

Design of RMS Model Order Reduction Automation Tool for Realisation and Stability Analysis of Reduced Order Model Pupillary Light Reflex System

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Abstract: New methods and design of Tool for the reduction of high-order, pupillary reflex systems is proposed. The methods are based on the different techniques and generates low order stable models retaining both the initial Markov parameters and time-moments of the original system. Theses models give a better approximation for both the steady-state as well as the transient part of the time response. The proposed procedures avoids the necessity of formulating Routh-type arrays, application of reciprocal transformation, finding the time-moments of the nth order original system before hand and the use of gain-factor, to generate the denominator and numerator of the models unlike other methods. The new procedures are simple, direct and computationally superior to other methods . The methods are well illustrated with pupillary light reflex system. These methods are implemented using RMS Model Order Reduction automation Tool and also responses are analyzed by Tool and simulation.

In this paper a MATLAB-based tool with a Graphical User Interface (GUI), to compute reduced models of a pupillary reflex system are presented. The model reduction techniques implemented in this tool are based on Different techniques. The methods selected in this paper are eleven . Upon execution of the toolbox, a GUI will appear with four frames named “METHODS”, “INPUT DATA”, “OUTPUT OPTIONS”, and “DISPLAY UNIT”.

Keywords: Modeling, Stability, order, pupillary light reflex system , MATLAB , RMS automation model order reduction tool, Reduction techniques.

1. Introduction:

Polynomial reduction methods are some important groups of the reduction techniques that are applied in the frequency domain. They are used to approximate lower-order transfer functions where the model coefficients are chosen according to various criteria. The frequency domain model-reduction techniques that are mainly based on polynomial manipulations are Pade -type approximations.

Model reduction based on the Routh table criterion use reduction algorithms with stability equations, mixed methods of approximation and energy-based methods.

The methods proposed in the paper for deriving a reduced order transfer function, R(s), from the original higher-order transfer function G(s) .

2. Description of the method:

Let the full n^{th} order system be represented by its transfer function

$$G(S) = \frac{B_0 + B_1 S + \dots + B_{n-1} S^{n-1}}{A_0 + A_1 S + \dots + A_n S^n} \dots(1)$$

Then the reduced-order model of order k which retain initial m Markov parameter and t time-moments of $G_n(s)$ using the proposed methods are defined as,

$$R_k(S) = \frac{b_0 + b_1 s + \dots + b_{k-1} S^{k-1}}{a_0 + a_1 S + \dots + a_k S^k} \dots(2)$$

3. Analysis Of Pupillary Light Reflex System:

The pupillary light reflex has been studied extensively using control system analysis, most notably by bioengineering pioneers Lawrence Stark and Manfred Clynes. The Purpose of this reflex is to regulate the total light flux reaching the retina, although the same pupil control system is also used to alter the effective lens aperture so as to reduce optical aberrations and increase depth of focus. The reflex follows the basic scheme shown in Fig. 1..

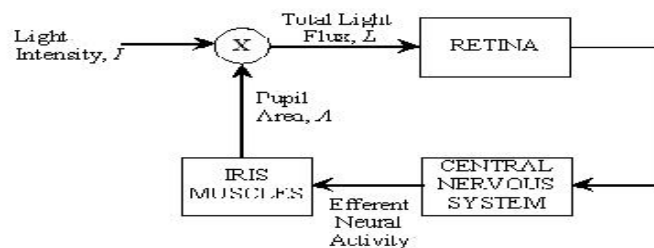
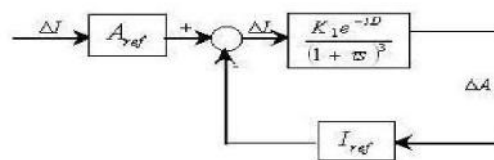
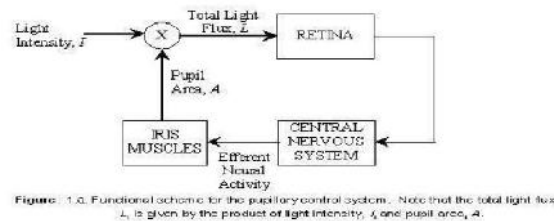


Fig : 1 A fundamental scheme for the pupillary control system

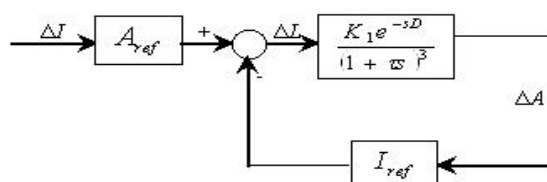


Fig : 2 Linearized (small signal) model of the pupillar light reflex system

An increase in the intensity (I) of ambient light elevates the total light flux (L) received by the retina, which converts the light into neural signals. The afferent neural information is sent via the optic nerve to the lateral geniculate body and then to the pretectal nucleus. Subsequently, the Edinger-Westphal nucleus sends efferent neural signals back toward the periphery to the iris sphincter and dilator muscles which, respectively, contract and relax to reduce the pupil area (A).

Not surprisingly, quantitative investigations into this feedback control scheme have revealed significant nonlinearities in each of the system components. However, Stark came up with a linear characterization that provides a reasonably good approximation of the under-lying dynamics when the changes involved are relatively small. This linearized model is schematized in Fig.2. Using an ingenious experimental design, he was able to functionally “open the loop” of this reflex and measure the dynamics of this system. He found that the dynamics could be modeled by a third-order transfer function with time constant T , in series with a pure

time delay D . In Fig.2, I represents a small change in light intensity from the reference intensity level, I_{ref} while A represents the corresponding change in pupil area from the reference value, A_{ref} and L is the change in total light flux reaching the retina. Based on the model, the closed-loop transfer function of the pupillary reflex can be deduced as

$$\frac{\Delta A}{\Delta I} = \frac{\frac{A_{ref} K_1 e^{-sD}}{(1 + \tau s)^3}}{1 + \frac{I_{ref} K_1 e^{-sD}}{(1 + \tau s)^3}} \dots(3)$$

By inspection of Fig.2 and Equation (3) above, one can readily infer that the *loop transfer function* of this model is given by

$$H_L(s) = \frac{K e^{-sD}}{(1 + \tau s)^3} \dots(4)$$

where $K = I_{ref} K_1$. Therefore, characteristic equation for the closed-loop model is

$$1 + \frac{K e^{-sD}}{(1 + \tau s)^3} = 0 \dots(5)$$

From his measurements on normal humans, Stark found the following values for the model parameters: $K = 0.16$, $D = 0.18$ s and $\tau = 0.1$ s.

4. Realisation Of Reduced Order Models and Results

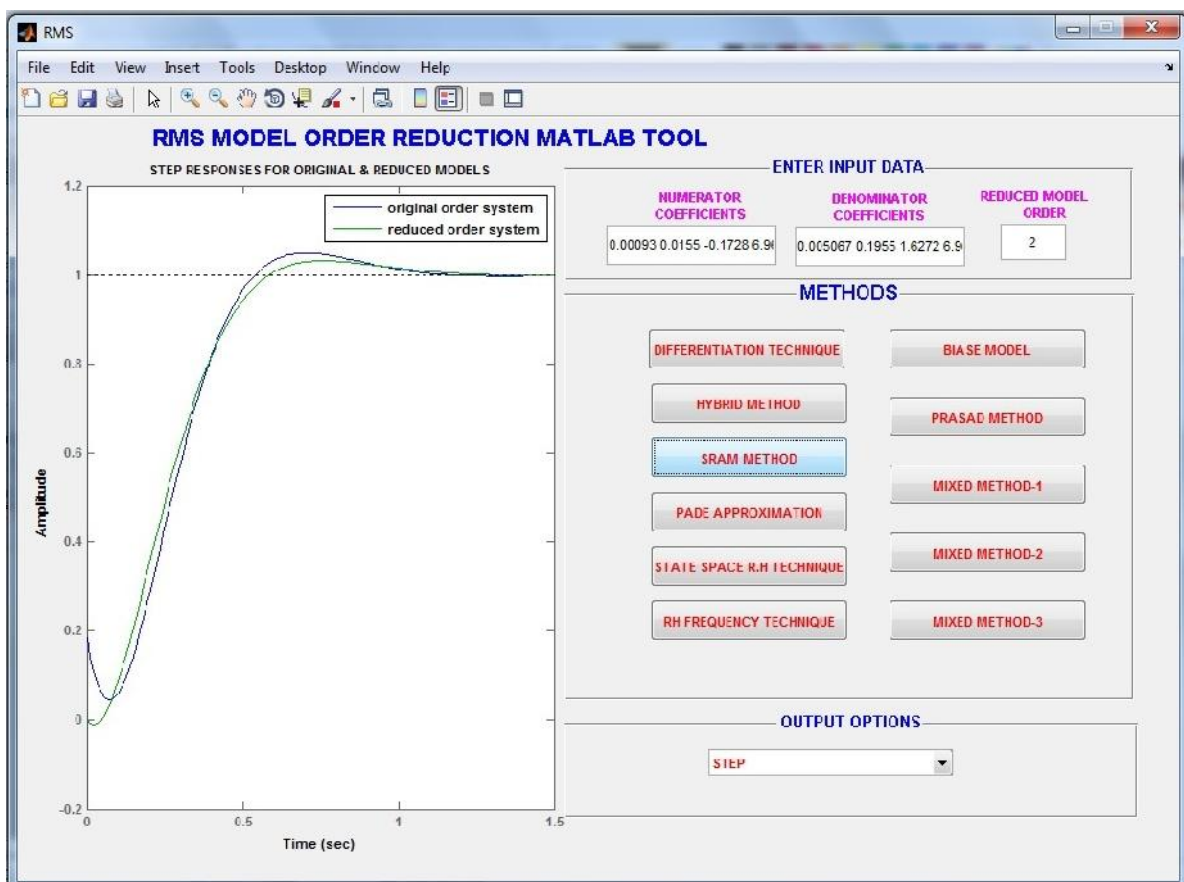


Fig.3 Frequency responses for original and reduced models by SRAM method of RMS Tool

Original transfer function $G(s) = \frac{0.00093s^3 + 0.0155s^2 - 0.1728s + 6.96}{0.005067s^3 + 0.1955s^2 + 1.6272s + 6.96}$		
S.No	Name of the Reduction technique	Reduced transfer function $R_k(s)$
1	Differentiation method	$R_2(s) = \frac{0.01196s + 6.96}{0.06517s^2 + 1.085s + 6.96}$
2	Hybrid method	$R_2(s) = \frac{-0.1028s + 1}{0.009363s^2 + 0.1559s + 1}$
3	SRAM method	$R_2(s) = \frac{-1.009s + 40.64}{s^2 + 9.502s + 40.64}$
4	PadeApproximationmethod	$R_2(s) = \frac{0.164s^2 - 2.634s + 46.99}{s^2 + 9.52s + 46.99}$
5	State Space R.H Technique	$R_2(s) = \frac{0.03746s - 0.0009s}{0.03746s^2 + 0.008758s}$
6	R.H Frequency Technique	$R_2(s) = \frac{0.328s^2 + 0.5495s + 47.31}{s^2 + 9.502s + 40.64}$
7	Biased Model method	$R_2(s) = \frac{-1.009s + 40.64}{s^2 + 9.502s + 40.64}$
8	Prasad method	$R_2(s) = \frac{-0.1728s + 5.301}{0.1955s^2 + 1.627s - 134.1}$
9	Mixed method-1	$R_2(s) = \frac{-1.009s + 40.64}{s^2 + 9.502s + 40.64}$
10	Mixed method-2	$R_2(s) = \frac{-1.009s + 40.64}{s^2 + 9.502s + 40.64}$
11	Mixed method-3	$R_2(s) = \frac{-1.009s + 40.64}{0.1955s^2 + 1.627s - 134.1}$

Table 1, Original and reduced model transfer functions of pupillary light reflex system

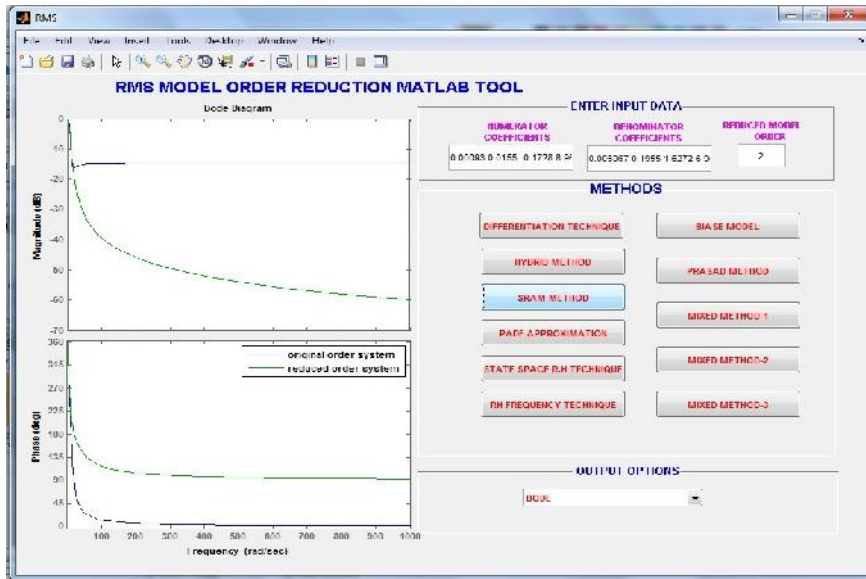


Fig.4 Unit step responses for original and reduced models by SRAM method of RMS Tool

The responses of the original system and reduced system represented in the above Fig3 & Fig 4. These can be inferred from these figures that the reduced order model obtained by the proposed method gives a better time-response. This is because of the fact that the proposed method retains both the time moments and Markov parameters

5. Result Analysis:

S. No.	Name of the method	Original System G(s) specifications					Reduced system R(s) specifications				
		Rise Time (Sec.)	Settling Time (Sec.)	Peak Over Shoot (%)	Pole / Zero Locations	Stability	Rise time (Sec.)	Settling Time (Sec.)	Peak Over Shoot (%)	Pole / Zero Locations	Stability
1.	Differentiation Technique	0.00022	0.00768	72.9	Left Half	Stable	1.07×10^{-5}	1.96×10^3	41.2×10^7	Left Half	Stable
2.	Hybrid Method	0.00022	0.00768	72.9	Left Half	Stable	0.0369	0.0669	0	Left Half	Stable
3.	SRAM Method	0.00022	0.00768	72.9	Left Half	Stable	0.11	0.198	0	Left Half	Stable
4.	Pade Approximation	0.00022	0.00768	72.9	Left Half	Stable	0	0	0	Left Half	Stable
5.	State space R.H. Technique	0.00022	0.00768	72.9	Left Half	Stable	5.5×10^7	9.79×10^7	0	Left & right Half	Un stable
6.	R. H. Frequency technique	0.00022	0.00768	72.9	Left Half	Stable	0.11	0.198	0	Left Half	Stable
7.	Biased model	0.00022	0.00768	72.9	Left Half	Stable	0.11	0.198	0	Left Half	Stable
8.	Prasad Method	0.00022	0.00768	72.9	Left Half	Stable	0.11	0.196	0	Left & Right Half	Un Stable
9.	Mixed Method –1	0.00022	0.00768	72.9	Left Half	Stable	0.11	0.198	0	Left Half	Stable
10.	Mixed Method –2	0.00022	0.00768	72.9	Left Half	Stable	0.11	0.198	0	Left Half	Stable
11.	Mixed Method –3	0.00022	0.00768	72.9	Left Half	Stable	0.11	0.195	0	Left & Right Half	Un stable

Table: 2, Time Response Analysis for original and reduced systems

S. No.	Name of the method	Original System G(s) specifications					Reduced system R(s) specifications				
		Gain margin (db)	Phase margin (degrees)	Gain cross-over frequency (\check{S}_{gc})	Phase cross-over frequency (\check{S}_{pc})	Stability	Gain margin (db)	Phase margin (degrees)	Gain cross-over frequency (\check{S}_{gc})	Phase cross-over frequency (\check{S}_{pc})	Stability
1.	Differentiation Technique	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	90.0001	Non	2.5012×10^7	Stable
2.	Hybrid Method	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	-180	Non	7.4	Stable
3.	SRAM Method	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	-180	Non	2.13	Stable
4.	Pade Approximation	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	-180	Non	12.8	Stable
5.	State space R.H. Technique	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	90.001	Non	1.35	Un Stable
6.	R. H. Frequency technique	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	-180	Non	2.13	Stable
7.	Biase model	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	-180	Non	2.13	Stable
8.	Prasad Method	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	Infinite	Non	1.84	Un Stable
9.	Mixed Method ₋₁	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	-180	Non	2.13	Stable
10.	Mixed Method ₋₂	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	-180	Non	2.13	Stable
11.	Mixed Method ₋₃	Infinite	16.25	Infinite	7.0002×10^3	Stable	Infinite	Infinite	Non	Non	Un Stable

Table:3, Frequency Response Analysis for original and reduced systems

6. Simulation and Results:

Simulation models of the original and reduced pupillary light reflex system are shown in Fig. 5. The simulation analysis for original and reduced models are carried out by LTI Viewer and shown in Fig.6 and result analysis in time and frequency domain as shown in table 4&5

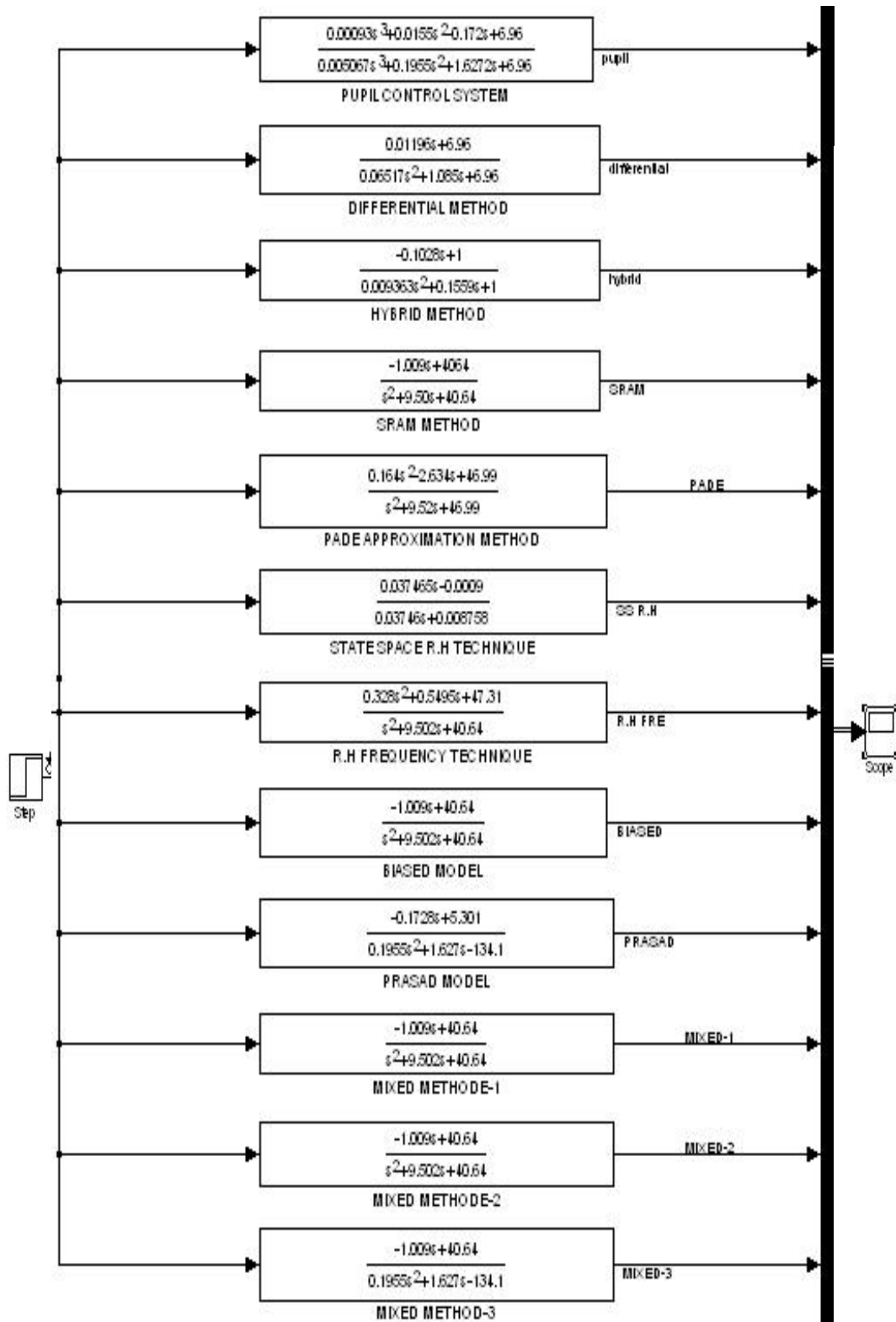


Fig : 5 Simulation models for Original and Reduced systems

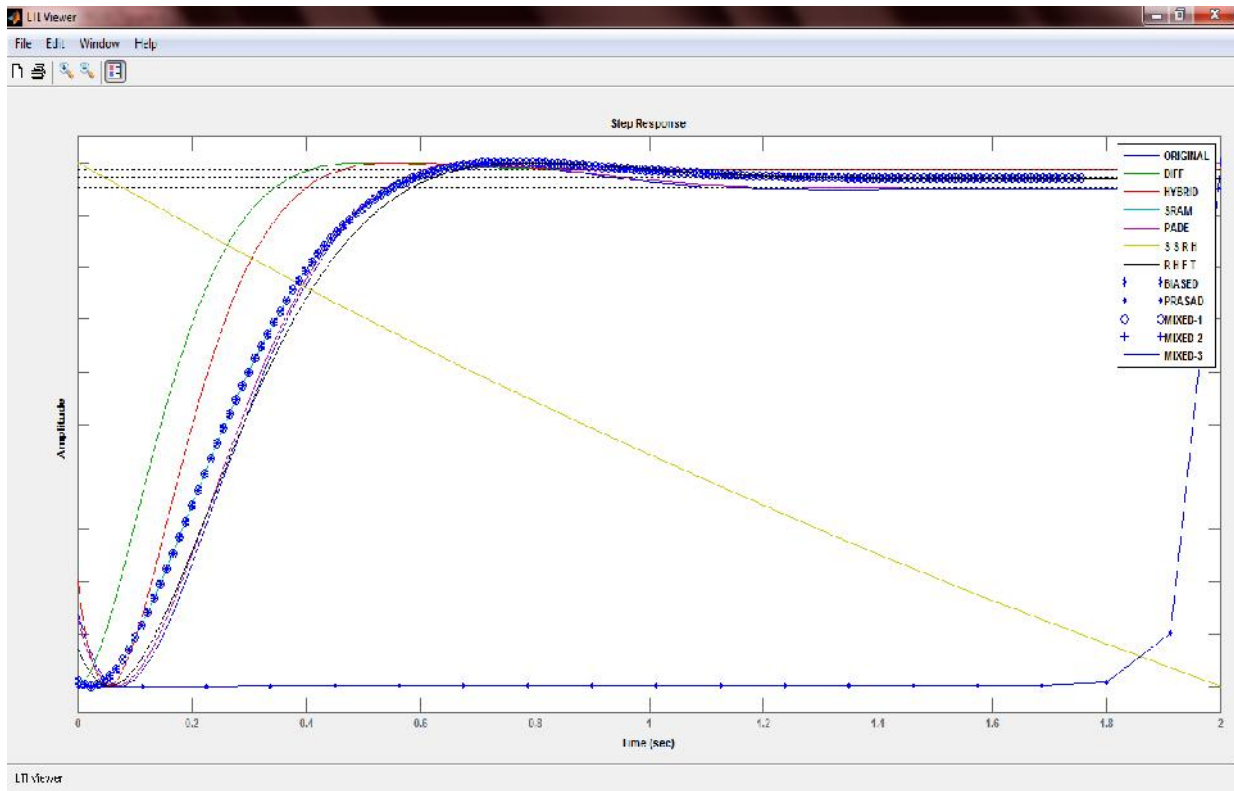


Fig : 6 Unit step responses for original and reduced models by LTI Viewer

7. Result Analysis By Simulation:

Table: 4, Time Response Analysis for original and reduced systems

S. No.	Name of the method	Original System G(s) specifications					Reduced system R(s) specifications				
		Rise time (Sec.)	Settling Time (Sec.)	Peak Over Shoot (%)	Pole / Zero Locations	Stability	Rise time (Sec.)	Settling Time (Sec.)	Peak Over Shoot (%)	Pole / Zero Locations	Stability
1.	Differentiation Technique	0.266	0.938	5.11	Left Half	Stable	0.241	0.366	1.4	Left Half	Stable
2.	Hybrid Method	0.266	0.938	5.11	Left Half	Stable	0.21	0.421	1.76	Left Half	Stable
3.	SRAM Method	0.266	0.938	5.11	Left Half	Stable	0.266	0.928	3.02	Left Half	Stable
4.	Pade Approximation	0.266	0.938	5.11	Left Half	Stable	0.273	0.942	4.66	Left Half	Stable
5.	State space R.H. Technique	0.266	0.938	5.11	Left Half	Stable	Non	Non	Non	Left & Right Half	Un stable
6.	R. H. Frequency technique	0.266	0.938	5.11	Left Half	Stable	0.335	0.938	2.69	Left Half	Stable
7.	Biased model	0.266	0.938	5.11	Left Half	Stable	0.353	0.929	3.02	Left Half	Stable
8.	Prasad Method	0.266	0.938	5.11	Left Half	Stability	Non	Non	Non	Left & Right Half	Un Stable
9.	Mixed Method –1	0.266	0.938	5.11	Left Half	Stable	0.353	0.929	3.02	Left Half	Stable
10.	Mixed Method –2	0.266	0.938	5.11	Left Half	Stable	0.353	0.929	3.02	Left Half	Stable
11.	Mixed Method –3	0.266	0.938	5.11	Left Half	Stable	Non	Non	Non	Left & Right Half	Un stable

Table: 5, Frequency Response Analysis for original and reduced systems

S. No.	Name of the method	Original System G(s) specifications					Reduced system R(s) specifications				
		Gain margin (db)	Phase margin (degrees)	Gain cross-over frequency (ω_{gc})	Phase cross-over frequency (ω_{pc})	Stability	Gain margin (db)	Phase margin (degrees)	Gain cross-over frequency (ω_{gc})	Phase cross-over frequency (ω_{pc})	Stability
1.	Differentiation Technique	10.8	-180	11.2	0	Stable	Infinite	-180	Non	0	Stable
2.	Hybrid Method	10.8	-180	11.2	0	Stable	3.62	73.9	16.4	7.54	Stable
3.	SRAM Method	10.8	-180	11.2	0	Stable	19.5	-180	20.6	0	Stable
4.	Pade Approximation	10.8	-180	11.2	0	Stable	11.2	-180	11.7	0	Stable
5.	State space R.H. Technique	10.8	-180	11.2	0	Stable	Infinite	Infinite	Non	Non	Un Stable
6.	R. H. Frequency technique	10.8	-180	11.2	0	Stable	Infinite	134	Non	3.42	Stable
7.	Biased model	10.8	-180	11.2	0	Stable	19.5	-180	20.6	0	Stable
8.	Prasad Method	10.8	-180	11.2	0	Stable	28.1	Infinite	0	Non	Un Stable
9.	Mixed Method – 1	10.8	-180	11.2	0	Stable	19.5	-180	20.6	0	Stable
10.	Mixed Method – 2	10.8	-180	11.2	0	Stable	19.5	-180	20.6	0	Stable
11.	Mixed Method – 3	10.8	-180	11.2	0	Stable	10.4	Infinite	0	Non	Un Stable

The time response and frequency response analysis of original system and reduced model are compared in table-4 & 5. With this analysis, it is observed that the reduced model obtained by proposed method SRAM is better response than the original system.

8. Conclusion:

Model reduction of pupillary light reflex system is considered in this work. The higher order original system is reduced to second order model using an eleven proposed methods. The responses obtained by the second order system using SRAM method is in good agreement with those of the original system. The proposed method does not require computation of the poles of the original system as for dominant pole retention is concerned. It is observed that the SRAM method is computationally efficient and gives an unique stable reduced-order model

S. No.	Best Method	Original System G(s) specifications					Reduced system R(s) specifications				
		Rise Time (Sec.)	Settling Time (Sec.)	Peak Over Shoot (%)	Pole / Zero Locations	Stability	Rise time (Sec.)	Settling Time (Sec.)	Peak Over Shoot (%)	Pole / Zero Locations	Stability
1	SRAM	0.266	0.938	5.11	Left Half	Stable	0.266	0.928	3.02	Left Half	Stable

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