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# A Hybrid System Simulation for Formation Control of Wheeled Mobile Robots: An Application of Artificial Potential Field and Kinematic Controller

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## ABSTRACT

*This paper proposes a novel hybrid formation control technique for a group of differentially driven wheeled mobile robots in the Leader-Follower framework. In the proposed method, the leader robot of the group plans its path of navigation by an artificial potential field and the follower robots plan their path in order to follow the leader robot by maintaining a particular formation employing the separation-bearing control  $(l-\epsilon)$ . Thus, the resulting control problem has been transformed as a trajectory tracking control. The path planning of the leader robot in artificial potential field is discrete in nature, whereas, the trajectory tracking of the desired path is continuous in nature. Therefore, a hybrid system, which is the combination of the discrete event and the continuous systems that will act together to perform formation control is been proposed in this paper. The overall formation control technique results into a robust formation control for a group of differentially driven wheeled mobile robots. The effective performance of the proposed method has been verified in simulation.*

## Keywords

**Artificial Potential Field based Navigation, Formation Control, Leader-Follower approach, Trajectory Tracking.**

## INTRODUCTION

Formation control is one of the active research issues in swarm robots. In nature, many examples of formations have been observed in herds of animals, flocks of birds, schools of fish and colonies of bacteria that are capable of maneuvering at high speeds while maintaining the prescribed formation. The most popular one is the V-shape formation formed by geese and various kinds of birds [1]. Swarm robotics is a coordination based approach for multiple robots to cooperatively achieve a single task [3]. In this approach, a group of robots may act together to achieve an assigned task. In addition, most of the times they perform the assigned task in a more reliable, faster, or cheaper way than that of a single robot [3].

Wang [4] developed a strategy for formations of mobile robots where individual position coordinates have been allotted to a particular robot to maintain a specific position in the group with respect to the leader and neighbor. From the Arkin's definition [2], the term formation means that a robot team forms a definite geometric pattern (or called formation shape) and maintains it while moving. In this paper, we have considered the aspects of formation control of ground-based mobile robots.

There are various control strategies, which are implemented to the leader-follower formation scheme which includes input-output linearization [5], Backstepping Based [9] [10], graph theoretic [14] [15], Direct Lyapunov method kinematic control [16], switching strategy [17] and many others. In this method, each robot takes another neighboring robot as a reference point to determine its motion. The referenced robot is called a leader, and the robot following it called a follower.

In all the approaches of leader-follower formation control, it has been assumed that the leader robot knows its path of navigation. That actually makes the fully autonomous operation during the formation control of the swarm difficult. In our approach, we are allowing the leader robot to autonomously plan its path using APF, and the follower robots will track the leader's path to maintain the desired separation distance  $l_{12}^d$  and the bearing angle  $\Theta_{12}^d$ . Therefore, the formation control problem now becomes a trajectory tracking problem. Numerical methods are used in [19], [20] for trajectory tracking of the single mobile robot, where the path the robot is pre-established (either a circular path or eight path). In our case, the orientation of the leader robot changes depending on the number of static obstacles placed in the environment, so that leader robot will adapt the optimum path to reach the goal position in the influence of artificial potential fields.

In practice, real time implementation of continuous-time control strategies in formation control is achieved on a processor with a fast-enough sampling frequency. However, a disadvantage of this approach is that the control depends on the sampling frequency. One way to overcome this problem is using discrete-time control strategies. Unlike the continuous-time case, to the best of our knowledge there are very few works utilizing discrete-time models in formation control [22]. Therefore, we have combined the discrete-time systems and continuous-time systems to perform formation control.

This is the extension of our previous work [21], to the best of the knowledge of the authors, leader-follower formation control employing APF for the fully autonomous path planning of the leader and tracking the leader's path by kinematic principles were never addressed in the literature.

## LEADER-FOLLOWER FRAMEWORK

We are considering the kinematic model described by [5], in which two scenarios for feedback control within the formation are described. In the first scenario, one robot follows another by controlling the relative distance and orientation between the two ( $l-\Theta$ ) and in another scenario, a robot maintains its position in the formation by maintaining a specified distance from two robots, or from one robot and an obstacle in the environment ( $l-l$ ).

We have considered the ( $l-\Theta$ ) formation scheme. In the  $l-\Theta$  control of the two mobile robots, the aim is to maintain a desired length (separation distance),  $l_{12}^d$  and a desired relative angle (bearing angle)  $\Theta_{12}^d$  between the two robots.

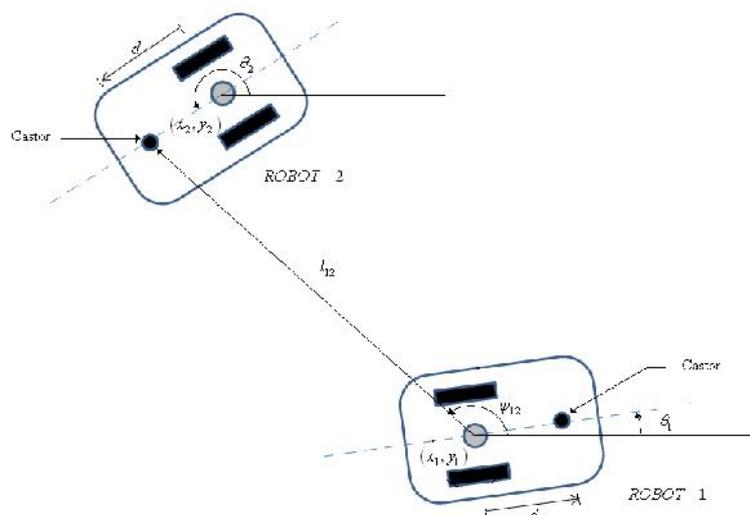


Figure 1: Notation for  $l-\Theta$  control

The kinematic equations for the system of two mobile robots shown in Figure 1 is given by,

$$\begin{aligned} \dot{x}_i &= v_i \cos \theta_i \\ \dot{y}_i &= v_i \sin \theta_i \\ \dot{\theta}_i &= \dot{\phi}_i \end{aligned} \quad (1)$$

The model for leader-follower formation using the  $l-\phi$  formation scheme is given as:

$$\begin{aligned} \dot{x}_2 &= v_2 \cos \alpha_1 - v_1 \cos(\phi_{12} + \alpha_1) + d \dot{\phi}_2 \sin \alpha_1 \\ \dot{y}_2 &= \frac{1}{l_{12}} \{v_1 \sin(\phi_{12} - \alpha_1) - v_2 \sin \alpha_1 + d \dot{\phi}_2 \cos \alpha_1 - l_{12} \dot{\phi}_1\} \\ \dot{\theta}_2 &= \dot{\phi}_2 \end{aligned} \quad (2)$$

Where,  $\alpha_1 = \theta_1 + \phi_{12} - \theta_2$  and  $v_i, \dot{\phi}_i (i=1,2)$  are the linear and angular velocities at the center of the axle of each robot. In order to avoid collisions between robots, we will require that  $l_{12} > d$ , where  $d$  is the distance between the castor wheel and the centre of rear wheels.

From Figure 1,  $(x_1, y_1)$  and  $(x_2, y_2)$  are the Cartesian coordinates of the leader and follower robot respectively. The desired separation distance  $l_{12}^d$  and the desired relative angle (bearing angle)  $\phi_{12}^d$  between the two robots are defined by,

$$\begin{aligned} l_{12}^d x &= x_1 - d \cos \theta_1 - x_2 \\ l_{12}^d y &= y_1 - d \sin \theta_1 - y_2 \\ l_{12}^d &= \sqrt{(l_{12}^d x)^2 + (l_{12}^d y)^2} \\ \phi_{12}^d &= \tan^{-1} \left( \frac{l_{12}^d y}{l_{12}^d x} \right) - \theta_1 + \phi \end{aligned} \quad (3)$$

Where,  $l_{12}^d x$  and  $l_{12}^d y$  are the  $x$  component and  $y$  component of  $l_{12}^d$ .

Similarly, for  $n$  number of leaders and  $m$  number of followers we can write equation (2) in the general form as given below.

$$\begin{aligned} \dot{x}_j &= v_j \cos \alpha_j - v_i \cos(\phi_{ij} + \alpha_j) + d \dot{\phi}_j \sin \alpha_j \\ \dot{y}_j &= \frac{1}{l_{ij}} \{v_i \sin(\phi_{ij} - \alpha_j) - v_j \sin \alpha_j + d \dot{\phi}_j \cos \alpha_j - l_{ij} \dot{\phi}_i\} \\ \dot{\theta}_j &= \dot{\phi}_j \end{aligned} \quad (4)$$

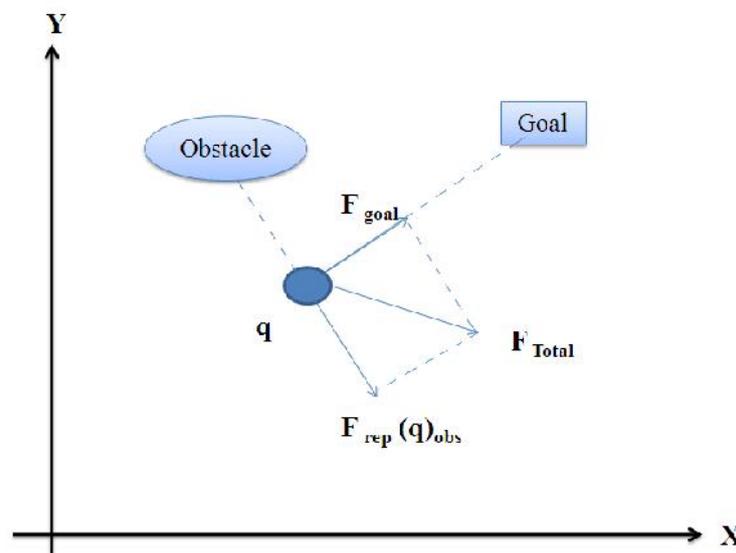
Where,  $\alpha_j = \theta_i + \phi_{ij} - \theta_j, i=1,2,\dots,n$  and  $j=1,2,\dots,m$ . There can be  $n$  number of leaders and  $m$  number of followers for each leader. In our work, we have considered one leader and two followers.

Then the actual position and orientation of the follower  $j$  with respect to leader  $i$  can be defined as.

$$\begin{aligned} x_j &= x_i - d \cos \theta_i + l_{ij} \cos(\phi_{ij} + \theta_i) \\ y_j &= y_i - d \sin \theta_i + l_{ij} \sin(\phi_{ij} + \theta_i) \\ \theta_j &= \theta_i \end{aligned} \quad (5)$$

## ARTIFICIAL POTENTIAL FUNCTION FOR PATH PLANNING OF THE LEADER ROBOT

The APF framework is widely adopted in the mobile robot navigation and control. The main advantage of using the APF based approach is that it is convenient for computation in real-time and can handle the dynamics of the robot [6]. In this work, a modified form of the simple artificial potential function, as described by Khatib [7], has been introduced. Here, a WMR has been mathematically modeled as a moving particle inside an artificial potential field that is generated by superposing an attractive potential which pulls the robot to a goal configuration and a repulsive potential that pushes the robot away from obstacles, similar to that described by Khatib [7]. However, the physical dimension of the WMR has been taken into account by introducing the term  $s$  as the distance between the centre of gravity of the obstacle and the centre of gravity of the WMR into the artificial potential function to avoid collision. The negative gradient of the generated global potential field is interpreted as an artificial force acting on the robot and dictating its motion.



**Figure 2: Moving direction of robot in Artificial Potential Field**

Artificial potential field (APF) two has two kinds of potential sources: gravitation pole and repulsion pole. The target is the gravitation pole, and the obstacle is the repulsion pole. They jointly produce the artificial potential field. As shown in Figure. 2, the negative gradient of the APF is the moving direction of the robot in the system.

The target gravitation and obstacle repulsion in APF are defined as:

Let  $q$  be the position of the robot,  $\dots(q, g)$  be the distance between the robot and the target  $g$ , the gravitational potential field  $U_g$  and gravitation  $F_g$  at robot  $q$  are defined as:

$$U_g(q) = \frac{1}{2} \langle \dots^2(q, g) \rangle \quad (6)$$

$$F_g(q) = \langle \dots(g, q) \rangle \quad (7)$$

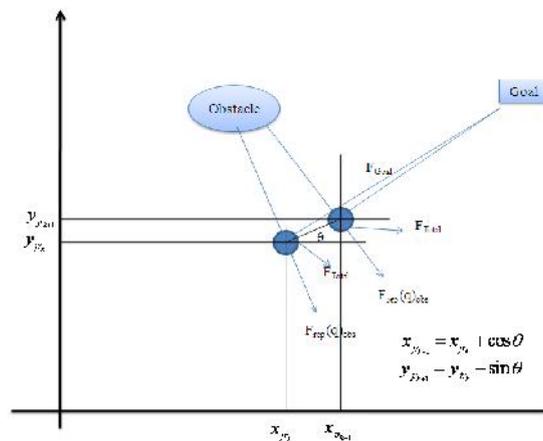
Let  $q_{obsj}$  ( $j=1, \dots, m$ ) be the position of the  $j^{\text{th}}$  obstacle,  $\dots(q, q_{obsj})$  be the distance between robot  $q$  and the  $q_{obsj}$ , the repulsive potential field  $U_{rep}(q)$  and gravitation  $F_{rep}(q_{obsj})$  at the robot  $q$  are defined as:

$$U_{rep}(q) = \begin{cases} \frac{1}{2} \left( \frac{1}{\dots(q, q_{obsj})} - \frac{1}{\dots_s} \right)^2 & \text{if } \dots(q, q_{obsj}) \leq \dots_s \\ 0 & \text{if } \dots(q, q_{obsj}) > \dots_s \end{cases} \quad (8)$$

$$F_{rep}(q_{obs}) = \begin{cases} \left( \frac{1}{\dots(q, q_{obsj})} - \frac{1}{\dots_s} \right)^2 \frac{1}{\dots^2(q, q_{obsj})} \nabla \dots(q, q_{obsj}) & \text{if } \dots(q, q_{obsj}) \leq \dots_s \\ 0 & \text{if } \dots(q, q_{obsj}) > \dots_s \end{cases} \quad (9)$$

So the resultant force of robot  $q$  in the APF is:

$$F_{total}(q) = F_g(q) + \sum_{j=1}^m F_{rep}(q_{obsj}) \quad (10)$$



**Figure 3: Geometric representation of motion of the robot in artificial potential field.**

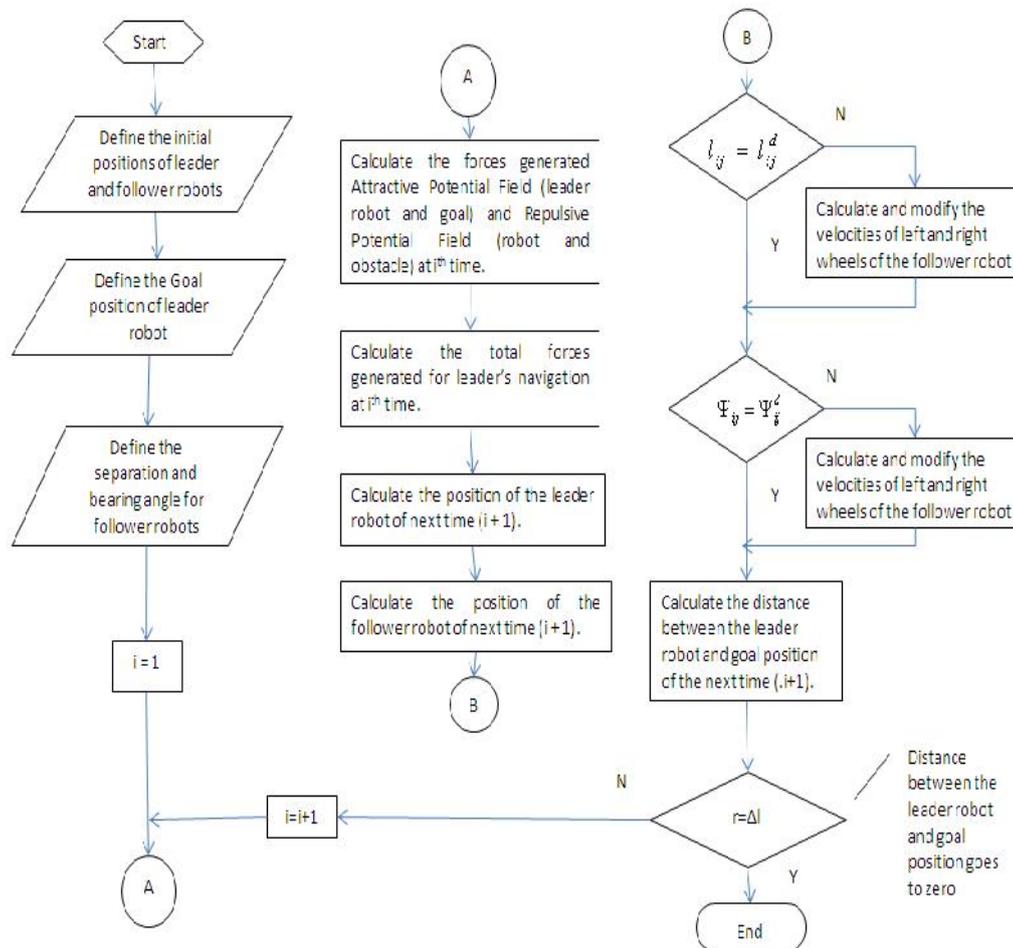
Figure 3 shows the analytical representation of how the position of the mobile robot at the next time step can be determined when it is navigating within the APF. From the above analytical interpretation, we can get the desired  $x_{j_{k+1}}$  and  $y_{j_{k+1}}$  positions of the leader robot for the next time step, and the orientation  $\theta_{j_{k+1}}$  can be generated kinematically. Therefore, we are having the desired  $(x_{j_{k+1}}, y_{j_{k+1}}, \theta_{j_{k+1}})$  of the leader robot for the next time step in discrete form.

By reducing the time step to the very small value, the desired or reference position and orientation  $(x_{j_{k+1}}, y_{j_{k+1}}, \theta_{j_{k+1}})$  of the leader robot can be approximated to as a continuous one as shown in the subsequent sections. The discrete form is useful for computer simulation, however, for the purpose of mathematical analysis and synthesis we have adopted the continuous time representation of the desired or reference position and orientation of the leader robot.

### TRAJECTORY TRACKING

The main issues found in mobile robot control is trajectory tracking. In general, the objective is that the wheeled mobile robot (WMR) reaches the Cartesian position  $(x, y)$  with a pre-established orientation (it is either a circular path or eight path as in [18],[19] and [20]) for each sampling period. These combined actions result in tracking the desired trajectory of the mobile robot. In order to achieve this objective, only two control variables are available: the linear and angular velocity of the robot,  $v$  and  $w$  respectively. In our work, the leader's path is dictated by the artificial potential fields, and the orientation may change as per the static obstacles placed in the environment, and the follower robots will track the leader's path by maintaining the desired separation distance  $l_{12}^d$  and the bearing angle  $\alpha_{12}^d$ . Assumptions are made that the follower robots can communicate with the leader robot to get the position  $(x, y)$  and orientation  $\theta$  of

the leader robot; so that they can maintain desired separation distance and relative bearing angle and the follower robots are also capable of avoiding collision with the obstacles and other robots.



**Figure 4: Flow Chart for Formation Control**

The formation control scheme is described in the flow chart as shown in Figure 4. Since, the next position and orientation of the leader robot are dictated by the artificial potential field, we have  $(x, y)$  i.e. position and  $\theta$  i.e. orientation of the leader robot  $l_{ij}$  and  $\Phi_{ij}$  i.e. the separation distance and the bearing angle respectively, which the follower robot has to maintain while following the leader robot.

### KINEMATIC CONTROLLER

To avoid collisions, separation distances are measured from the back of the leader to the front of the follower, and the kinematic equations for the front of the  $j^{th}$  follower robot can be written as:

$$\begin{bmatrix} \dot{x}_j \\ \dot{y}_j \\ \dot{\theta}_j \end{bmatrix} = S_j(q_j)v_j = \begin{bmatrix} \cos \theta_j & -d \sin \theta_j \\ \sin \theta_j & -d \cos \theta_j \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_j \\ \dot{\theta}_j \end{bmatrix} \quad (11)$$

Where  $d$  is the distance from the rear axle to the front of the robot,

Now, we can write the following error system model [9] from the reference generated for the leader WMR, and consequently for the follower WMRs, depending upon attraction and repulsion forces experienced by the leader WMR in the APF as:

$$\begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} \cos \theta_j & \sin \theta_j & 0 \\ -\sin \theta_j & \cos \theta_j & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{jr} - x_j \\ y_{jr} - y_j \\ \theta_{jr} - \theta_j \end{bmatrix} \quad (12)$$

$$\dot{x}_j = v_j \cos \theta_j, \dot{y}_j = v_j \sin \theta_j, \dot{\theta}_j = \dot{\theta}_j, \dot{\phi}_j = [\dot{x}_j \quad \dot{y}_j \quad \dot{\theta}_j]^T \quad (13)$$

where  $x_j, y_j,$  and  $\theta_j$  are actual position and orientation of the robot, and  $x_{jr}, y_{jr},$  and  $\theta_{jr}$  are the positions and orientation of the reference or leader robot  $j$  depending upon attraction and repulsion forces experienced by the leader WMR in the APF.

And the error rate becomes

$$\begin{bmatrix} \dot{e}_{j1} \\ \dot{e}_{j2} \\ \dot{e}_{j3} \end{bmatrix} = v \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + w \begin{bmatrix} e_{j2} \\ -e_{j1} \\ 1 \end{bmatrix} + \begin{bmatrix} v_r \cos e_{j3} \\ v_r \sin e_{j3} \\ w_r \end{bmatrix} \quad (14)$$

Following the pioneering works of Fierro and Lewis [9] and Kanayama and Kimura [24], the velocity control input for the kinematic model of WMR, as given in equation (11), to achieve stable tracking can be computed as following:

$$v_c = \begin{bmatrix} v_r \cos e_{j3} + k_1 e_1 \\ w_r + k_2 v_r e_{j2} + k_3 v_r \sin e_{j3} \end{bmatrix} \quad (15)$$

The necessary proof of stability of the control input of equation (15) can be found in [9],[24] in detail. Now, let the WMR  $i$  is acting as the leader and the WMR  $j$  is acting as the follower, and  $(x_{jr}, y_{jr})$  are defined as points at a distance  $l_{ijd}$  and a desired angle  $\Psi_{ijd}$  from the leader robot.

Therefore, the basic tracking control problems can be extended to a formation control as follows:

$$\begin{bmatrix} \dot{x}_j \\ \dot{y}_j \\ \dot{\theta}_j \end{bmatrix} = \begin{bmatrix} \cos \theta_i & -d \sin \theta_i \\ \sin \theta_i & d \cos \theta_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix} \quad (16)$$

$$x_j = x_i - d \cos \theta_i + l_{ij}^d \cos(\Psi_{ij}^d + \theta_i)$$

$$y_j = y_i - d \sin \theta_i + l_{ij}^d \sin(\Psi_{ij}^d + \theta_i)$$

$$\theta_j = \theta_i \quad (17)$$

and

$$v_j = [v_i \quad \dot{\theta}_i]^T \quad (18)$$

Then the actual position and orientation of the follower  $j$  with respect to leader  $i$  can be defined as:

$$\begin{aligned}
 x_j &= x_i - d \cos \theta_i + l_{ij} \cos(\Psi_{ij} + \theta_i) \\
 y_j &= y_i - d \sin \theta_i + l_{ij} \sin(\Psi_{ij} + \theta_i) \\
 \theta_j &= \theta_i
 \end{aligned}
 \tag{19}$$

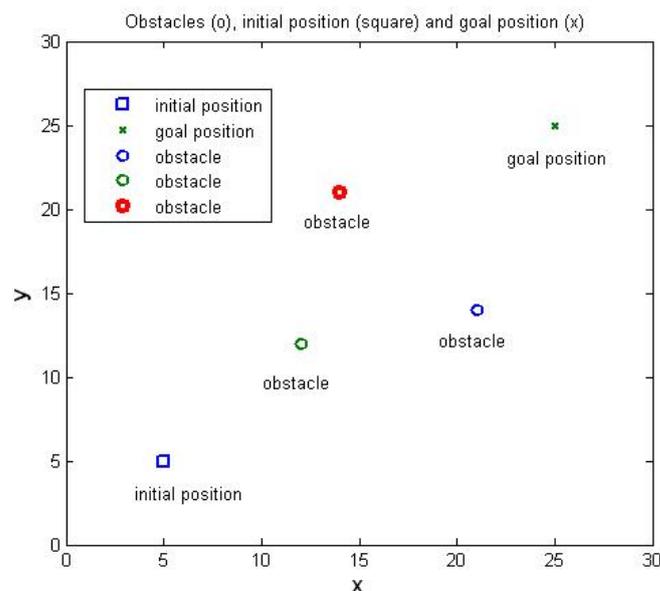
Where  $l_{ij}$  and  $\Psi_{ij}$  is the actual separation and bearing of the follower  $j$ . In our work we have considered one leader robot and two follower robots.

### SIMULATION RESULTS

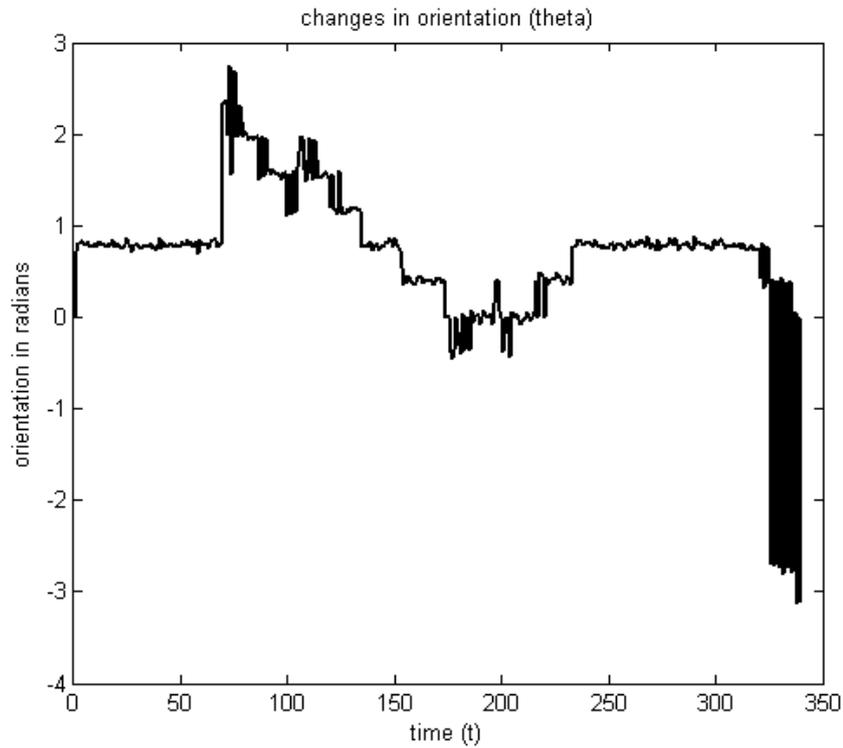
A triangular formation of three identical mobile robots is considered where the leader's path of navigation is dictated by the artificial potential field and is considered as the desired formation trajectory. The simulation of the formation control has been carried out in MATLAB 7.10.0(R2010a) with three obstacles in the environment. The initial position of the leader and two followers are defined as (5, 5), (5, 3) and (3, 5) units respectively. The goal position of the leader robot is defined as (25, 25) units. The positions of the obstacles are defined as (21, 14), (12, 12) and (14, 21) units. The leader's path is dictated by the artificial potential fields as described in section III is shown in red line in figure 7, The path of the follower robots, while they chase the leader, is shown in blue lines. Robots positions after regular time intervals are also shown in figure 7, these paths of leader and followers are the desired trajectory, and the kinematic controller will make the group of the mobile robots to track the desired trajectory.

Based on the simulation results, it can be seen from figure 7, that as the leader robot navigates itself by artificial potential field, its locomotion control is stable and robust against collision while reaching to the goal position and the followers are following the leader's path effectively.

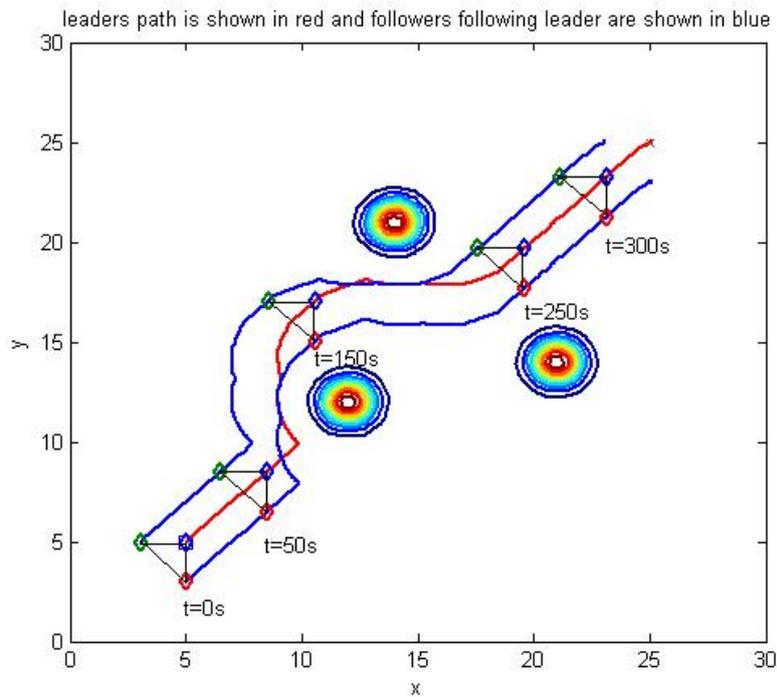
In figure 6, we can see how the orientation of the leader robot is changing whenever there are obstacles in the environment. The trajectory tracking of WMR, by using kinematic controller can be seen in figure 8. In case of the static obstacles the robots will move in the uniform speed, this can be seen in the velocity profile as shown in figure 9 (the linear velocities  $v$  are almost constant and the angular velocities  $w$  are somewhat fluctuating, whenever there is change in the orientation of the wheeled mobile robot). Figure 10 shows the convergence of error of equation (12).



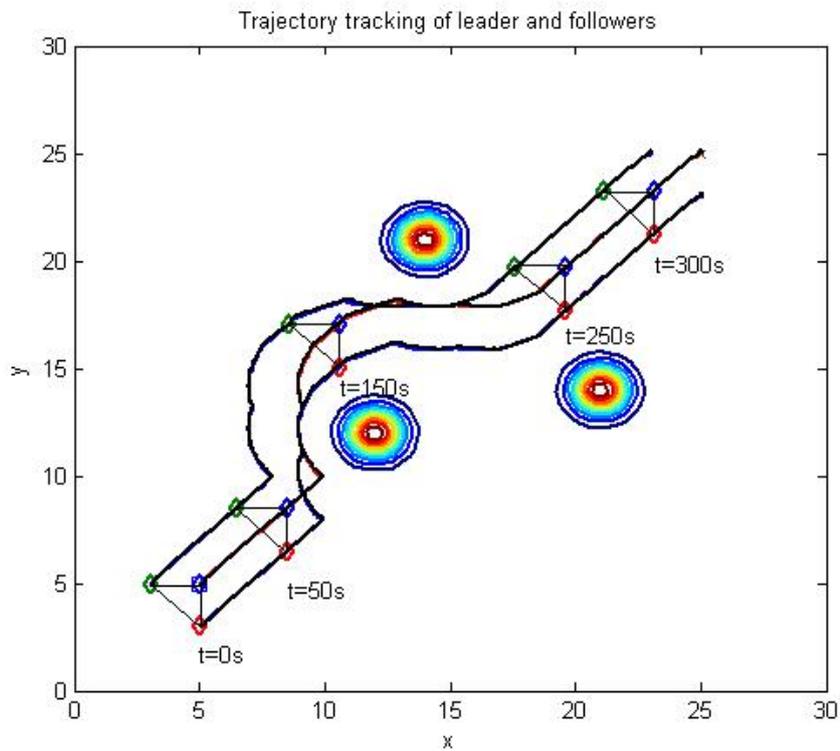
**Figure 5: Plot showing initial position, goal position and obstacles**



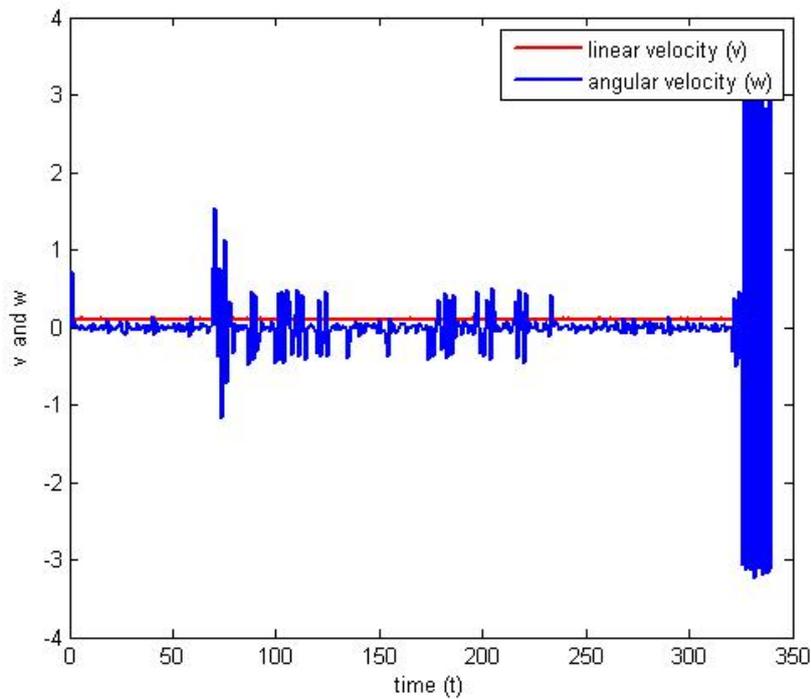
**Figure 6: Plot showing the changes in orientation of the leader robot while moving in the environment with obstacles.**



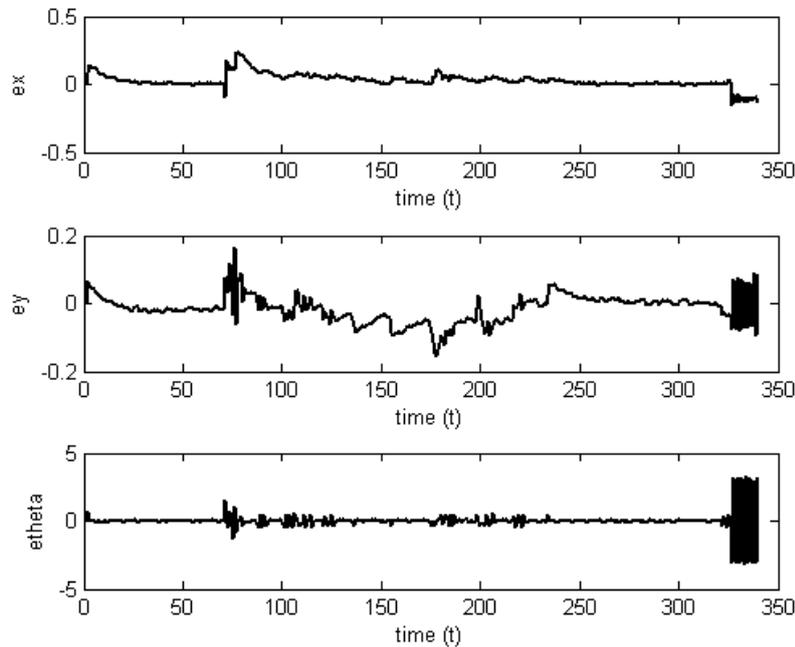
**Figure 7: Plot showing the desired trajectories**



**Figure 8: Trajectory tracking of leader and followers**



**Figure 9: Velocity profile of leader robot**



**Figure 10: Convergence of error variables**

## CONCLUSION AND FUTURE WORK

Based on the simulation results, it can be seen that as the leader robot navigates itself by artificial potential field, its locomotion control is stable and robust against collision while reaching to the goal position and the followers are following the leader's path effectively.

Thus, from the simulation results, we can see that the desired formation control using leader-follower scheme is effective. From figure 10, it can also be seen that the error dynamics described in this paper tends to zero as time tends to infinity. In this work we have considered only the kinematics of the differential-drive wheeled mobile robot, so in future we will include the dynamics as it is well known that due to non-holonomic constraint of the differential-drive mobile robot, the perfect velocity tracking will not hold, we will have to consider the torque as well.

## ACKNOWLEDGMENT

This work was inspired by the prior research work of Kevin Passino and O. Khatib [7]. The authors wish to acknowledge their contributions. The authors are also grateful to All India Council for Technical Education, Government of India for allowing the corresponding author to pursue PhD. under Quality improvement program in Jadavpur University, Kolkata, India. The authors are also thankful to the head of department, Electrical Engineering, Jadavpur University, Kolkata, India for allowing this research work to be conducted in its Mechatronics Laboratory.

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