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Pisot Numbers Property by Fibonacci sequence

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ABSTRACT

Unit quadratic pisot number is a pisot number for which the product with its conjugates is ï 1, and they must have the minimal polynomials of degree 2.These numbers will satisfy the polynomials of the form $x^2 – cx - 1$ for c Č { 1,2,3….} or $x^2 – cx + 1$ for c Č {3,4,5….} Let q be a unit quadratic pisot number l^m (q) = | Dq- c | where 'm' a particular integer and C/D is the best approximation of q.In this paper, we present the best approximation by Fibonacci sequence by considering golden number is 1.366

KEYWORDS

Pisot Numbers: Golden Ratio:Fibonacci sequence:

INTRODUCTION

Pisot numbers have a long history, being studied as early as 1912. Salem first got interested in Pisot numbers q because of their property that $q^n \to 0 \pmod 1$ as $n \to \infty$. Salem shows that the set of pisot numbers is infinite and are the only algebraic numbers that have this property.

$l^m(q)$ is given by

$l^m(q) = \inf \{|y| : y = e_n q^n + e_{n-1} q^{n-1} + \ldots + e_0, e_i \in \{\pm m, \pm (m-1) .., \pm 1, 0\}\ y \neq 0\}$ [3]

Definition: An algebraic number is a root of a polynomial with integer coefficients.[1]

Definition: An algebraic integer is a root of a monic polynomial with integer coefficients.[1]

Definition: The Conjugates of an algebraic number are the other roots of the algebraic numbers minimal polynomial.[1]

Definition:A pisot number is a positive real algebraic integer greater than 1, all of whose conjugates are of modulus strictly less than 1.[1]

Definition: A Salem number is a positive real algebraic integer greater than 1, all of whose conjugates are of modulus less than or equal to 1. And at least one of the conjugates must be of modulus 1.[1]

Definition: The Fibonacci sequence (F_k) defined as follows.

$$F_0 = 0,\ F_1 = 1,\ F_k = F_{k-1} + F_{k-2},\ K = 2, 3\ldots..$$

THEOREM:

If $A^{k-2} < m \leq A^{k-1,}$ for some integer $K \geq 1$,then $l^m(A) = |F_kA - F_{k+1}|$

Proof: To prove the theorem, first we prove following two lemmas.

Lemma: If $m > A^{k-2}$, then $l^m(A) \leq |F_{k+1} – F_kA|$

Lemma: If $m \leq A^{k-1}$, then $l^m(A) \geq | F_{k+1} – F_kA|$

We use the following identities

$A^n = F_n A + F_{n-1}$ for n = 0,1…….

$F_{a+1}F_b - F_a F_{b+1} = (-1)^a F_{b-a}$ if $b \geq a \geq 0$,

$F_{a+1} F_{b+1} + F_a F_b = F_{a+b+1}$ if $a, b \geq 0$.

and the problem has a solution if and only if $m > A^{k-2}$. The solutions are given by the formulas

$$e_n = F_{n+k-1} - (F_{n+1}-1)m \; , e_{n-1} = (F_{n+2} - 2)\, m - F_{n+k}$$

with $m \geq F_{k+1}$ $\begin{cases} & \text{if } n = 1 \\ m \geq \frac{F_{k+2}}{2} & \text{if } n = 2 \\ \frac{F_{n+k}}{F_{n+2}-1} \leq m \leq \frac{F_{n+k}}{F_{n+2}-3} & \text{if } n \geq 3 \end{cases}$

The proof Lemma**is** given in three cases.

Case I: Supposethat $K \geq 3$ and assume on the contrary that$l^m(A) < | F_{k+1} - F_k A |$

Case II: The number Z has necessarily the form

$$Z = e_n A^n + e_{n-1} A^{n-1} + m (A^{n-2} - A^{n-3} + \ldots + (-1)^n) \text{with } n \geq 3$$

and $-m \leq e_{n-1} \leq -e_n$

Case III: If $m \leq \frac{F_{k+2}}{2}$, then$l^m (A) \geq | F_{k+1} - F_k A |$

Now consider the number $Y_i = F_{i+1} - F_i A$, $0 < | y_{k+1} | < | y_k | < | y_{k-1} |$

and$y_k$and y_{k-1} have opposite signs

we have $y_{k+1} = y_k + y_{k-1}$

By the definition of the Fibonacci sequence $0 < | Z | < |y_k|$

By the definition of z, $m \leq \frac{F_{k+3}}{2}$

$| Z | \geq | F_{k+2} - F_{k+1} A | = |y_{k+1}|$

Since $m \leq A^{k-1}$ and $m < F_{k+2}$

$|y_{k+1}| \neq | Z |$ and therefore$|y_{k+1}| < | Z | < | y_k |$

Now we distinguish two cases.If Z and $(-1)^n y_k$ have the same sign, then

$$Z' = Z + (-1)^{n+1} y_k$$
$$= e_n A^n + e_{n-1} A^{n-1} + m (A^{n-2} - A^{n-3} + \ldots + (-1)^n A^2)$$
$$+ (-1)^{n+1} (m - F_k) A + (-1)^n (F_{k+1} - m)$$

Satisfies $0 < | Z' | < | y_k |$

further Z' belongs to $\lambda^m$and $\frac{F_{k+1}}{2} \leq F_k \leq F_{k+1}$

Finally, since the equalities $| m - F_k | = m$ and $| F_{k+1} - m | = m$ cannot hold Simultaneously, Z' is smaller than Z. This contradicts the definition of Z.

If Z and $(-1)^n y_k$ have different signs, then Z and $(-1)^n y_{k-1}$ have the same sign. Then it follows that

$$Z' = Z + (-1)^{n+1} y_{k-1}$$

$= e_n A^n + e_{n-1} A^{n-1} + m (A^{n-2} - A^{n-3} + .. + (-1)^n A^2) + (-1)^{n+1}(m-F_{k-1}) A + (-1)^n (F_k - m)$

satisfies $0 < |Z'| < |y_{k-1} - y_{k+1}| = |y_k|$

Further Z′belongs to $\lambda^m$and since theequation $|m-F_{k-1}|$=mand$|F_k - m| = m$ cannot hold simultaneously. Z′is smallerthan Z. This contradicts the definition Z again

- Form≥ 2, under the condition $F_k \leq m < F_{k+1}$the theorem holds

REFERENCES

[1] Pisot numbers and the Spectra of Real numbers –from the Ph.D. thesis of Kevin G. Hare..

[2] Kevin G. Hare, The Structure of the Spectra of Pisot numbers, 17 July 2003.

[3] Erdos. I. Joo, and V.Komornik On the sequence ofnumbers of the form $\varepsilon 0 + \varepsilon 1 q + \ldots\ldots + \varepsilon_n q_n$, $\varepsilon i \in \{0,1\}$.Acta.Arithmetica 83 (1998). 3.201 210.

[4] B. Krishna Gandhi and H. Anita. “On Explorations of the Spectra of Pisot Numbers”. Indian Journal of Mathematics and Mathematical Sciences, Vol.2, No.1 (June 2006) 81-91.