International Journal of Engineering Technology Science and Research
IJETSR
www.ijetsr.com
ISSN 2394 – 3386
Volume 4, Issue 11
November 2017

Optimization based Consensus Model for Multi-Criteria Group Decision Making under 2-Tuple Linguistic Framework

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ABSTRACT

In this paper, multi criteria large group decision making (MCLGDM) problems under 2-tuple linguistic framework are studied. Since, in large groups, creating consensus for unanimous and harmonious solution is a daunting task. So, in this contribution, firstly the efficiency values of experts corresponding to each alternative are evaluated using a data envelopment analysis (DEA) inspired model and hence grouped into smaller sets of most efficient experts for each alternative. The second phase of the algorithm constitutes an optimization model to evaluate the upgraded opinion of the efficient experts in the interest of maximizing consensus degree. Further, a consensual linguistic matrix is contrived and the best alternative is achieved. To demonstrate the practicality of the algorithm, a faculty appointment system selection problem is adopted. The illustration reviews the applicability and practicality of the proposed methodology in real world.

Keywords

Multi criteria group decision making, consensus, 2-tuple linguistic variables, optimization model.

INTRODUCTION

Multi criteria decision making (MCDM) problems comprises of reviewing the subjective evaluations of finite alternatives under certain precogitated and conflicting criteria by a decision maker (DM) or a group of decision makers (DMs). The latter case is known as multi-criteria group decision making (MCGDM) problem, see [1]. Various MCGDM techniques have been designed under diversified frameworks over the last few decades, for ready reference see [2, 3]. The amelioration in MCGDM algorithms is extensively influenced by numerous applications to real life problems. Since the real life decision scenarios involve precarious and significant decisions for the society, sometimes it require large number of experts from diversified fields of interest and expertise, resulting in multi criteria large group decision making (MCLGDM) problems [4, 5]. However, the accessible literature on LGDM is meager and sparse, but in last few years, researchers have shown extensive interest in inspecting methodologies for LGDM problems. In [6], Liu et. al. have classified the available literature of LGDM in four categories. One can analyze the intricate study of the categories of LGDM in [6]. In decision making problems, always imparting exact information of each alternative is inconceivable and impractical. The experts may however be cautious or unable to provide exact numeric assessments due to intrinsic subjectivity of human thinking. Hence quantitative valuations are not adequate to represent such decision problems. The linguistic approach is a pertinent way to surmount the uncertainty in real world decision situations wherein the information is depicted in qualitative terms by means of linguistic variables. In 2000, Herrera and Martinez [7] proposed 2-tuple linguistic model that can represent any counting of information in the universe. Among various existing linguistic models, the 2-tuple linguistic model has been widely applied in various fields of decision science due to its rectitude and interpretability (see, [7, 8, 9]). Hence, in last few years MCLGDM problems under linguistic framework have engaged attention of numerous researchers [10, 11].

Supplementary to uncertainty of information, the other aspect of MCLGDM problems is achieving a unanimous solution. Usually, the group of experts may not always harmoniously coincide with the same solution and may believe that their evaluations have not been amply considered in the final decision [12, 13]. Hence, to reach at a consolidated outcome accepted by the complete group, consensus reaching process (CRP) in decision science has evolved as a field of immense interest.

The term consensus in decision science signifies a complete and unified agreement [14, 15], generally not attainable in real world. To improve its applicability in real world, Kacprzyk and Fedrizzi defined soft consensus that signifies the human perception of consensus [16]. Herrera-Viedma et.al. presented a detailed review of soft consensus models in uncertain environment [17]. One can see [17] for extensive study of contributions in the literature of consensus, consensus approaches in MCGDM and the prevailing trends in the consensus models. Generally, the extant consensus models are contingent on iterative process involving a feedback mechanism [18, 19, 20]. The available models enforce various rounds of analysis and discussions where the experts need to modify their assessments for the convergence of the consensus process [18, 19, 20]. Xu [21] defined an automatic iterative algorithm that improves the opinion of the experts. However, the model eliminates the issue of improving the opinions repeatedly but the iterative process was the elementary feature of the model. Hence, the main disadvantage of the generally existing consensus models is the iterative procedure, a time consuming practice and the feedback mechanism depending on the repeated improvements of the experts' opinion.

So, in this paper, a fractional programming problem with linguistic coefficient is defined in view of DEA to determine the efficiency of the experts. Then, the large group is partitioned in over-lapping sets of efficient experts as achieving consensus in small groups is easier than creating consensus in large groups. An optimization based consensus model is then proposed to evaluate the optimized linguistic opinion of the efficient experts for each alternative and best alternative is determined. The methodology is illustrated via a hypothetical situation of selecting the best faculty appointment system in a technological university to depict the practicality and adaptability of the algorithm.

The remainder of this paper is as follows. Section 2 briefs the rudiments of linguistic variables with some arithmetic operations. Section 3 introduces the proposed methodology. In section 4, an example depicting an application of university faculty selection system is presented to illustrate the proposed method. The paper concludes in Section 5.

PRELIMINARIES

In this section, the rudiments of 2-tuple linguistic variables are reviewed and the arithmetic given by Herrera and Martinez [7] are discussed. The 2-tuple linguistic model is entrenched on the symbolic method and employs the conception of symbolic translation as the fundamental of its representation. Let $\mathrm{LT} = \{l_i : i = 0,1,\ldots,g\}$ be a finite and totally ordered linguistic term set, where l_i denotes a linguistic variable with following features [7].

(i) $l_i \geq l_j$ if $i \geq j$ (the set LT is ordered);

(ii) $\max(l_i, l_j) = l_i$, if $l_i \geq l_j$ and $\min(l_i, l_j) = l_i$, if $l_i \leq l_j$;

(iii) $\mathrm{neg}(l_i) = l_j$, where $j = g - i$.

Definition 1 [7]. Suppose $\in [0, g]$ be the result of an aggregation of the indices of a set of labels assessed in LT. Let i = round (β), where round (·) is the usual round operation, and α = β–i , α ∈ [−0.5, 0.5). Then α is called the symbolic translation.

Herrera and Martinez [7] proposed a 2-tuple linguistic model $(l_i, \sim_i)$ with $l_i \in \mathrm{LT}$ and $\sim_i \in [-0.5, 0.5)$. They also explained the lexicographic ordering of the 2-tuple linguistic variables.

Definition 2 [7]. Let $(l_i, \sim_i)$ and $(l_j, \sim_j)$ be two 2-tuple linguistic variables. Then,

International Journal of Engineering Technology Science and Research
IJETSR
www.ijetsr.com
ISSN 2394 – 3386
Volume 4, Issue 11
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(i) If $i < j$ then $(l_i, \sim_i) < (l_j, \sim_j)$.

(ii) If $i = j$, i.e. l_i and l_j are same linguistics, then

a. if $\sim_i = \sim_j$, then $(l_i, \sim_i) = (l_j, \sim_j)$ that is $(l_i, \sim_i)$ and $(l_j, \sim_j)$ represent same information;

b. if $\sim_i > \sim_j$, then $(l_i, \sim_i) > (l_j, \sim_j)$;

c. if $\sim_i < \sim_j$, then $(l_i, \sim_i) < (l_j, \sim_j)$.

Definition 3 [7] Let β ∈ [0, g] be the valuation depicting the resultant of symbolic aggregation. An equivalent 2-tuple linguistic variable can be acquired using the function Δ: [0, g] → LT × [−0.5, 0.5), given as

$$\Delta\,(\beta) = (l_i, \sim_i) \text{ where } i = \text{round}\,(\beta) \text{ and } \sim_i = \beta - i\,.$$

The function Δ is a bijection and its inverse exists and is given by

$$\Delta^{-1}\text{: LT} \times [-0.5, 0.5) \rightarrow [0, g] \text{ as } \Delta^{-1}(l_i, \sim_i) = i + \sim_i = \beta.$$

The literature of diverse operators on 2-tuple linguistic variables is proliferated and extensive. Here, we specify only the arithmetic and weighted average operators.

Definition 4 [22] Let $\{ (l_{r_i}, \sim_{r_i}),\, i = 1,...,q,\; r_i \in \{0,1,...,g\} \}$, be a set of 2-tuple linguistic variables. Then

$$AM\,((l_{r_i}, \sim_{r_i}) : i = 1,...,q) = (l_k, \sim_k),$$

where

$$k = \text{round}\left(\frac{\sum_{i=1}^{q} r_i + \sum_{i=1}^{q} \sim_{r_i}}{q}\right), \quad \sim_k = \left(\frac{\sum_{i=1}^{q} r_i + \sum_{i=1}^{q} \sim_{r_i}}{q}\right) - k.$$

Definition 5 [22]. Let $\{ (l_{r_i}, \sim_{r_i}),\, i = 1,...,q,\; r_i \in \{0,1,...,g\} \}$, be a set of 2-tuple linguistic variables and $\mu = \{w_1,..., w_q\}^T$ be the weight vector with $0 \le w_i \le 1,\; i = 1,...,q$ and $\sum_{i=1}^{q} w_i = 1$. Then,

$$LWA[(l_{r_i}, \sim_{r_i}) : i = 1,...,q] = (l_{r_1}, \sim_{r_1})w_1 \oplus (l_{r_2}, \sim_{r_2})w_2 \oplus ... \oplus (l_{r_q}, \sim_{r_q})w_q$$

$$= \text{V}\left(\sum_{i=1}^{q} w_i \text{V}^{-1}(l_{r_i}, \sim_{r_i})\right)$$

Consequently,

$$\Delta^{-1}\left(\bigoplus_{i=1}^{q} (\ell_{r_i}, \alpha_{r_i}) w_i\right) = \sum_{i=1}^{q} w_i \Delta^{-1}(\ell_{r_i}, \alpha_{r_i}). \tag{1}$$

ALGORITHM

The multi criteria group decision making implicates experts from diversified fields ruminating over the best alternative assessed under numerous conflicting criteria. In this section, we consider an MCGDM problem with t experts to rank m alternatives under n criteria. Since, in real world scenario, it is not always a cinch to quantify the assessments for the alternatives, so in this paper, the experts are extricated from this stringent restriction and can evaluate the alternatives using linguistic or 2-tuple linguistic frameworks. Suppose $\overset{\circ}{P}_k = (\overset{\circ}{p}{}_{ij}^{(k)})$ be the linguistic assessment matrix provided by expert $e_k \in \{e_1, e_2, ..., e_t\}$ for m alternatives analyzed under n criteria. Let $I = \{1, 2, ..., m\}$ be the indices set of alternatives. The criteria set is classified into cost criteria with index set $J_1 = \{\ (1),\ (2), ...,\ (n_1)\}$ and beneficial criteria having index $J_2 = \{\ (n_1 + 1),\ (n_1 + 2), ...,\ (n)\}$. The proposed algorithm involves two phases as described below.

International Journal of Engineering Technology Science and Research
IJETSR
www.ijetsr.com
ISSN 2394 – 3386
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1. Grouping of the experts using efficiency measure

The methodology establishes with the estimation of efficiency of each expert for given alternatives. In this context, the 'efficiency' construes the quantitative measure of competency of the expert in evaluating each alternative. The initial phase involves the following steps.

Step 1.1: Consider the linguistic assessment matrix $A_i = (\tilde{a}_{kj}^{(i)})_{t\times n}$; $i = 1, 2, ..., m,$ where $\tilde{a}_{kj}^{(i)}$ represents the linguistic evaluation of alternative *i* under criteria *j* from the predefined linguistic term set $LT = \{l_0, l_1, ..., l_g\}$ given by expert k .

Step 1.2: The efficiency of each decision maker is determined for each alternative using the fundamentals of data envelopment analysis (DEA). By virtue of output maximization DEA model, the cost criteria are treated as input while the beneficial criteria are considered as output. Hence, the DEA model to evaluate the efficiency of each expert for given alternatives is given below.

$$\max\ \widetilde{E}_k^{(i)} = \frac{\Delta^{-1}\Big(\bigoplus_{j\in J_2} v_{\beta(j)}\widetilde{a}_{k\beta(j)}^{(i)}\Big)}{\Delta^{-1}\Big(\bigoplus_{j\in J_1} u_{\beta(j)}\widetilde{a}_{k\beta(j)}^{(i)}\Big)} \quad ;\ i\in I \ \text{ and } \ k=1,2,\dots,t \qquad \text{(M1)}$$

$$\text{subject to}\quad 0\le \widetilde{E}_s^{(i)} \le 1 \quad ;\ s=1,2,\dots,t.$$

Here, $u_{(j)}$ and $v_{(j)}$ are the weights corresponding to the evaluations of cost and benefit criteria respectively.

Model M1 can be simplified in the light of eq.(1) as follows.

$$\max\ \widetilde{E}_k^{(i)} = \frac{\sum_{j=n_1+1}^{n} v_{\beta(j)}\Big(\Delta^{-1}(\widetilde{a}_{k\beta(j)}^{(i)})\Big)}{\sum_{j=1}^{n_1} u_{\beta(j)}\Big(\Delta^{-1}(\widetilde{a}_{k\beta(j)}^{(i)})\Big)} \quad ;\ i\in I \ \text{ and } \ k=1,2,\dots,t \qquad \text{(M2)}$$

$$\text{subject to}\quad 0\le \widetilde{E}_s^{(i)} \le 1 \quad ;\ s=1,2,\dots,t.$$

Since in real case studies, the competency and proficiency of the experts in assessing each alternative consistently is ideological, therefore in the following step, the experts with same efficiency index for each alternative are categorized.

Step 1.3: For alternative i , define set G_i of experts with efficiency value 1 as follows.

$$G_i = \{e_k \mid E_k^{(i)} = 1 \forall i \in I\}$$

To manifest the real world scenario, one may also use different thresholds other than 1 for classifying the experts.

So, the first phase of the method evaluates the efficiency of each experts for given alternatives and the experts with efficiency value 1 in each alternative are selected for set G_i . It is an important step of the algorithm as in group decision situations with large number of experts, it is an arduous and gradual effort to reach a unanimous solution, hence the larger group is divided into parts with less number of experts for each alternative. Obviously, such division assists in achieving a consensual conclusion proficiently.

2. Consensus and aggregation

Consensus is an extrusive component of recent studies of multi criteria group decision making with large number of experts. In large groups, reaching to a soft consensus is also highly difficult. Thus, in the second phase, we suggest to reach the consensus only among the efficient experts corresponding to each alternative using optimization model. The optimization model expedites the gradual process of achieving consensus. It involves the following steps.

Step 2.1: Compute the aggregated linguistic assessment vector for alternative i given as, $\tilde{A}^{(i)}_{comb} = [\tilde{a}^{(i)}_1, \tilde{a}^{(i)}_2, ..., \tilde{a}^{(i)}_n]$ by using the following.

$$\tilde{a}^{(i)}_j = AM[\tilde{a}^{(i)}_{kj}; k \in G_i]; i \in I, j \in J_1 \cup J_2$$

Step 2.2: Determine the consensus between the opinion of expert $e_k \in G_i$ and the combined assessment of alternative i. Since the aggregated evaluation reflects the outlook of the entire group G_i for alternative i, the consensus degree of expert $e_k \in G_i$ is characterized in view of cosine similarity between the combined assessment and the evaluation of expert $e_k \in G_i$ for alternative i. The cosine similarity is pertinent in the context as it analyzes the homogeneity of direction of the two vectors rather than the precise values. Now, define the consensus degree of expert $e_k \in G_i$ as follows.

$$c^{(i)}_k = \frac{\sum_{j=1}^{n} \Delta^{-1}(\tilde{a}^{(i)}_{kj})\Delta^{-1}(\tilde{a}^{(i)}_j)}{\sqrt{\sum_{j=1}^{n} \left(\Delta^{-1}(\tilde{a}^{(i)}_{kj})\right)^2}\sqrt{\sum_{j=1}^{n} \left(\Delta^{-1}(\tilde{a}^{(i)}_j)\right)^2}}; i \in I$$

The evaluated $c^{(i)}_k$ is the initial consensus degree of expert $e_k \in G_i$ based on their assessment.

Step 2.3: The customarily available consensus models facilitate with an iterative procedure for the feedback mechanism to upgrade consensus. In this paper, an optimization model is designed to modify the assessment of each expert to increase the cosine similarity. The model is as follows.

$$\max\ C^{(i)}_k = \frac{\sum_{j=1}^{n} \left(\Delta^{-1}(\tilde{a}^{(i)}_j)\right)x^{(i)}_{kj}}{\sqrt{\sum_{j=1}^{n} \left(\Delta^{-1}(\tilde{a}^{(i)}_j)\right)^2}\sqrt{\sum_{j=1}^{n} x^{(i)2}_{kj}}} \quad ; i \in I_2 \text{ and } k \in G_i \qquad \text{(M3)}$$

subject to

$$\min\{\Delta^{-1}(\tilde{a}^{(i)}_{kj}), \Delta^{-1}(\tilde{a}^{(i)}_j)\} \le x^{(i)}_{kj} \le \max\{\Delta^{-1}(\tilde{a}^{(i)}_{kj}), \Delta^{-1}(\tilde{a}^{(i)}_j)\} \ ; j = 1, 2, \ldots, n.$$

Here, $x^{(i)}_{kj}$ is the modified optimized evaluation of expert $e_k \in G_i$ for alternative i under criteria j. Here, the corresponding transformed 2-tuple linguistic variable of $x^{(i)}_{kj}$ is considered for further processing. The foregoing model to assess the optimized consensus degree for given alternative is efficient and robust in contrast to the extant models. The advantage of employing optimization theory in consensus model is that it forthwith assigns the best attainable values to maximize the consensus. The available models improvise the consensus degree using a feedback mechanism with each iteration that results in a protracted and slow process. The suggested model exhibits excellent competency and efficiency in real world situations that require immediate reactions are required.

Step 2.4: Now, aggregate the modified evaluations of each alternative in view of Step 2.1 to acquire the optimized consolidated assessment with full consensus and depicted as $\mathring{S}_i = AM[\tilde{a}_j^{(i)}; j = 1,2,..,n]; i \in I.$ for each alternative.

Step 2.5: The combined linguistic score for alternative *i* is defined as aggregation of its criteria values as follows.

$$\mathring{S}_i = AM[\tilde{a}_j^{(i)}; j = 1,2,..,n]; i \in I.$$

The alternative with largest linguistic score, $\mathring{S}_i$ is recognized as the best alternative. One may also employ the predefined weight vector for the criteria set or can determine the weights using existing methods like AHP, entropy etc.

ILLUSTRATION

In this section, the proposed algorithm is practiced on faculty appointment system enhancement in a technical university. University is a worldwide renowned platform with 10 departments focusing on engineering, management and science in India. To further improvise the proficiency of teaching, research and participation in the amelioration of the society, university intends to reform the current implemented teacher appointment system. The officials have identified 5 prospective alternatives A_1, A_2, A_3, A_4, A_5 that could be implemented in the university. To select the best alternative, the university officials request two representatives from each department to participate in a group decision. The group decision associates 20 experts to cogitate over selecting the best alternative unanimously under 3 criteria C_1, C_2, C_3 given as:

C_1 is the emphasis given to extracurricular activities of the candidate. It involves contribution of the candidate in the corporate life and management of the university in conjunction with the professional development activity.

C_2 is the cost involved in implementing the alternative.

C_3 involves the time period required to perform the process as suggested in alternative.

Here, the criteria C_1 is beneficial criteria and C_2 along with C_3 serve as cost criteria. The experts are required to assess each potential alternative under the foregoing criteria using the linguistic term set $LT = \{l_0 : ExtremelyLow, l_1 : VeryLow, l_3 : Low, l_4 : High, l_5 : VeryHigh, l_6 : Perfect\}$. The linguistic assessment matrices $\mathring{P}_k$ provided by 20 experts are reviewed and the linguistic matrices A_i ; $i = 1,2,...,5$ for each alternative are deduced for further processing. Now, evaluate the efficiency of each expert e_k using the DEA model M1 for each alternative. In light of model M1, the optimization problem in context of alternative A_2 to establish the efficiency of expert e_k is as follows.

$$\max\ \widetilde{E}_1^{(2)} = \frac{\Delta^{-1}(v_1 a_{11}^{(2)})}{\Delta^{-1}(u_1 a_{12}^{(2)} \bigoplus u_2 a_{13}^{(2)})} = \frac{\Delta^{-1}(v_1(l_1, 0.3333))}{\Delta^{-1}(u_1(l_1, 0.3333) \bigoplus u_2(l_3, -0.3333))}$$

$$\text{subject to} \quad 0 \le \widetilde{E}_s^{(2)} \le 1 \quad ; \ s = 1,2,\ldots,20.$$

The simplified model M2 corresponding to above problem is as

$$\max\ \widetilde{E}_1^{(2)} = \frac{v_1\Delta^{-1}(l_1, 0.3333)}{u_1\Delta^{-1}(l_1, 0.3333) + u_2\Delta^{-1}(l_3, -0.3333)} = \frac{1.333v_1}{1.333u_1 + 2.6667u_2}$$

$$\text{subject to}\quad 0 \le \widetilde{E}_s^{(2)} \le 1 \quad ;\ s = 1, 2, \ldots, 20.$$

follows.

Analogously, the model to compute efficiency of each expert considering every alternative could be easily constructed and efficiency value are calculated as given in Table1.

Table 1: Efficiency of each expert for given alternatives

	A_1	A_2	A_3	A_4	A_5
e_1	0.0556	0.6667	0.2573	1.0000	0.4398
e_2	0.7000	0.5556	0.6690	0.1622	0.3519
e_3	0.7778	0.6667	0.8736	0.2500	0.5523
e_4	0.7179	0.5556	0.6690	0.0714	0.3654
e_5	0.0897	1.0000	1.0000	0.2632	1.0000
e_6	1.0000	0.8333	0.6680	1.0000	1.0000
e_7	0.7000	0.8333	0.6667	0.2857	0.9828
e_8	0.7000	0.8333	0.5000	0.2857	0.9828
e_9	0.5385	0.1333	0.0710	1.0000	0.1250
e_{10}	1.0000	0.4000	0.1727	0.5000	0.2317
e_{11}	0.3333	0.5833	0.2872	1.0000	0.2500
e_{12}	0.2222	0.3333	0.7422	0.4286	0.4130
e_{13}	0.8077	0.8333	0.4086	0.1250	0.7500
e_{14}	0.9333	0.5556	0.3838	0.5000	0.4672
e_{15}	0.2722	1.0000	0.5846	0.5000	0.2007
e_{16}	0.9333	0.5556	0.3838	0.5000	0.4672
e_{17}	0.2160	0.6667	0.8333	0.1667	0.7125
e_{18}	0.9333	0.5556	0.3838	0.5000	0.4672
e_{19}	0.0365	1.0000	1.0000	0.4444	0.4453
e_{20}	0.5104	0.4444	0.4497	0.4615	0.8382

Now, for each alternative i, obtain the set G_i of experts with full efficiency i.e. efficiency value as 1. The set reviews the experts that are dexterous in assessing the given alternative. The set of efficient experts for each alternative is given in Table 2.

Table 2: Set of most efficient experts for given alternative

Alternative	Set of most efficient experts
A_1	$G_1 = \{e_6, e_{10}\}$
A_2	$G_2 = \{e_5, e_{15}, e_{19}\}$
A_3	$G_3 = \{e_5, e_{19}\}$
A_4	$G_4 = \{e_1, e_6, e_9, e_{11}\}$
A_5	$G_5 = \{e_5, e_6\}$

The set G_i partitions the large group into smaller sets of experts as reaching consensus in a smaller group is attainable and feasible. It is evident that the complete efficiency of the expert in assessing an alternative is a rigorous and stringent restraint in real world scenario. So, one may also use a threshold of 95% or 90% efficiency or an interval to obscure the efficient experts.

Compute the aggregated linguistic assessment for each alternative i using the opinion of experts in set G_i in view of Step 2.1.

$$A_{comb}^{(1)} = [(l_4, 0),\ (l_2, -0.3333),\ (l_1, 0)]$$

$$A_{comb}^{(2)} = [(l_5, 0.3333),\ (l_4, -0.4444),\ (l_3, -0.1111)]$$

$$A_{comb}^{(3)} = [(l_4, 0.3333),\ (l_2, 0.3333),\ (l_3, 0)]$$

$$A_{comb}^{(4)} = [(l_4, 0),\ (l_2, -0.3333),\ (l_2, 0)]$$

$$A_{comb}^{(5)} = [(l_4, 0),\ (l_2, 0),\ (l_2, -0.3333)]$$

Employing the opinions provided by the experts in set G_i and the combined linguistic evaluation of each alternative i, compute the initial consensus degree using the model suggested in step 2.2. The initial consensus degree for efficient expert corresponding to each alternative is given in Table 3.

Table 3: Initial consensus degree of experts in set G_i for each alternative i

Alternative	Initial consensus degree
A_1	$c_6^1 = 0.9997$ $c_{10}^1 = 0.9997$
A_2	$c_5^2 = 0.9999$ $c_{15}^2 = 1$ $c_{19}^2 = 0.9999$
A_3	$c_5^3 = 0.9997$ $c_{19}^3 = 0.9996$
A_4	$c_1^4 = 0.9999$ $c_6^4 = 0.9997$ $c_9^4 = 0.9997$ $c_{11}^4 = 0.9999$
A_5	$c_5^5 = 0.9972$ $c_6^5 = 0.9992$

The initial consensus value suggests the similarity between the assessment of the efficient experts and the combined opinion. Now, to improvise the consensus among the opinions of experts in set G_i, implement the model M3 and evaluate the optimized assessments of the experts. The modified evaluations reflect the projected values of the alternatives in view of the efficient experts for the complete consensus. The corresponding model M3 for the 5th expert corresponding to alternative A_2 is as follows.

$$\max\ C_5^{(2)} = \frac{\Delta^{-1}(l_5,0.3333)x_1 + \Delta^{-1}(l_4,-0.4444)x_2 + \Delta^{-1}(l_3,-0.1111)x_3}{49.4321\sqrt{x_1^2+x_2^2+x_3^2}}$$

$$\text{subject to}\quad \begin{aligned} &\Delta^{-1}(l_5,0.3333) \le x_1 \le \Delta^{-1}(l_6,0)\\ &\Delta^{-1}(l_4,-0.4444) \le x_2 \le \Delta^{-1}(l_4,0)\\ &\Delta^{-1}(l_3,-0.1111) \le x_3 \le \Delta^{-1}(l_5,0.056)\end{aligned}$$

The solution of the above optimization problem is $[(l_6,-0.0348), (l_4,-0.025), (l_3,0.2307)]$. In similar fashion, the optimized values for the opinion of each efficient expert in group G_i corresponding to each alternative A_i is determined and hence, the aggregated unified assessment of each alternative is evaluated using step 2.4. The modified unanimous evaluations of each alternative are given in following matrices.

$$\underbrace{\begin{array}{c} \\ e_6 \\ e_{10} \\ \tilde{A}^{(1)}_{comb} \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} (l_4,0) & (l_2,-0.3333) & (l_1,0) \\ (l_4,0) & (l_2,-0.3333) & (l_1,0) \\ (l_4,0) & (l_2,-0.3333) & (l_1,0) \end{bmatrix}\end{array}}_{A_1} \quad \underbrace{\begin{array}{c} \\ e_5 \\ e_{15} \\ e_{19} \\ \tilde{A}^{(2)}_{comb} \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} (l_6,-0.0348) & (l_4,-0.025) & (l_3,0.2307) \\ (l_4,0.0541) & (l_3,-0.2972) & (l_2,0.1960) \\ (l_5,0.3333) & (l_4,-0.4445) & (l_3,-0.1113) \\ (l_5,0.1176) & (l_3,0.4117) & (l_3,-0.228) \end{bmatrix}\end{array}}_{A_2}$$

$$\underbrace{\begin{array}{c} \\ e_5 \\ e_{19} \\ \tilde{A}^{(3)}_{comb} \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} (l_4,0.3333) & (l_2,0.3333) & (l_3,0) \\ (l_4,0.3333) & (l_2,0.3333) & (l_3,0) \\ (l_4,0.3333) & (l_2,0.3333) & (l_3,0) \end{bmatrix}\end{array}}_{A_3} \quad \underbrace{\begin{array}{c} \\ e_1 \\ e_6 \\ e_9 \\ e_{11} \\ \tilde{A}^{(4)}_{comb} \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} (l_3,-0.3190) & (l_1,0.1171) & (l_1,0.3407) \\ (l_4,0) & (l_2,-0.3333) & (l_2,0) \\ (l_4,0) & (l_2,-0.3333) & (l_2,0) \\ (l_3,0.3439) & (l_1,0.3933) & (l_2,-0.3287) \\ (l_4,-0.4938) & (l_1,0.4609) & (l_2,-0.2467) \end{bmatrix}\end{array}}_{A_4}$$

$$\underbrace{\begin{array}{c} \\ e_5 \\ e_6 \\ \tilde{A}^{(5)}_{comb} \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} (l_4,0) & (l_2,0) & (l_2,-0.3333) \\ (l_4,0) & (l_2,0) & (l_2,-0.3333) \\ (l_4,0) & (l_2,0) & (l_2,-0.3333) \end{bmatrix}\end{array}}_{A_5}$$

It is noteworthy that in each case, the improved consensus degree is 1.
Now evaluate the combined harmonious decision matrix as given below.

$$\widetilde{R}_{comb} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \left[\begin{array}{ccc} (l_4,0) & (l_2,-0.3333) & (l_1,0) \\ (l_5,0.1176) & (l_3,0.4117) & (l_3,-0.2280) \\ (l_4,0.3333) & (l_2,0.3333) & (l_3,0) \\ (l_4,-0.4938) & (l_1,0.4609) & (l_2,-0.2467) \\ (l_4,0) & (l_2,0) & (l_2,-0.3333) \end{array}\right] \end{array}$$

The combined linguistic score of each alternative is determined in view of step 2.5.

$$S_1 = (l_2, 0.2222);\ S_2 = (l_4, -0.2329);\ S_3 = (l_3, 0.2222);\ S_4 = (l_2, 0.2401);\ S_5 = (l_3, -0.4444)$$

So, in light of the views of experts, alternative A_2 is the best to employ for faculty appointment system.

CONCLUSION

The real world decision making problems can be easily modeled as MCLGDM problems under uncertainty. In large groups, achieving unanimous and unified solution is a daunting task. So, in this paper, we have designed an optimization based consensus model that achieves the harmonious linguistic assessment of the alternatives. The proposed method is competent in comparison to the existing model as the suggested model provides the consensual decision matrix by solving few optimization problems whereas the existing consensus models involve iterative procedures with a feedback mechanism. The existing models support the gradual iterative process when in fact the proposed algorithm is direct and competent.

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