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# Development of User Position Algorithm for Terrestrial Navigation System

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## ABSTRACT

*Terrestrial navigation systems are being developed as an alternative to Global Navigation Satellite Systems (GNSS). Using signals received from ground-based transmitters, a user receiver can estimate its position co-ordinates very accurately. Terrestrial navigation systems are basically ground-based systems which transmit the signals from a very short range, therefore, user position algorithms used in GNSS systems which are based on linearization of pseudorange equations are no longer applicable. The iterative algorithms are based on Taylor series linearization method and essentially requires an initial guess which fails to converge when the initial guess is far away. Therefore, in this paper, a new algorithm is developed which is based on single difference of the pseudorange measurements with highest elevation angle as the reference transmitter selection criteria. This algorithm does not use linearization technique and does not require any initial guess which is an essential requirement for terrestrial navigation systems which are very sensitive to the initial guess. This algorithm is tested using simulated data and position accuracy of 2.5 cm is achieved with Position Dilution of Precision (PDOP) value of 10.37.*

## KEYWORDS

*Terrestrial navigation, ground based transmitter, iterative algorithm, non-iterative solution, SVD*

## INTRODUCTION

The satellite-based navigation systems, popularly called Global Navigation Satellite Systems (GNSS), are used in many services requiring position, navigation and time (PNT). However, alternate positioning systems using ground-based transmitters are being developed worldwide to provide positioning services in specific limited coverage areas. Ground-based navigation systems can potentially provide a standalone navigation service independent of satellite systems and may become a candidate for Alternate Positioning System (APNT).

Ground-based navigation systems aim to provide better position accuracy than GNSS since terrestrial systems do not travel through ionosphere which is one of the largest source of error in GNSS for single frequency users. In addition to that, locations of transmitters can be known with mm level accuracy so orbit /ephemeris errors will also be absent in such systems.

Terrestrial systems are well suited for localized areas such as testing accuracy of unmanned vehicles, mining, aviation etc. using code phase and carrier-phase measurements.

Position determination algorithm used in GNSS is based on linearization of pseudo-range equations. This algorithm is an iterative algorithm and requires an initial guess for converging near to the true position of the user. This position determination algorithm works very well for GNSS signals and converges to true position solution after 3 to 4 iterations.

Authors have tested the traditional iterative PVT (Position Velocity Time) algorithm with the simulated dataset shown in Table 1. having co-ordinates of 10 ground-based transmitters and their pseudoranges. The center of the earth i.e. (0,0,0,0) was taken as an initial guess just like GNSS. We observed that it failed to converge for even after 5 iterations as per the results shown in Table 2.

**Table 1. Terrestrial transmitter co-ordinates and Pseudoranges**

No.	X co-ordinate (m)	Y co-ordinate (m)	Z co-ordinate (m)	Pseudorange (m)
1	1764617.08	5601651.00	2479307.18	60.66
2	1764552.72	5601695.76	2479268.98	65.65
3	1764673.49	5601662.91	2479238.59	65.27
4	1764583.64	5601708.63	2479193.91	73.21
5	1764583.17	5601707.12	2479205.27	64.25
6	1764581.87	5601702.99	2479215.46	55.65
7	1764580.56	5601698.86	2479225.65	48.07
8	1764579.26	5601694.73	2479235.85	42.08
9	1764577.96	5601690.59	2479246.04	38.41
10	1764576.66	5601686.46	2479256.23	37.76
<b>Reference point: (1764609.4583,5601668.9198,2479249.7374)</b>				

**Table 2. Results of Iterative Algorithm**

Iteration	RSS Error (m)
1	7.22e+011
2	2.11e+016
3	7.37e+010
4	8.56e+015
5	1.03e+017

## METHODOLOGY

The idea for the new terrestrial positioning algorithm is inspired from the single difference approach used for carrier phase measurements in GNSS. The pseudorange  $\rho_i$  between user receiver and the  $i^{th}$  ground-based transmitter is evaluated by the equation,

$$\begin{aligned}
 (\rho_i - b)^2 &= (x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 \\
 &\vdots \\
 (\rho_n - b)^2 &= (x_n - x)^2 + (y_n - y)^2 + (z_n - z)^2
 \end{aligned} \tag{1}$$

where  $(x_i, y_i, z_i)$  are the co-ordinates of  $i^{th}$  ground-based transmitter,  $(x, y, z)$  are the co-ordinates of user receiver,  $b$  is the clock bias and  $n$  is the total number of ground-based transmitters.

In this paper, an attempt has been made where one of the  $n$  ground-based transmitters is taken as a reference station and the pseudorange is differenced from the rest of the transmitter's pseudorange. This will eliminate all the possible errors in the pseudorange measurements in the determination of accurate user position.

On simplifying the equations and representing them in matrix form, we get

$$A = 2 \begin{bmatrix} x_i - x_r & y_i - y_r & z_i - z_r & -(r_i - r_r) \\ \vdots & \vdots & \vdots & \vdots \\ x_n - x_r & y_n - y_r & z_n - z_r & -(r_n - r_r) \end{bmatrix} \tag{2}$$

$$B = \begin{bmatrix} (x_i^2 + y_i^2 + z_i^2) - (x_r^2 + y_r^2 + z_r^2) - (r_i^2 - r_r^2) \\ \vdots \\ (x_n^2 + y_n^2 + z_n^2) - (x_r^2 + y_r^2 + z_r^2) - (r_n^2 - r_r^2) \end{bmatrix} \tag{3}$$

$$X = \begin{bmatrix} x \\ y \\ z \\ b \end{bmatrix} \tag{4}$$

where  $(x_r, y_r, z_r)$  are the co-ordinates of the selected reference ground-based transmitter,  $r_r$  is the pseudorange between the reference ground-based transmitter and user receiver and  $i$  varies from 1 to 10. The user position can be calculated by,

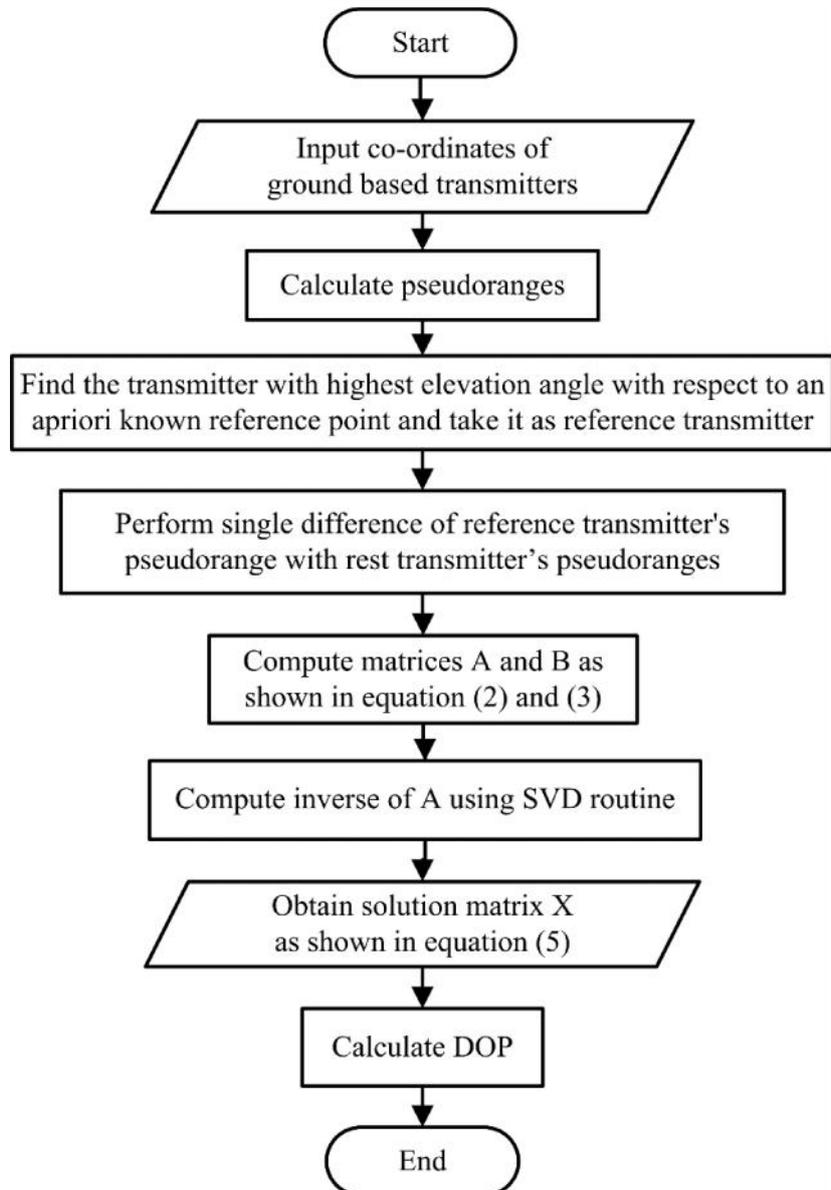
$$X = A^{-1}B \tag{5}$$

The reference transmitter can be selected depending upon the elevation angle between the transmitters and the receiver or by the range residuals measured by the receiver. Here, we have chosen the transmitter with highest elevation angle as reference transmitter with respect to a reference position which is known priory.

## IMPLEMENTATION

Authors have implemented the algorithm as shown in Fig. I in C language. Also a Singular Value Decomposition (SVD) routine for computing the inverse of observation matrix  $A$  has been coded. Using SVD, inverse of any singular matrix can be calculated very efficiently. SVD can be defined for any order of matrices i.e. rectangular or square matrices unlike the common decomposition methods used in Linear Algebra hence making this algorithm applicable for any number of transmitters.

After determining the user position, we have calculated DOP (Dilution of Precision). DOP signifies the impact of the geometric distribution of the transmitters, with respect to the receiver's position, on the position error, and hence on the accuracy of the user positioning [1]. It may be noted that smaller the DOP, better will be the accuracy of the estimated user position.



**Fig. 1: Flowchart of the user position determination**

## RESULTS

From the results shown in Table 2, it can be clearly observed that iterative algorithm is very sensitive to the initial guess in case of ground-based navigation system. Therefore, we propose an algorithm with elevation angle based reference transmitter selection criteria, which is independent of initial guess and converge to the user position with better accuracy.

On testing the proposed algorithm using the dataset shown in Table 1, we observed that it gives better position accuracy compared to the iterative algorithm and major problem of convergence is also resolved. An apriori known reference point is taken for validating the user position obtained by the algorithm. It is to be noted that the highest elevation transmitter is selected with respect to this apriori known reference point as the reference transmitter for the computation.

**Table 3. User Position & Error**

<b>X co-ordinate (m)</b>	<b>Y co-ordinate (m)</b>	<b>Z co-ordinate (m)</b>	<b>RSS Error (m)</b>	<b>PDOP</b>	<b>UERE (m)</b>
1764609.4675	5601668.9437	2479249.7420	0.025	10.37	0.0025

As shown in Table 3, RSS error obtained with respect to the reference location is only 2.5 cm along with Position Dilution of Precision (PDOP) value of 10.37 for the data set shown in Table 1. This shows that a much accurate solution is obtained in comparison to the iterative PVT algorithm. Testing is done with other different datasets as well and the results are found to be consistent.

## CONCLUSION

In this paper, authors present a new positioning algorithm as an alternative to the iterative algorithm based on the elevation angle based reference transmitter selection criteria. It can be observed from the analysis that algorithm works very well and provide very good position accuracy of 2.5 cm with PDOP of 10.37. The algorithm is independent of initial guess as well as it guarantees convergence for nonlinear systems which can be used in terrestrial navigation systems. This algorithm can be used for terrestrial systems developed for coastal regions, aviation, inter-planetary missions etc.

## FUTURE PROSPECTS

Considering the feasibility of the proposed algorithm, a hybrid approach can be considered where the user position determined by the proposed algorithm can be given as an initial guess to the iterative algorithm in order to ensure it converges to the true position.

## ACKNOWLEDGMENTS

The authors would like to thank Director SAC Shri Tapan Mishra for continuous encouragement to carry out this work at SAC. Authors express their sincere gratitude to Associate Director Shri D K Das and support of Division Head, SNTD, Ms Saumi De and Group Director SNGG, Shri S N Satasia. Authors also acknowledge SNTD and SNGG engineers who have provided their support in this activity.

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