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On e-connectedness in Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper the concept of types of intuitionistic fuzzy e-connected and intuitionistic fuzzy e-extremally disconnected in intuitionistic fuzzy topological spaces are introduced and studied. Here we introduce the concepts of intuitionistic fuzzy eC_5 -connectedness, intuitionistic fuzzy eC_S -connectedness, intuitionistic fuzzy eC_M -connectedness, intuitionistic fuzzy e-strongly connectedness, intuitionistic fuzzy e-super connectedness, intuitionistic fuzzy eC_i -connectedness $(i = 1,2,3,4)$, and obtain several properties and some characterizations concerning connectedness in these spaces.

Keywords and phrases: IFe ***-connected,*** $IFeC_5$ ***-connected,*** IFe ***-strongly connected,*** $IFeC_M$ ***-disconnected,*** $IFeC_s$ ***-connected,*** IFe ***-extremally disconnected.***

AMS (2000) subject classification: 54A40, 54A99, 03E72, 03E99.

1. Introduction

Ever since the introduction of fuzzy sets by Zadeh [14], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by Change [2]. Atanassov [1] introduced the notion of intuitionistic fuzzy sets, Coker [3] introduced the intuitionistic fuzzy topological spaces. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Turnali and Coker [13]. The initiations of e-open sets, e-continuity and e-compactness in topological spaces are due to Ekici [5, 6, 7]. In fuzzy topology, e-open sets were introduced by Seenivasan in 2014 [10]. Sobana et.al [11] were introduced the concept of fuzzy e-open sets, fuzzy e-continuity and fuzzy e -compactness in intuitionistic fuzzy topological spaces (briefly., IFTS's). In this paper we have introduced some types of intuitionistic fuzzy e-connected and intuitionistic fuzzy e-extremally disconnected spaces and studied their properties and characterizations.

2. Preliminaries

Definition 2.1 [1] Let X be a nonempty fixed set and I be the closed interval in $[0,1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form $A = \{< x, «_A(x),¬_A(x) >; x \in X\}$ where the mappings $«_A(x) : X \to I$ and $¬_A(x) : X \to I$ denote the degree of membership (namely) $«_A(x)$ and the degree of non membership (namely) $¬_A(x)$ for each element $x \in X$ to the set A respectively, and $0 \le «_A(x) + ¬_A(x) \le 1$ for each $x \in X$.

Definition 2.2 [1] Let A and B are intuitionistic fuzzy sets of the form $A = \{< x, «_A(x),¬_A(x) >: x \in X\}$ *and* $B = \{< x, «_B(x),¬_B(x) >: x \in X\}$. *Then*

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1. $A \subseteq B$ if and only if $«_A(x) \le «_B(x)$ and $¬_A(x) \ge ¬_B(x)$;
2. $\overline{A}\,(or A^c) = \{< x, ¬_A(x), «_A(x) >: x \in X\}$;
3. $A \cap B = \{< x, «_A(x) \wedge «_B(x), ¬_A(x) \vee ¬_B(x) >: x \in X\}$;
4. $A \cup B = \{< x, «_A(x) \vee «_B(x), ¬_A(x) \wedge ¬_B(x) >: x \in X\}$;
5. $[\,]A = \{< x, «_A(x), 1 - «_A(x) >: x \in X\}$;
6. $\langle\,\rangle A = \{< x, 1 - ¬_A(x), ¬_A(x) >: x \in X\}$;

We will use the notation $A = \{< x, «_A, ¬_A >: x \in X\}$ instead of $A = \{< x, «_A(x), ¬_A(x) >: x \in X\}$.

Definition 2.3 [3] $0_{\sim} = \{< x, 0, 1 >; x \in X\}$ and $1_{\sim} = \{< x, 1, 0 >; x \in X\}$. Let $\sim,\ \in [0,1]$ such that $\sim + \ \le 1$. An intuitionistic fuzzy point $(IFP)_{p_{(\sim,\ )}}$ is intuitionistic fuzzy set defined by $p_{(\sim,\ )}(x) = \begin{cases} (\sim,\ ) & \text{if } x = p \\ (0,1) & \text{otherwise} \end{cases}$

Definition 2.4 [3] An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family T of intuitionistic fuzzy sets in X satisfying the following axioms:

1. $0_{\sim}, 1_{\sim} \in T$;
2. $G_1 \cap G_2 \in T$, for any $G_1, G_2 \in T$;
3. $\cup G_i \in T$ for any arbitrary family $\{G_i; i \in J\} \subseteq T$.

In this paper by (X,T) or simply by X we will denote the intuitionistic fuzzy topological space(IFTS). Each IFS which belongs to T is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.5 [8] Let $p_{(\sim,\ )}$ be an IFP in IFTS X. An IFS A in X is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(\sim,\ )}$ if there exists an IFOS B in X such that $p_{(\sim,\ )} \in B \subseteq A$.

Let X and Y are two non-empty sets and $f : (X,T) \to (Y,\pm)$ be a function [3]. If $B = \{< y, «_B(y), ¬_B(y) >; y \in Y\}$ is an IFS in Y, then the pre-image of B under f is denoted and defined by $f^{-1}(B) = \{< x, f^{-1}(«_B(x)), f^{-1}(¬_B(x)) >; x \in X\}$. Since $«_B(x), ¬_B(x)$ are fuzzy sets, we explain that $f^{-1}(«_B(x)) = «_B(x)(f(x)), f^{-1}(¬_B(x)) = ¬_B(x)(f(x))$.

Definition 2.6 [3] Let (X,T) be an IFTS and $A = \{< x, «_A, ¬_A >; x \in X\}$ be an IFS in X. Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior of A are defined by

1. $cl(A) = \bigcap \{C : C \text{ is an IFCS in } X \text{ and } C \supseteq A\}$;

2. $int(A) = \bigcup\{D : D \text{ is an IFOS in } X \text{ and } D \subseteq A\}$;

It can be also shown that $cl(A)$ is an IFCS, $int(A)$ is an IFOS in X and A is and IFCS in X if and only if $cl(A) = A$; A is an IFOS in X if and only if $int(A) = A$

Proposition 2.1 [3] Let (X,T) be an IFTS and A, B be intuitionistic fuzzy sets in X. Then the following properties hold:

1. $cl(\overline{A}) = \overline{(int(A))}, int(\overline{A}) = \overline{(cl(A))}$;
2. $int(A) \subseteq A \subseteq cl(A)$.

Definition 2.7 *Let A be IFS in an IFTS (X,Ψ). A is called an*

1. intuitionistic fuzzy regular open set [12] (briefly $IFROS$) if $A = intcl(A)$ and intuitionistic fuzzy regular closed set (briefly $IFRCS$) if $A = clint(A)$

2. intuitionistic fuzzy e-regular open set [11] (briefly $IFeROS$) if $A = eint(ecl(A))$ and intuitionistic fuzzy e-regular closed set (briefly $IFeRCS$) if $A = ecl(eint(A))$

Definition 2.8 [12] Let (X,Ψ) be an IFTS and $A =< x, \mu_A(x), \nu_A(x) >$ be a IFS in X. Then the fuzzy δ closure of A are denoted and defined by $cl_\delta(A) = \cap\{K : K$ is an IFRCS in X and $A \subseteq K\}$ and $int_\delta(A) = \cup\{G : G$ is an IFROS in X and $G \subseteq A\}$.

Definition 2.9 [11] Let A be an IFS in an IFTS (X,Ψ). A is called an intuitionistic fuzzy e-open set (IFeOS, for short) in X if $A \subseteq clint_\delta(A) \cup intcl_\delta(A)$

Definition 2.10 [11] Let (X,Ψ) be an IFTS and $A =< x, \mu_A, \nu_A >$ be an IFS in X. Then the intuitionistic fuzzy e-interior and intuitionistic fuzzy e-closure are defined and denoted by:

$cl_e(A) = \cap\{K : K$ is an IFeCS in X and $A \subseteq K\}$

and

$int_e(A) = \cup\{G : G$ is an IFeOS in X and $G \subseteq A\}$.

It is clear that A is an IFeCS (IFeOS) in X iff $A = cl_e(A)(A = int_e(A))$.

Definition 2.11 [3] Let (X,Ψ) and (Y,Φ) be IFTS's. A function $f : (X,\Psi) \to (Y,\Phi)$ is called intuitionistic fuzzy continuous(resp., e-continuous [11]) if $f^{-1}(B)$ is an IFOS(resp., IFeOS)in X for every $B \in \Phi$.

Lemma 2.1 [13]

1. $A \cap B = 0_\sim \Rightarrow A \subseteq \overline{B}$.

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2. $A \not\subseteq B \Rightarrow A \cap B \neq 0_{\sim}$

Definition 2.12 [4] Two intuitionistic fuzzy sets A and B are said to be q-coincident (AqB) if and only if there exists an element $x \in X$ such that $«_A(x) > \neg_B(x)$ or $\neg_A(x) < «_B(x)$. If A and B are said to be not q-coincident $(A\bar{q}B)$ if and only if $A \subseteq B$.

Definition 2.13 [9] An IFTS (X, T) is called intuitionistic fuzzy C_5-connected between two intuitionistic fuzzy sets A and B if there is no IFOS E in (X, T) such that $A \subseteq E$ and $E\bar{q}B$.

3. Types of intuitionistic fuzzy e-connectedness in intuitionistic fuzzy topological spaces

Definition 3.1 An IFTS (X, T) is IF e-disconnected if there exists intuitionistic fuzzy e-open sets P, Q in X, $P \neq 0_{\sim}, Q \neq 0_{\sim}$ such that $P \cup Q = 1_{\sim}$ and $P \cap Q = 0_{\sim}$. If X is not IF e-disconnected then it is said to be IF e-connected.

Example 3.1 Let $X = \{a, b\}, T = \{0_{\sim}, 1_{\sim}, P\}$ where
$P = \{< x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.7}, \frac{b}{0.5}) >, x \in X\}, Q = \{< x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.8}, \frac{b}{0.5}) >, x \in X\}$, P and Q are intuitionistic fuzzy e-open sets in X, $P \neq 0_{\sim}, Q \neq 0_{\sim}$ and $P \cup Q = P \neq 1_{\sim}, P \cap Q = Q \neq 0_{\sim}$. Hence X is IF e-connected.

Example 3.2 In Example 3.1, Consider the intuitionistic fuzzy sets
$Q = \{< x, (\frac{a}{0}, \frac{b}{1}), (\frac{a}{1}, \frac{b}{0}) >, x \in X\}, R = \{< x, (\frac{a}{1}, \frac{b}{0}), (\frac{a}{0}, \frac{b}{1}) >, x \in X\}$, Q and R are intuitionistic fuzzy e-open sets in X, $Q \neq 0_{\sim}, R \neq 0_{\sim}$ and $Q \cup R = 1_{\sim}, Q \cap R = 0_{\sim}$. Hence X is IF e-disconnected.

Definition 3.2 An IFTS (X, T) is IF eC_5-disconnected if there exists IFS P in X, which is both IF eOS and IF eCS such that $P \neq 0_{\sim}$, and $P \neq 1_{\sim}$. If X is not IF eC_5-disconnected then it is said to be IF eC_5-connected.

Example 3.3 In Example 3.1, Consider the intuitionistic fuzzy sets
$Q = \{< x, (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.3}) >, x \in X\}$ Q is an IF eOS in X, but Q is not IF eCS since
$int(cl_{¢}(Q)) \cap cl(int_{¢}(Q)) \not\subseteq Q$

Example 3.4 In Example 3.1, Consider the intuitionistic fuzzy sets $Q = \{< x, (\frac{a}{0}, \frac{b}{1}), (\frac{a}{1}, \frac{b}{0}) >, x \in X\}$, Q

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is an intuitionistic fuzzy e-open sets in X. Also Q is IF e CS since $int(cl_{¢}(Q)) \wedge cl(int_{¢}(Q)) = 0_{\sim} \leq Q$. Hence there exists an IFS Q in X such that $1_{\sim} \neq Q \neq 0_{\sim}$ which is both IF e OS and IF e CS in X. Thus X is IF eC_5-disconnected.

Proposition 3.1 IF eC_5-connectedness implies IF e-connectedness.

Proof. Suppose that there exists nonempty intuitionistic fuzzy e-open sets P and Q such that $P \cup Q = 1_{\sim}$ and $P \cap Q = 0_{\sim}$ (IF e-disconnected) then $\mu_P \vee \mu_Q = 1, \nu_P \wedge \nu_Q = 0$ and $\mu_P \vee \mu_Q = 0, \nu_P \wedge \nu_Q = 1$. In other words $\overline{Q} = P$. Hence P is IF e-clopen which implies X is IF eC_5-disconnected.

But the converse need not be true as shown by the following example.

Example 3.5 In Example 3.1, Consider the intuitionistic fuzzy sets

$Q = \{< x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.8}, \frac{b}{0.5}) >, x \in X\}$, P and Q are IF e OS in X. Also $P \cup Q \neq 1_{\sim} = P, P \cap Q \neq 0_{\sim} = Q$. Hence X is IF e-connected. Since IFS P is both IF e OS and IF e CS in X, X is IF eC_5-disconnected.

Proposition 3.2 Let $f : (X,T) \to (Y,S)$ be a IF e-irresolute surjection, (X,T) is an IF e-connected, then (Y,S) is IF e-connected.

Proof. Assume that (Y,S) is not IF e-connected then there exists nonempty intuitionistic fuzzy e-open sets P and Q in (Y,S) such that $P \cup Q = 1_{\sim}$ and $P \cap Q = 0_{\sim}$. Since f is IF e-irresolute mapping, $R = f^{-1}(P) \neq 0_{\sim}, U = f^{-1}(Q) \neq 0_{\sim}$ which are intuitionistic fuzzy e-open sets in X. And $f^{-1}(P) \cup f^{-1}(Q) = f^{-1}(1_{\sim}) = 1_{\sim}$ which implies $R \cup U = 1_{\sim}$. $f^{-1}(P) \cap f^{-1}(Q) = f^{-1}(0_{\sim}) = 0_{\sim}$ which implies $R \cap U = 0_{\sim}$. Thus X is IF e-disconnected, which is a contradiction to our hypothesis. Hence Y is IF e-connected.

Proposition 3.3 (X,T) is IF eC_5-connected iff there exists no nonempty intuitionistic fuzzy e-open sets P and Q in X such that $P = \overline{Q}$

Proof. Suppose that P and Q are intuitionistic fuzzy e-open sets in X such that $P \neq 0_{\sim} \neq Q$ and $P = \overline{Q}$. Since $P = \overline{Q}, \overline{Q}$ is an IF e OS and Q is an IF e CS, and $P \neq 0_{\sim}$ implies $Q \neq 1_{\sim}$. But this is a contradiction to the fact that X is IF eC_5-connected. Conversely, let P be both IF e OS and IF e CS in X such that $0_{\sim} \neq P \neq 1_{\sim}$. Now take $Q = \overline{P}$. Q is an IF e OS and $P \neq 1_{\sim}$ which implies $Q = \overline{P} \neq 0_{\sim}$ which is a contradiction.

Definition 3.3 An IFTS (X,T) is IF e-strongly connected if there exists no nonempty IF e CS P and Q in X such that $\mu_P + \mu_Q \subseteq 1, \nu_P + \nu_Q \supseteq 1$

In otherwords, an IFTS (X,T) is IF e-strongly connected if there exists no nonempty IF e CS P and Q in

X such that $P \cap Q = 0_{:}$.

Proposition 3.4 An IFTS (X, T) is IF e-strongly connected if there exists no IF eOS P and Q in X, $P \neq 1_{:} \neq Q$ such that $\text{«}_P + \text{«}_Q \supseteq 1, \neg_P + \neg Q \subseteq 1$

Example 3.6 In Example 3.1, Consider the intuitionistic fuzzy sets $Q = \{< x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.8}, \frac{b}{0.5}) >, x \in X\}$, P is an IF eOS in X. P and Q is an IF eOS in X since $Q \subseteq cl(int_{¢}(Q) \cup int(cl_{¢}(Q))$. Also $\text{«}_P + \text{«}_Q \subseteq 1, \neg_P + \neg_Q \supseteq 1$. Hence X is IF e-strongly connected.

Proposition 3.5 Let $f : (X, T) \to (Y, S)$ be a IF e-irresolute surjection. If X is an IF e-strongly connected, then so is Y.

Proof. Suppose that Y is not IF e-strongly connected then there exists IF eCS C and D in Y such that $C \neq 0_{:}, D \neq 0_{:}, C \cap D = 0_{:}$. Since f is IF e-irresolute, $f^{-1}(C), f^{-1}(D)$ are IF e CSs in X and $f^{-1}(C) \cap f^{-1}(D) = 0_{:}, f^{-1}(C) \neq 0_{:}, f^{-1}(D) \neq 0_{:}$. (If $f^{-1}(C) = 0_{:}$, then $f(f^{-1}(C)) = C$ which implies $f(0_{:}) = C$. So $C = 0_{:}$ a contradiction) Hence X is IF e-strongly disconnected, a contradiction. Thus (Y, S) is IF e-strongly connected.

IF e-strongly connected does not imply IF eC_5-connected, and IF eC_5-connected does not imply IF e-strongly connected. For this purpose we see the following examples:

Example 3.7 In Example 3.1, Consider the intuitionistic fuzzy sets $Q = \{< x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.8}, \frac{b}{0.5}) >, x \in X\}$, X is IF eC_5-connected. Since $Q \subseteq cl(int_{¢})(Q) \cup int(cl_{¢}(Q))$. Also $\text{«}_P + \text{«}_Q \subseteq 1, \neg_P + \neg_Q \supseteq 1$. Hence X is IF e-strongly connected. But X is not IF eC_5-connected, since Q is both IF eOS and IF eCS in X.

Example 3.8 In Example 3.1, Consider the intuitionistic fuzzy sets $Q = \{< x, (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.3}) >, x \in X\}, R = \{< x, (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}) >, x \in X\}$, X is IF eC_5-connected. But X is not IF e-strongly connected since Q and R are intuitionistic fuzzy e-open sets in X such that $\text{«}_Q + \text{«}_R \supseteq 1, \neg_Q + \neg_R \subseteq 1$.

Definition 3.4 P and Q are non-zero intuitionistic fuzzy sets in (X, T). Then P and Q are said to be

1. IF e-weakly separated if $eclP \subseteq \overline{Q}$ and $eclQ \subseteq \overline{P}$.
2. IF e-q-separated if $(eclP) \cap Q = 0_{:} = P \cap (eclQ)$.

Definition 3.5 An IFTS (X,T) is said to be IF eC_s-disconnected if there exists IF e-weakly separated non-zero intuitionistic fuzzy sets P and Q in (X,T) such that $P \cup Q = 1_{\sim}$.

Example 3.9 In Example 3.1, Consider the intuitionistic fuzzy sets $Q = \{< x, (\frac{a}{0}, \frac{b}{1}), (\frac{a}{1}, \frac{b}{0}) >, x \in X\}, R = \{< x, (\frac{a}{1}, \frac{b}{0}), (\frac{a}{0}, \frac{b}{1}) >, x \in X\}$, Q and R are intuitionistic fuzzy e-open sets in X, $ecl(Q) \subseteq R$ and $ecl(R) \subseteq Q$. Hence Q and R are IF e-weakly separated and $Q \cup R = 1_{\sim}$. So X is IF eC_s-disconnected.

Definition 3.6 An IFTS (X,T) is said to be IF eC_M-disconnected if there exists IF e-q-separated non-zero intuitionistic fuzzy sets P and Q in (X,T) such that $P \cup Q = 1_{\sim}$.

Example 3.10 In Example 3.1, Consider the intuitionistic fuzzy sets $Q = \{< x, (\frac{a}{0}, \frac{b}{1}), (\frac{a}{1}, \frac{b}{0}) >, x \in X\}, R = \{< x, (\frac{a}{1}, \frac{b}{0}), (\frac{a}{0}, \frac{b}{1}) >, x \in X\}$, Q and R are intuitionistic fuzzy e-open sets in X, $(ecl(Q)) \cap R = 0_{\sim}$ and $Q \cap (ecl(R)) = 0_{\sim}$ which implies Q and R are IF e-q-separated and $Q \cup R = 1_{\sim}$. Hence X is IF eC_M-disconnected.

Remark 3.1 An IFTS (X,T) is IF eC_S-connected if and only if (X,T) is IF eC_M-connected.

Definition 3.7 An IFTS (X,T) is said to be IF e-super disconnected if there exists an IF e-regular open set P in X such that $0_{\sim} \neq P \neq 1_{\sim}$. X is called IF e-super connected if X is not IF e-super disconnected.

Example 3.11 In Example 3.1, Consider the intuitionistic fuzzy sets $Q = \{< x, (\frac{a}{1}, \frac{b}{0}), (\frac{a}{0}, \frac{b}{1}) >, x \in X\}, R = \{< x, (\frac{a}{0}, \frac{b}{1}), (\frac{a}{1}, \frac{b}{0}) >, x \in X\}$, Q and R are intuitionistic fuzzy e-open sets in X and $eint(ecl(Q)) = Q$. This implies Q is an IF e-regular open set in X. Hence X is an IF e-super disconnected.

Proposition 3.6 Let (X,T) be an IFTS. Then the following are equivalent:

1. X is IF e-super connected
2. For each IF e OS $P \neq 0_{\sim}$ in X, we have $eclP = 1_{\sim}$
3. For each IF e CS $P \neq 1_{\sim}$ in X, we have $eintP = 0_{\sim}$
4. There exists no IF e OS's P and Q in X such that $P \neq 0_{\sim} \neq Q$ and $P \subseteq \overline{Q}$
5. There exists no IF e OS's P and Q in X such that $P \neq 0_{\sim} \neq Q$, $Q = \overline{eclP}$ and $P = \overline{eclQ}$

6. There exists no IF eCS's P and Q in X such that $P \neq 1_\sim \neq Q$, $Q = \overline{eintP}$ and $P = \overline{eintQ}$

Proof. (i) $\Rightarrow$ (ii): Assume that there exists an $P \neq 0_\sim$ such that $eclP \neq 1_\sim$. Take $P = eint(eclP)$. Then P is proper e-regular open set in X which contradicts that X is IF e-super connectedness.

(ii) $\Rightarrow$ (iii): Let $P \neq 1_\sim$ be an IF eCS in X. If we take $Q = \overline{P}$ then Q is an IF eOS in X and $Q \neq 0_\sim$. Hence by (ii) $eclQ = 1_\sim \Rightarrow \overline{eclQ} = 0_\sim \Rightarrow eint(\overline{Q}) = 0_\sim \Rightarrow eintA = 0_\sim$.

(iii) $\Rightarrow$ (iv): Let P and Q are IF eOS in X such that $P \neq 0_\sim \neq Q$ and $P \subseteq \overline{Q}$. Since $\overline{Q}$ is an IF eCS in $X, \overline{Q} \neq 1_\sim$ by (iii) $eint\overline{Q} = 0_\sim$. But $P \subseteq \overline{Q}$ implies $0_\sim \neq P = eint(P) \subseteq eint(\overline{Q}) = 0_\sim$ which is a contradiction.

(iv) $\Rightarrow$ (i): Let $0_\sim \neq P \neq 1_\sim$ be an IF e-regular open set in X. If we take $Q = \overline{eclP}$, we get $Q \neq 0_\sim$. (If not $Q = 0_\sim$ implies $\overline{eclP} = 0_\sim \Rightarrow eclP = 1_\sim \Rightarrow P = eint(eclP) = eint(1_\sim) = 1_\sim \Rightarrow P = 1_\sim$ a contradiction to $P \neq 1_\sim$). We also have $P \subseteq \overline{Q}$ which is also a contradiction. Therefore X is IF e-super connected.

(i) $\Rightarrow$ (v): Let P and Q be two IF eOS in (X,T) such that $P \neq 0_\sim \neq Q, Q = \overline{eclP}$ and $P = \overline{eclQ}$. Now we have $eint(eclP) = eint(\overline{Q}) = \overline{eclQ} = P, P \neq 0_\sim$ and $P \neq 1_\sim$, since if $P = 1_\sim$ then $1_\sim = \overline{eclQ} \Rightarrow eclQ = 0_\sim \Rightarrow Q = 0_\sim$. But $Q \neq 0_\sim$. Therefore $P \neq 1_\sim \Rightarrow P$ is proper IF e-regular open set in (X,T) which is contradiction to (i). Hence (v) is true.

(v) $\Rightarrow$ (i): Let P be IF eOS in X such that $P = eint(eclP), 0_\sim \neq P \neq 1_\sim$. Now take $Q = \overline{eclP}$. In this case, we get $Q \neq 0_\sim$ and Q is an IF e OS in X and $Q = \overline{eclP}$ and $\overline{eclQ} = \overline{ecl(\overline{eclP})} = \overline{(\overline{eint(eclP)})} = eint(eclP) = P$. But this is a contradiction to (v). Therefore (X,T) is IF e-super connected space.

(v) $\Rightarrow$ (vi): Let P and Q be IF e-closed sets in (X,T) such that $P \neq 1_\sim \neq Q, Q = \overline{eintP}$ and $P = \overline{eintQ}$. Taking $C = \overline{P}$ and $D = \overline{Q}$, C and D become IF e-open sets in (X,T) and $C \neq 0_\sim \neq D, \overline{eclC} = \overline{ecl(\overline{P})} = \overline{\overline{(eintP)}} = eintP = \overline{Q} = D$ and similarly $\overline{eclD} = C$. But this is a contradiction to (v). Hence (vi) is true.

(vi) $\Rightarrow$ (i): We can prove this by the similar way as in (v) $\Rightarrow$ (vi).

Proposition 3.7 Let $f:(X,T) \to (Y,S)$ be a IF e-irresolute surjection. If X is an IF e-super connected, then so is Y.

Proof. Suppose that Y is IF e-super disconnected. Then there exists IF eOS's C and D in Y such that $C \neq 0_\sim \neq D, C \subseteq \overline{D}$. Since f is IF e-irresolute, $f^{-1}(C)$ and $f^{-1}(D)$ are IF eOS's in X and $C \subseteq \overline{D}$ implies $f^{-1}(C) \subseteq f^{-1}(\overline{D}) = \overline{f^{-1}(D)}$. Hence $f^{-1}(C) \neq 0_\sim \neq f^{-1}(\overline{D})$ which means that X is IF e-super disconnected which is a contradiction.

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Definition 3.8 An IFTS (X,T) is called intuitionistic fuzzy eC_5-connected between two intuitionistic fuzzy sets P and Q if there is no IFOS E in (X,T) such that $P \subseteq E$ and $E\overline{q}Q$.

Example 3.12 In Example 3.1, Consider the intuitionistic fuzzy sets $Q = \{< x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.8}, \frac{b}{0.6}) >, x \in X\}, R = \{< x, (\frac{a}{0.9}, \frac{b}{0.6}), (\frac{a}{0.1}, \frac{b}{0.2}) >, x \in X\}$, P is IF e OS in (X,T). Then (X,T) is intuitionistic fuzzy e-connected between P and Q.

Theorem 3.1 If an IFTS (X,T) is an intuitionistic fuzzy eC_5-connected between two intuitionistic fuzzy sets P and Q, then it is intuitionistic fuzzy C_5-connected between two intuitionistic fuzzy sets P and Q.

Proof. Suppose (X,T) is not intuitionistic fuzzy C_5-connected between two intuitionistic fuzzy sets P and Q then there exists an IFOS E in (X,T) such that $P \subseteq E$ and $E\overline{q}Q$. Since every IFOS in IF e OS, ther exists an IF e OS E in (X,T) such that $P \subseteq E$ and $E\overline{q}Q$ which implies (X,T) is not intuitionistic fuzzy e-connected between P and Q, a contradiction to our hypothesis. Therefore, (X,T) is intuitionistic fuzzy C_5-connected between P and Q.

However, the converse of the above Theorem is need not be true, as shown by the following example.

Example 3.13 In Example 3.1, Consider the intuitionistic fuzzy sets $Q = \{< x, (\frac{a}{0.1}, \frac{b}{0.2}), (\frac{a}{0.9}, \frac{b}{0.6}) >, x \in X\}, R = \{< x, (\frac{a}{0.7}, \frac{b}{0.4}), (\frac{a}{0.3}, \frac{b}{0.4}) >, x \in X\}$ P is IFOS in (X,T). Then (X,T) intuitionistic fuzzy C_5-connected between Q and R. Consider IFS $D = \{< x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.8}, \frac{b}{0.5}) >, x \in X\}$, D is an IF e OS such that $Q \subseteq D$ and $D \subseteq \overline{R}$ which implies (X,T) is intuitionistic fuzzy e-disconnected between Q and R.

Theorem 3.2 Let (X,T) be an IFTS and P and Q be intuitionistic fuzzy sets in (X,T). If PqQ then (X,T) is intuitionistic fuzzy eC_5-connected between P and Q.

Proof. Suppose (X,T) is not intuitionistic fuzzt eC_5-connected between P and Q. Then there exists an IF e OS E in (X,T) such that $P \subseteq E$ and $E \subseteq \overline{Q}$. This implies that $P \subseteq \overline{Q}$. That is $P\overline{q}Q$ which is a contradiction to our hypothesis. Therefore (X,T) is intuitionistic fuzzy eC_5-connected between P and Q.

However, the converse of the above Theorem is need not be true, as shown by the following example.

Example 3.14 In Example 3.1, Consider the intuitionistic fuzzy sets $Q = \{< x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.8}, \frac{b}{0.6}) >, x \in X\}, R = \{< x, (\frac{a}{0.9}, \frac{b}{0.6}), (\frac{a}{0.1}, \frac{b}{0.2}) >, x \in X\}$, P is IF e OS in

(X,T). Then (X,T) is intuitionistic fuzzy e-connected between P and Q. But P is not q-coincident with Q, since $\ll_P(x) < \neg_Q(x)$.

Definition 3.9 Let N be an IFS in IFTS (X,T)

(a) If there exists intuitionistic fuzzy e-open sets M and W in X satisfying the following properties, then N is called IF eC_i-disconnected (i=1,2,3,4):

$C_1 : N \subseteq M \cup W, M \cap W \subseteq \overline{N}, N \cap M \neq 0_{\sim}, N \cap W \neq 0_{\sim},$

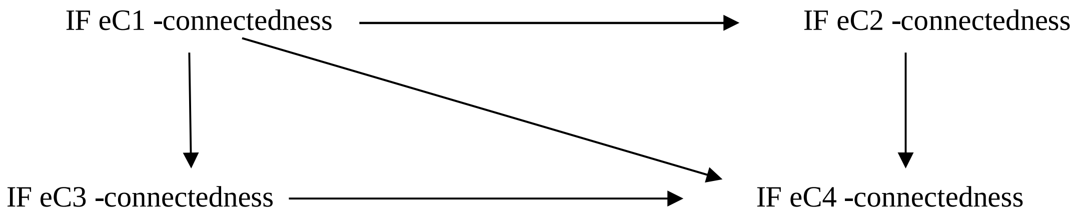
$C_2 : N \subseteq M \cup W, N \cap M \cap W = 0_{\sim}, N \cap M \neq 0_{\sim}, N \cap W \neq 0_{\sim},$

$C_3 : N \subseteq M \cup W, M \cap W \subseteq \overline{N}, M \not\subseteq \overline{N}, W \not\subseteq \overline{N},$

$C_4 : N \subseteq M \cup W, N \cap M \cap W = 0_{\sim}, M \not\subseteq \overline{N}, W \not\subseteq \overline{N},$

(b) N is said to be IF eC_i-connected $(i = 1,2,3,4)$ if N is not IF eC_i-disconnected $(i = 1,2,3,4)$.

Obviously, we can obtain the following implications between several types of IF eC_i-connected $(i = 1,2,3,4)$:



Example 3.15 In Example 3.1,

$M = \{< x, (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.3}) >, x \in X\}, W = \{< x, (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}) >, x \in X\}$, be IF e OS.

Consider the IFS $N = \{< x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.6}, \frac{b}{0.5}) >, x \in X\}$, N is IF eC_2-connected, IF eC_3-connected, IF eC_4-connected but IF eC_1-disconnected.

Example 3.16 In Example 3.1,

$M = \{< x, (\frac{a}{0.3}, \frac{b}{0}), (\frac{a}{0.4}, \frac{b}{1}) >, x \in X\}, W = \{< x, (\frac{a}{0}, \frac{b}{1}), (\frac{a}{1}, \frac{b}{0}) >, x \in X\}$, be IF e OS. Consider the IFS $N = \{< x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.6}, \frac{b}{0.5}) >, x \in X\}$, N is IF eC_2-disconnected but IF eC_4-connected.

Example 3.17 In Example 3.1,

$M = \{< x, (\frac{a}{0.8}, \frac{b}{0.7}), (\frac{a}{0.2}, \frac{b}{0.1}) >, x \in X\}, W = \{< x, (\frac{a}{0.6}, \frac{b}{0.8}), (\frac{a}{0.4}, \frac{b}{0.4}) >, x \in X\}$, be IF e OS.

Consider the IFS $N = \{< x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.6}, \frac{b}{0.5}) >, x \in X\}$, N is IF eC_3-disconnected but IF eC_4 -connected.

4. Intuitionistic Fuzzy e-Extremally Disconnected ness in Intuitionistic Fuzzy Topological Spaces

Definition 4.1 Let (X,T) be any IFTS. X is called IFe-extremally disconnected if the e-closure of every IFeOS in X is IFeOS.

Theorem 4.1 For an IFTS (X,T) the following are equivalent:

1. (X,T) is an IFe-extremally disconnected space.
2. For each IFeCS P, $eint(P)$ is an IFeCS.
3. For each IFeOS P, $ecl(P)=\overline{ecl(\overline{ecl(P)})}$ is an IFeCS.
4. For each intuitionistic fuzzy e-open sets P and Q with $ecl(P)=\overline{Q}, ecl(P)=\overline{eclB}$.

Proof. (i) $\Rightarrow$ (ii): Let P be any IFeCS. Then $\overline{P}$ is an IFeOS. So $ecl(\overline{P})=\overline{eint(P)}$ is an IFeOS. Thus $eint(P)$ is an IFeCS in (X,T).

(ii) $\Rightarrow$ (iii): Let P be an IFeOS. Then $ecl(\overline{ecl(P)})=ecl(eint(\overline{P})).\overline{ecl(\overline{ecl(P)})}=\overline{ecl(eint(\overline{P}))}$. Since P is an IFeOS, $\overline{P}$ is an IFeCS. So by (ii) $eint(\overline{P})$ is an IFeCS. That is $ecl(eint(\overline{P}))=eint(\overline{P})$. Hence $\overline{ecl(eint(\overline{P}))}=\overline{eint(\overline{P})}=ecl(P)$.

(iii) $\Rightarrow$ (iv): Let P and Q be any two intuitionistic fuzzy e-open sets in (X,T) such that $ecl(P)=\overline{Q}$. (iii) implies $ecl(P)=\overline{ecl(\overline{ecl(P)})}=\overline{ecl(\overline{Q})}=\overline{ecl(Q)}$.

(iv) $\Rightarrow$ (i): Let P be any IF e OS in (X,T). Put $Q=\overline{ecl(P)}$. Then $ecl(P)=\overline{Q}$. Hence by (iv) $ecl(P)=\overline{ecl(Q)}$. Therefore $ecl(P)$ is IF e OS in (X,T). That is (X,T) is an IF e-extremally disconnected space.

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