Fast and Accurate Model to Determine the Resonant Frequency of Tunable Rectangular Patch Antenna

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ABSTRACT: An improved model for accurate computation of resonant frequency for a tunable rectangular patch antenna with varying aspect ratio printed on single, composite and suspended substrate is presented. The computed results are compared with experimental and simulation results.

KEY WORDS: rectangular patch antenna, composite and suspended substrate, CAD.

I. INTRODUCTION
The suspended substrate patch antenna has received a great deal of attention due to its frequency tenability feature, longer bandwidth and improved radiation efficiency for a patch on high permittivity substrate (Alumina, GaAs, etc.) [1-10]. The suspended substrate patch antenna is a special type of a patch antenna on the composite substrate. The patch on the composite substrate is encountered in many microstrip designs like i) the patch in multilayered media for improving the antenna performances and protect the patch from environmental hazards [11-17] ii) an electromagnetically coupled patch antenna on a two layer [18] and three layer composite substrates [19] for improving the bandwidth and radiation characteristics: and iii) patch antenna made on a multilayered MMIC [20-21].

A number of theoretical methods have been reported to compute the resonant frequency, input impedance and radiation pattern of patch antennas on single layer or multiple layers using the variational technique [22], the multiport network approach [23], the spectral domain analysis (SDA) and other full-wave analysis methods [24-30]. According to a special variant of the discrete-mode-matching method, [31] and [32] analyzed patch antennas and feed networks embedded in multilayer structures of arbitrary shape. Neural network methods were employed in [33-34] to analyze resonant frequencies and design parameters of various patch antennas of regular geometries. All of these efforts produced reasonably accurate results. However, the numerical methods with high accuracy are too complex, rigorous and time consuming for direct use in CAD programs.

This article presents a set of closed form expressions to predict accurately the resonant frequency for a rectangular patch with varying thickness, permittivity and aspect ratio. This model relatively simple and fast even for more complicated models [22-34] and commercially available softwares based on them. The accuracy of the present model is verified against experimental results and experimental results available in open literature. The model has also been compared against the results obtained from commercial simulator (HFSS).

The expressions for these models are introduced in section II. Section III, includes the experimental tests. In section IV, we have presented the predicted results employing the present model and experimental results.

II. MATHEMATICAL FORMULATION
This section introduces the expressions for this model including effective permittivity, effective patch length and resonant frequency.
The rectangular microstrip patch is modeled as a cavity with a magnetic wall along the edge. This is a two layer cavity: the upper layer has thickness $d_2$ with relative permittivity $\varepsilon_{r2}$ and lower layer has thickness $d_1$ with permittivity $\varepsilon_{r1}$. In order to give a general formulation for both single- and two-layer structures, the two-layer structure is modeled as the single-layer one having substrate thickness of $d = d_1 + d_2$ and an equivalent substrate relative permittivity of $\varepsilon_{re}$ determined under the cavity model approximations as

$$\varepsilon_{re} = \frac{\varepsilon_{r1} \varepsilon_{r2} d}{\varepsilon_{r1} d_2 + \varepsilon_{r2} d_1}$$  \hspace{1cm} (1)$$

Following the analytical model by [35], an improved formulation is presented by introducing a new effective permittivity $\varepsilon_{r,eff}$ in place of the dynamic permittivity $\varepsilon_{r,dyn}$ of the medium below the patch to calculate the resonant frequency of a two dielectric layer rectangular patch antenna (Fig. 1) as

$$f_{r,nn} = \frac{c}{2 \sqrt{\varepsilon_{r,eff}}} \left[ (n/L_{eff})^2 + (m/W_{eff})^2 \right]^{1/2}$$  \hspace{1cm} (2)$$

where, $c$ is the velocity of light in free space, $L_{eff}$ and $W_{eff}$ are the effective length and effective width of the rectangular patch, incorporating the effect of fringing field, and are defined as (18) and (19), $\varepsilon_{r,eff}$ defined as [36]

$$\varepsilon_{r,eff} = \frac{4 \varepsilon_{re} \varepsilon_{r,dyn}}{(\sqrt{\varepsilon_{re}} + \sqrt{\varepsilon_{r,dyn}})^2}$$  \hspace{1cm} (3)$$

Fig. 1: (a) A schematic diagram of probe-fed Rectangular Microstrip Patch Antenna

The term $\varepsilon_{r,eff}$ introduced to take into account the effect of $\varepsilon_{re}$, the equivalent permittivity of the medium below the patch in combination with the dynamic permittivity $\varepsilon_{r,dyn}$ to improve the model. $\varepsilon_{r,eff}$ is deduced as (3) to yield the resonant frequency as an average of the frequencies resulting from (2) by substituting $\varepsilon_{re}$ and $\varepsilon_{r,dyn}$ separately in place of $\varepsilon_{r,eff}$.

The dynamic permittivity $\varepsilon_{r,dyn}$ depends on the dimensions ($W$, $L$, $d$), equivalent substrate relative permittivity $\varepsilon_{re}$, and field configuration of the mode under study [35]. It can be expressed as

$$\varepsilon_{r,dyn} = \frac{C_{dyn}(\varepsilon = \varepsilon_0 \varepsilon_{re})}{C_{dyn}(\varepsilon = \varepsilon_0)}$$  \hspace{1cm} (4)$$
In equation (4), \( C_{\text{dyn}}(\varepsilon) \) is the total dynamic capacitance of the condenser formed by the conducting patch and the ground plane separated by a dielectric of permittivity \( \varepsilon \). It takes into account the influence of the fringing field at the edge of the rectangular microstrip patch. \( C_{\text{dyn}}(\varepsilon) \) is the total dynamic capacitance when \( \varepsilon = \varepsilon_0 \). The \( C_{\text{dyn}}(\varepsilon) \) can be written as

\[
C_{\text{dyn}}(\varepsilon) = C_{0,\text{dyn}}(\varepsilon) + 2C_{e1,\text{dyn}}(\varepsilon) + 2C_{e2,\text{dyn}}(\varepsilon)
\]

where, \( C_{0,\text{dyn}} \) is the dynamic main field capacitance without considering the fringing field. This can be calculated as

\[
C_{0,\text{dyn}}(\varepsilon) = \frac{\varepsilon_0 \varepsilon_r e W L}{d \delta_n \delta_m} = \frac{C_{0,\text{stat}}(\varepsilon)}{\delta_n \delta_m}
\]

where, \( C_{0,\text{stat}}(\varepsilon) \) represents the static main field capacitance without fringing field and \( \delta_n \) and \( \delta_m \) are in the form

\[
\delta_i = 1 \quad \text{for} \quad i = 0
\]
\[
= 2 \quad \text{for} \quad i \neq 0
\]

Then, a dynamic edge capacitance for each side of the patch taking into account the influence of the fringing field is calculated. The dynamic edge capacitance \( C_{e1,\text{dyn}}(\varepsilon) \) on one side of patch length \( L \) and the dynamic edge capacitance \( C_{e2,\text{dyn}}(\varepsilon) \) on one side of patch width \( W \) can be computed as

\[
C_{e1,\text{dyn}}(\varepsilon) = \frac{1}{\delta_n} C_{e1,\text{stat}}(\varepsilon)
\]

\[
C_{e2,\text{dyn}}(\varepsilon) = \frac{1}{\delta_m} C_{e2,\text{stat}}(\varepsilon)
\]

where, \( C_{e1,\text{stat}}(\varepsilon) \) represents the static edge capacitance on one side of patch length \( L \) and \( C_{e2,\text{stat}}(\varepsilon) \) represents the static edge capacitance on one side of patch width \( W \).

The static edge capacitance for a circular disk was reported in [37]. Based on this concept the static edge capacitance for rectangular patch can be computed as

\[
C_e = \frac{\varepsilon_0 \varepsilon_r e W L}{d} (1 + p_L + p_W)
\]

In (10), the first term represents the static main capacitance \( C_{0,\text{stat}}(\varepsilon) \), \( p_L \) and \( p_W \) arises due to the fringing field at the edge of patch length \( L \) and patch width \( W \) respectively. Thus, \( C_{e1,\text{stat}}(\varepsilon) \) and \( C_{e2,\text{stat}}(\varepsilon) \) are defined as

\[
C_{e1,\text{stat}}(\varepsilon) = 0.5 C_{0,\text{stat}}(\varepsilon) p_L
\]

and

\[
C_{e2,\text{stat}}(\varepsilon) = 0.5 C_{0,\text{stat}}(\varepsilon) p_W
\]

The \( p_L \) and \( p_W \) are computed from originally formulated circular geometry of radius \( a \) [8, 37]. The \( p_L \) for this structure can be more explicitly written as

\[
p_L = p_1 + (1 + p_1)(p_2 + p_3)
\]

\[
p_1 = (1 + \varepsilon_{re}^{-1}) \frac{4d}{\pi a}
\]
Equation (17) is derived from an equivalence relation between rectangular patch (resonating length $L$ and width $W$) and a circular patch (radius $a$) resonating at the same frequency, that is $L = 2a$ and $W = (\pi - 2)a$ [38]. Likewise the $p_w$ can be computed from equations (13) by replacing $L = W$ and $a = W/ (\pi - 2)$.

Equation (10) also defines the effective patch length $L_{eff}$ and width $W_{eff}$ of the rectangular patch

$$L_{eff} = L (1 + p_L)^{1/2}$$  \hspace{1cm} (18)

$$W_{eff} = W (1 + p_W)^{1/2}$$  \hspace{1cm} (19)

### III. RESULTS AND DISCUSSIONS

Fig. 2 shows the theoretical values of $\varepsilon_{r,eff}$ and $\varepsilon_{r,dyn}$ as a function of $W/d$ with $\varepsilon_r$ as a parameter. The quantity $\varepsilon_{r,eff}$, though, becomes closer to $\varepsilon_{r,dyn}$ at large values of $W/d$, and differs significantly as $W/d$ decreases. The parameter $\varepsilon_{r,eff}$ introduced in the present theory thus becomes significant for all large and small values of $W/d$.

In Fig. 3 authors compare the computed resonant frequencies with the corresponding HFSS simulation results for a rectangular patch with varying aspect ratio ($W/L$) printed on the single substrate whose permittivity and thickness are varying. In this study $W$ is taken as a constant value and $L$ is a variable. From Fig. 3 it is observed that the resonant frequency increases with the increase of $W/L$. The validity of the present model is further verified in Table I. In Table I authors compare the experimental dominant mode resonant frequencies of rectangular patch with fixed aspect ratio ($W/L \approx 1.5$) on low permittivity substrate from [44] with predicted values obtained from [16, 38, 43, 45-50] and in this work. This comparison shows the present model accurately compute the resonant frequency.

![Graph showing the relationship between $\varepsilon_{r,dyn}$ and $\varepsilon_{r,eff}$ versus $W/d$.](image)

**Fig. 2** Relationship of $\varepsilon_{r,dyn}$ and $\varepsilon_{r,eff}$ versus $W/d$. $L = 30$ mm, $W$ is a variable, $d_1 = 0.0$ mm, $\varepsilon_{r,l} = 1.0$, $d_2 = 1.575$ mm, $d = d_1 + d_2 = 1.575$ mm.
A comparison of theoretically predicted resonant frequencies with HFSS simulation and new experimental results for rectangular patches on composite and suspended substrates is presented in Fig. 4. This study shows that present model accurately compute the resonant frequencies. The resonant frequency increases with the increase of $d_1$ for suspended substrate whereas the resonant frequency decreases with the increase of $d_1$ for composite substrate. So, the frequency tunability is achieved with the change of $d_1$ without altering the antenna parameters.

### TABLE I

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Total Average % Error

$\epsilon_r^1=1.0, \epsilon_r^2=2.33, d_1=0.0 \text{mm}, d_2=3.175 \text{mm}$

Average % Error = [(exp. - theory)/exp.]*100
IV. CONCLUSION

In this article, an improved model for the resonant frequency of a rectangular patch antenna printed on single, composite and suspended substrate is presented. The influence of fringing fields, and the effect of suspended and composite substrate on resonant frequency is taken into consideration by introducing effective width $W_{\text{eff}}$ and effective length $L_{\text{eff}}$, equivalent relative permittivity $\varepsilon_{\text{re}}$ and dynamic permittivity $\varepsilon_{\text{r,dyn}}$. A new way to calculate the static edge capacitance $C_{\text{e,stat}}(\varepsilon)$, $W_{\text{eff}}$ and $L_{\text{eff}}$ is reported. The formulation overcomes the limitations of the earlier models in predicting the resonant frequency and input impedance for small patch and higher permittivity of the substrate and is thus applicable to a wide range of patch dimensions from very large to very small values of $L/d$ printed on the substrate covering the entire range of permittivity. This model is very accurately compute the resonant frequency for wide range of variation of composite and suspended substrate parameters. The advantage of these models is the mathematical simplicity and low computation cost which is faster than the numerical techniques and the commercially available softwares and directly applied to the CAD programs. The computed values employing the present model are compared with new experimental results and results available in open literature and excellent agreement is revealed. This model will be very useful for designing the MIC on semiconductor materials with $\varepsilon_r \geq 10.0$.

REFERENCES:


