivity ratio.

Fig. 4 show the variation of $\overline{Nu}$ with Gr for thermal conductivity ratio 5, 25, 50 and ∞, with, t/l = 0.2, Pr = 0.7 for uniform temperature case. It is seen that $\overline{Nu}$ increases monotonically with increase of thermal conductivity.

Fig.5 represent the effect of the different types of boundary conditions on conjugate heat transfer with respect to Gr for t/l = 0.2, $k_w l/k_f\ t = 25$, and Pr = 0.7. It is evident that heat transfer rate is minimum in

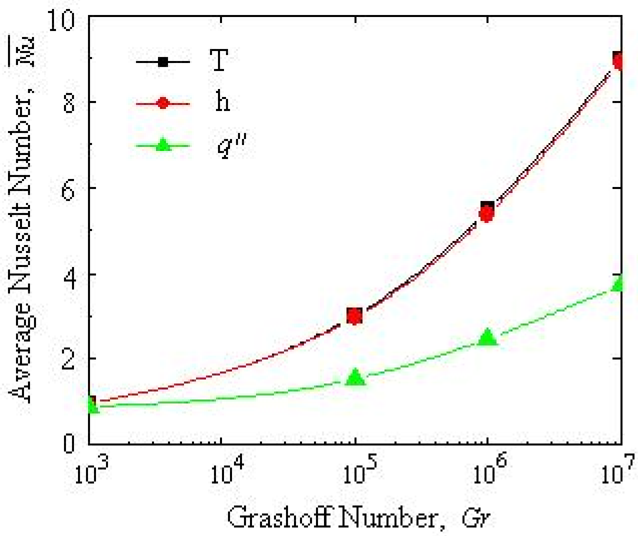


Fig. 5 shows the variation of $\overline{Nu}$ for different types of boundary conditions

Case of heat flux boundary condition. Also, it is noted that $\overline{Nu}$ is almost same for both uniform temperature condition and convection boundary condition cases as expected when h ≥ 50 W/m^{2}K.

However the $\overline{Nu}$ is higher for these two cases as compared to uniform heat flux case.

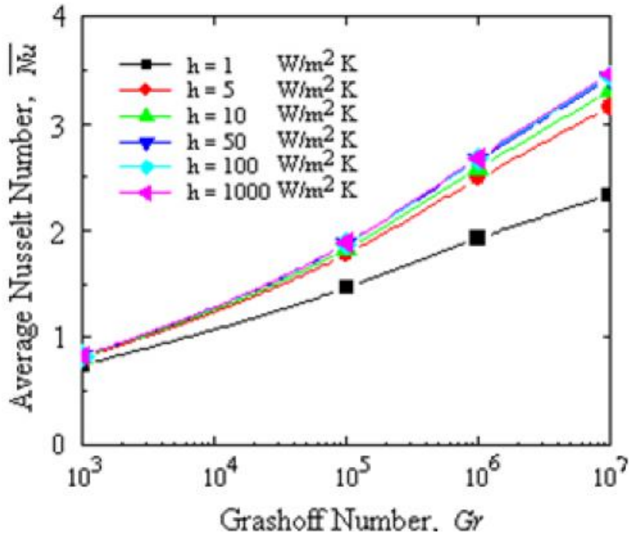


Fig. 6 shows the variation of $\overline{Nu}$ with *Gr* for range of heat transfer co-efficient values considered.

Fig.6 shows the effect of heat transfer co-efficient with *Gr* on $\overline{Nu}$ in conjugate heat transfer in a square enclosure. The variation in $\overline{Nu}$ is observed for values of heat transfer co-efficient ranging from 1-50 W/m^{2}K. However for h ≥ 50 W/m^{2}K, the variation is negligible. Hence for the further computation of $\overline{Nu}$ with Gr, the value of h = 100 W/m^{2}K is considered.

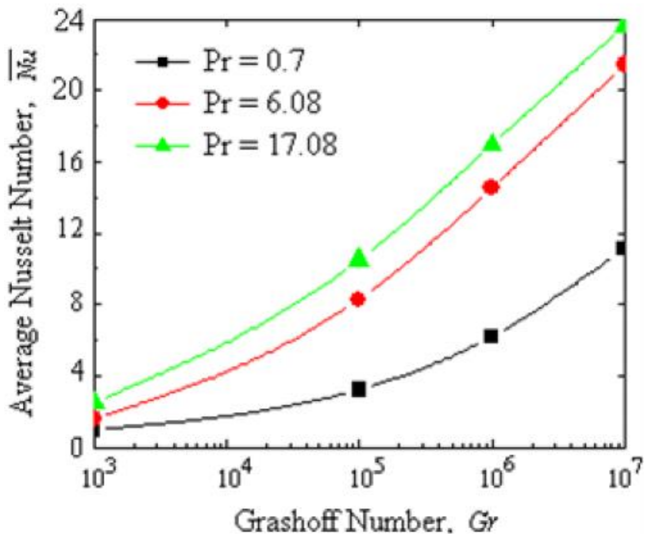


Fig. 7 shows the variation of the $\overline{Nu}$ with Pr.

Fig. 7 shows the variation of $\overline{Nu}$ with *Gr* for different *Pr* by keeping *t/l* = 0.4, $k_w l / k_f\ t = 50$, constant when solid wall subjected to uniform temperature. Three fluids whose Pr ranges from 0.7 to 17 is considered for study. It is observed that as the Prandtl number increases the $\overline{Nu}$ also increases.

7. CONCLUSIONS

The conjugate natural convection in a square enclosure with three different types of boundary conditions is analyzed. The following observations are noted during the investigations.

a) It is seen that $\overline{Nu}$ increases monotonically with increase of *Gr* for the different boundary conditions.

b) The $\overline{Nu}$ increases with increase of h up to 50 W/m^{2}K. However it remains same for values of h >50 W/m^{2}K.

c) The average Nusselt number increases, if either the conductivity ratio or the Prandtl number increases.

d) For a given Grashof number, the average Nusselt number is minimum for heat flux boundary condition than the uniform temperature or convective boundary conditions.

e) The effect of wall thickness to cavity length ratio on the average Nusselt number is negligible.

8. REFERENCES

1. Issa, R.I., 1985. Solution of the implicitly discretized fluid flow equations by operator-splitting, J. Comput. Phys. 62 pp. 40-65.
2. Kaminski, D.A and Prakash, C., 1986. Conjugate natural convection in a square enclosure: effect of conduction in one of the vertical walls, Int. J. of Heat and Mass Transfer, 29 (12)pp. 1979-1988.
3. Kuznetsov, G.V and Sheremet, M.A. , 2011. Conjugate natural convection in an enclosure with a heat source of constant heat transfer rate, Int. J. of Heat and Mass Transfer, 54, Issues 1-3, pp. 260-280.
4. Liaqat, A and Baytas,A.C., 2001.Conjugate natural convection in a square enclosure containing volumetric sources, Int. J. of Heat and Mass Transfer, 44 pp.3273–3280.
5. Sharif,A.R and Taquiur Rahman Mohammad.,2005.Natural convection in cavities with constant flux heating at the bottomwall and isothermal cooling from the sidewalls, Int. J. of Thermal Sciences, 44 pp. 865-878.