
Intelligent Controller for Inverted Pendulum System

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ABSTRACT

Artificial Neural Network (ANN) is used to predict the controller model of the Inverted Pendulum (IP) System ANN has been proved a very useful and successful tool in predicting highly nonlinear and complex processes with an impressive accuracy. As an artificial intelligence technique is used in this paper to control the angle with position of a non-linear inverted pendulum system. The ANN controller here is a specified three layered feed forward network having, Input, Hidden and Output layers. 'Trainlm' network function that is used to update weights and bias states according to Levenberg- Marquardt (LM) back-propagation, is used here for training. The dynamic modelling of Inverted pendulum system is done using Euler-Langerange equation and the main task is to control the angle with position of non-linear system by the neural network controller and to compare the response with response of conventional controller using MATLAB simulations and to show the improved response.

KEYWORDS

Artificial Intelligence (AI); Inverted pendulum (IP); Artificial Neural Network (ANN); Artificial Neural Network Four Hidden Layer (ANN-4HL); Feed-Forward Network (FFN); Levenberg- Marquardt (LM) back-propagation

INTRODUCTION

The inverted pendulum system is a classic control problem that is used for nonlinear systems which are generally very complex systems in research. Artificial Neural Network (ANN) is used to predict the controller model of the Inverted Pendulum (IP) System ANN has been proved a very useful and successful tool in predicting highly nonlinear and complex processes with an impressive accuracy. The reason why ANN attracted researchers is due to its capability of quickly learn the dynamics of the process, apart from that ANN also capable of handling the uncertain and noisy data [1]. In process industries there are several applications of nonlinear systems, the control of which is a very complex task and as conventional controllers are generally used for linear or approximately linear systems, there are very less amount of accuracy when used by conventional controllers. For the task of controlling a nonlinear system ANN has been seen a very good controller with a specific amount of accuracy. Artificial Neural Networks (ANNs) are human brain based non-linear mapping structures. An ANN consists of simple highly connected computational units called neurons. ANNs have wide range of applicability and skill to treat complicated problems easily. ANNs are parallel computational models which consist of interconnected processing units. ANN replaces 'learning by example' into 'programming' in solving problems which is its very important feature. Backpropagation algorithm is the most widely used algorithm in Hidden layer based ANN. Various types of ANNs are Multilayered Perceptron, Radial Basis Function and Kohonen networks. In the sense that these have been inspired by neuroscience so they may be called neural. Ni (1996) proposed a method for identification and control of nonlinear dynamic system a recurrent model was applied as identifier [2]. Gupta (1999) presented an improvement to back-propagation algorithm based on the use of an independent, adaptive learning rate parameter for each weight with adaptive nonlinear function [3]. LM algorithm is an effective optimization technique that can guide the weight optimization [4].

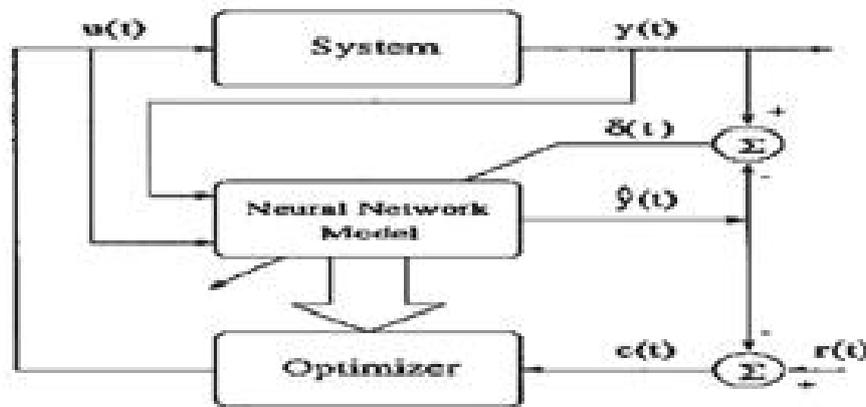


Fig.1 : Neural Network Controller

For state feedback controller design, nonlinear system control, nonlinear dynamical system identification, optimal control synthesis and three-dimensional medical image [5-9], neural network has been widely used.

By introducing a momentum term in the LMS learning algorithm the neural network generalized in [10]. The definition of momentum term is to wake up the learning process and to decrease the zigzag effect during learning. The task of this paper is to develop neural network-based nonlinear controller for IP system and provide a rapid, reliable solution for the control algorithm both online and offline.

BACKPROPAGATION ALGORITHM DESCRIPTION

The back propagation algorithm is used to train the neural network in this work. Assuming that the unknown nonlinear system to be considered is expressed by

$$y(t+1) = f(y(t), y(t-1), \dots, y(t-n), u(t), u(t-1), \dots, u(t-m)) \dots \dots (1)$$

where, $y(t)$ is the scalar output of the system, $u(t)$ is the scalar input to the system $f(\dots)$ is the unknown nonlinear function to be estimated by a neural network, and n and m are the known structure orders of the system. The purpose of this control algorithm is to select a control signal $u(t)$, such that the output of the system $y(t)$ is made as close as possible to a predefined set-point $r(t)$.

Fig. 1 shows the overall structure of the closed-loop control system which consists of:

- 1) The system as shown in Fig.1;
- 2) A feed forward neural network which estimates $f(\dots)$;
- 3) A controller realized by an optimizer.

A two-layer network is used to learn the system and the standard back propagation algorithm [11] and [12] is employed to train the weights. The activation functions are hyperbolic tangent for the first layer and linear for the second layer. Since the input to the neural network is

$$p = [y(t), y(t-1), \dots, y(t-n), u(t), u(t-1), \dots, u(t-m)] \dots \dots (2)$$

The neuro model for the unknown system (1) can be expressed as,

$$\hat{y}(t+1) = \hat{f}(y(t), y(t-1), \dots, y(t-n), u(t), u(t-1), \dots, u(t-m)) \dots \dots (3)$$

Where $\hat{y}(t+1)$ is the output of the neural network and \hat{f} is the estimate off. Since the backpropagation training algorithm guarantees that,

$$[y(t+1) - \hat{y}(t+1)]^2 = \min \dots \dots (4)$$

For this purpose, we define an objective function J as follows:

$$J = \frac{1}{2}e^2(t + 1) \dots\dots(5)$$

Where,

$$e(t + 1) = r(t + 1) - \hat{y}(t + 1)\dots\dots (6)$$

The control signal $u(t)$ should therefore be selected to minimize J . Using the neural-network structure, (3) can be rewritten to give,

$$\hat{y}(t + 1) = w_2[\tanh(w_1p + b_1)] + b_2\dots\dots (7)$$

Where w_1, w_2, b_1, b_2 are the weights and biases matrices of the neural network [13]. To minimize J , the $u(t)$ is recursively calculated via using a simple gradient descent rule

$$u(t + 1) = u(t) - \eta \frac{\partial J}{\partial u(t)} \dots\dots\dots (8)$$

Where, $\eta > 0$ is a learning rate. It can be seen that the controller relies on the approximation made by the neural network. Therefore it is necessary that $\hat{y}(t + 1)$ approaches the real system output $y(t + 1)$ asymptotically. This can be achieved by keeping the neural-network training online. Differentiating (5) with respect to $u(t)$, it can be obtained that

$$\frac{\partial J}{\partial u(t)} = -e(t + 1) \frac{\partial \hat{y}(t+1)}{\partial u(t)} \dots\dots\dots(9)$$

Where $\frac{\partial \hat{y}(t+1)}{\partial u(t)}$ is known as the gradient of the neural network model (or sensitivity derivatives) with respect to $u(t)$.

Substituting (9) into (8), we have

$$u(t + 1) = u(t) + \eta e(t + 1) \frac{\partial \hat{y}(t+1)}{\partial u(t)} \dots\dots\dots(10)$$

The gradient can then be analytically evaluated by using the known neural-network structure (7) as follows

$$\frac{\partial \hat{y}(t+1)}{\partial u(t)} = w_2 [\sec^2(w_1p + b_1) w_1 \frac{d}{d}] \dots\dots\dots(11)$$

Where,

$$\frac{d}{d} = [0, 0, \dots, 0, 1, 0, \dots, 0] \dots\dots\dots (12)$$

Is the derivative of the input vector p respect to $u(t)$.

Finally, (10) becomes

$$u(t + 1) = u(t) + \eta e(t + 1) w_2 [\sec^2(w_1p + b_1) w_1 \frac{d}{d}] \dots\dots\dots(13)$$

MODELING OF INVERTED PENDULUM AND CONTROLLERS

The inverted pendulum is a highly nonlinear and open-loop unstable system. This means that standard linear techniques cannot model the nonlinear dynamics of the system. Before the inverted pendulum model can be developed in simulink, the system dynamical equations are derived using 'Lagrange Equations'. [14] The Lagrangian equations are one of many methods of determining the system equations. Using this method it is possible to derive dynamical system equations for a complicated mechanical system such as the inverted pendulum.

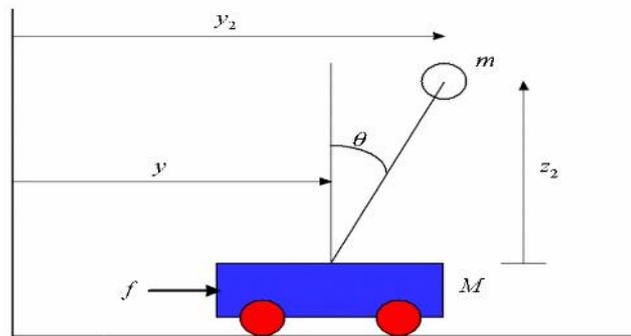


Fig.2: Free-bodied diagram of the pendulum system

Where,

M = Mass of the cart

m = Mass of the pole

l = Length of the pole

f = control force

The Lagrange equations use the kinetic and potential energy in the system to determine the dynamical equations of the cart-pole system. The kinetic energy of the system is the sum of the kinetic energies of each mass.

Kinetic energy of cart,

$$K_1 = \frac{1}{2} M \dot{x}^2$$

Kinetic energy of pole,

$$K_2 = \frac{1}{2} M \dot{x}_2^2 + \frac{1}{2} m \dot{p}^2$$

Where,

$$x_2 = l \sin \theta + x$$

$$\Rightarrow \dot{x}_2 = l \cos \theta \dot{\theta} + \dot{x}$$

and,

$$p = l \cos \theta$$

$$\Rightarrow \dot{p} = -l \sin \theta \dot{\theta}$$

$$\Rightarrow \dot{p}^2 = \dot{\theta}^2 l^2 \sin^2 \theta$$

Total the kinetic energy of the system can thus be formulated as

$$K = K_1 + K_2$$

$$K = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m (\dot{x}^2 + l^2 \dot{\theta}^2 \cos^2 \theta + 2 \dot{x} l \dot{\theta} \cos \theta + l^2 \dot{\theta}^2 \sin^2 \theta)$$

Potential energy,

$$P = mgl \cos \theta$$

In classical mechanics, the natural form of the Lagrangian is defined as the kinetic energy, K , of the system minus its potential energy, P .

$$L = K - P$$

To obtain a closed-form dynamic model of the pendulum, the energy expressions are used to formulate the Lagrangian $L = K - P$. Let the generalized forces corresponding to the generalized displacements $\bar{q} = \{x, \theta\}$ be $F = \{F_x, 0\}$. Using Lagrangian's equation,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = F_j; \quad j = 1, 2$$

the equation of motion is obtained as below,

$$(M + m)\ddot{x} + ml\cos\theta\ddot{\theta} - ml\dot{\theta}^2\sin\theta = F$$

$$ml\cos\theta\ddot{x} - ml\sin\theta\dot{\theta}^2 - mgl\sin\theta + ml^2\ddot{\theta} = 0$$

As θ is very small, for linearization, we take

$$\sin\theta \approx \theta$$

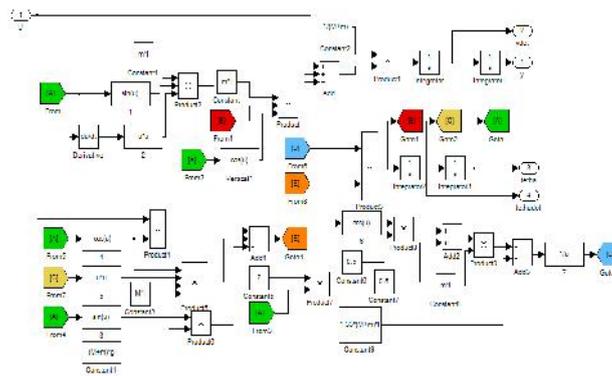


Fig.3: Simulink model of linear pendulum system

And $\cos\theta \approx 1$

Now Linear dynamic equations are,

$$\ddot{x} = -\frac{mg}{M}\theta + \frac{F}{M}$$

And

$$\ddot{\theta} = -\frac{F}{Ml} + \frac{(M + m)g}{Ml}\theta$$

Now, we have a set of equations (linear and non linear) describing the inverted pendulum. The next stage is constructing a simulink model of the inverted pendulum system. There is no procedure for developing simulink models from dynamical state equation. The diagram below is the linear pendulum model.

This model is constructed using integrators, gain blocks, etc. The model (Fig.3) is simply a simulink representation of the linear state equations.

And the non linear model of inverted pendulum system is given below

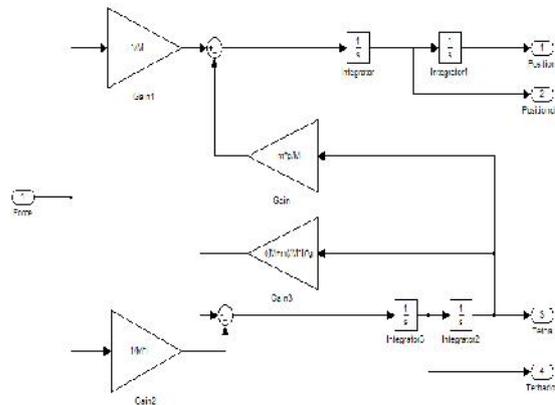


Fig.4: Simulink model of nonlinear pendulum system

DESIGN OF CONTROL LAW FOR NON-LINEAR IP MODEL

Now, here the control law for nonlinear inverted pendulum is shown

The main equation given below calculates the required force, U to keep the pendulum stable.

$$h_1 = \frac{3}{4l} g \sin \theta$$

$$h_2 = \frac{3}{4l} \cos \theta$$

$$f_1 = m \left(l \sin \theta \dot{\theta}^2 - \frac{3}{8} g \sin 2\theta \right) - f_x$$

$$f_2 = M + m \left(1 - \frac{3}{4} \cos^2 \theta \right)$$

$$u = \frac{f_2}{h_2} \left[h_1 + k_1 \theta + k_2 \dot{\theta} + c_1 x + c_2 \dot{x} \right] - f_1$$

For the simulations M, m, l, g are set to the values of the pendulum model. The following numeric values are used: M = 1.2 Kg, m = 0.109 Kg, l = 0.25m, g = 9.81 m/s, k₁=25, k₂=10, c₁=1, c₂= 2.6 [4].

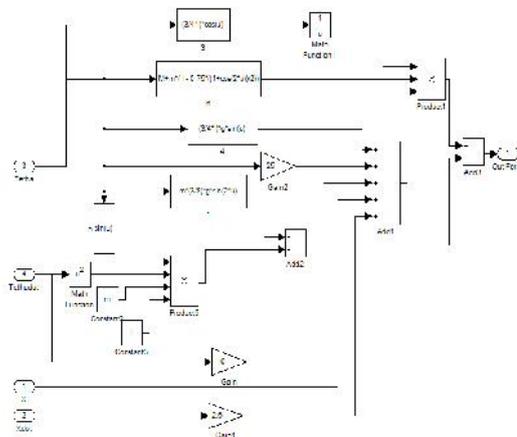


Fig.5: Simulink model of nonlinear control law

The following Fig.6 shows the set-up of the non-linear pendulum with control law.

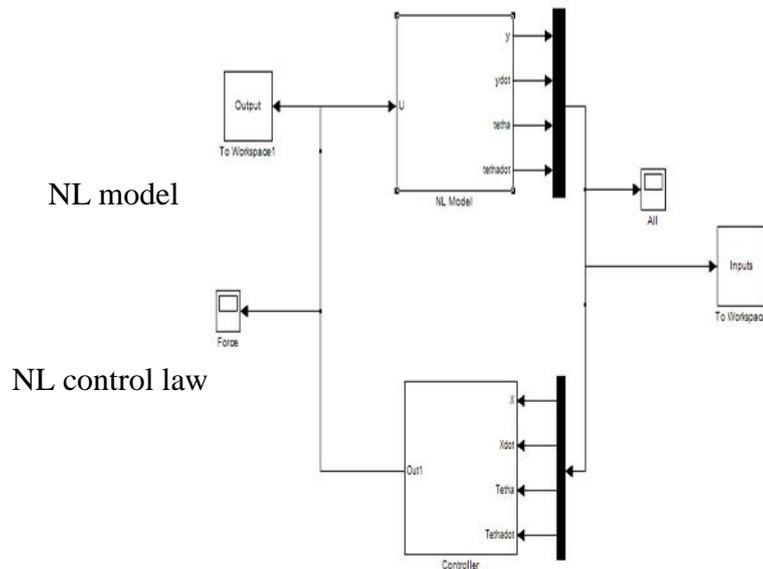


Fig.6: Simulink blocks of non-linear model of IP and controller

NEURAL CONTROLLER OF THE INVERTED PENDULUM

The main task of this project is to design a controller which keeps the pendulum system inverted. There are a few important points to remember when designing a controller for the inverted pendulum. The inverted pendulum is open-loop unstable, non-linear and a multi-output system. Nonlinear system: Standard linear PID controllers cannot be used for this system because they cannot map the complex nonlinearities in the pendulum process. ANN's have shown that they are capable of identifying complex nonlinear systems. They should be well suited for generating the complex internal mapping from inputs to control actions. Multi-output system: The inverted pendulum has four outputs, in order to have full state feedback control four PID controllers would have to be used. Neural networks have a big advantage here due to their parallel nature. One ANN could be used instead of four PID's. Open-loop unstable: The inverted pendulum is open-loop unstable. As soon as the system is simulated the pendulum falls over. Neural networks take time to train so the pendulum system will have to be stabilized somehow before a neural network can be trained. Before the actual neuro-controller is developed in Matlab, the main types of neuro-control are discussed. The five types of neural network control methods that have been researched are supervised, model reference control, direct inverse, internal model control and unsupervised [15].

DESIGN OF THE TRAINED ANN CONTROLLER FOR INVERTED PENDULUM

The offline ANN has been trained by using the training data generated by the feedback control law from the setup shown in the Fig.6, the offline ANN training specification is given as: the number of layers, three layer network (input Hidden and output), network type is feed forward network, network weight optimization technique is Levenberg-Marquardt (trainlm network functions). Input activation function is sigmoid (non linear), the output activation function is pureline (linear). Learning rate is 0.1. Number of maximum iteration (epochs) is 1300. The simulation time is 10 sec and the sample time is 0.01 sec. the error goal is 0. After training the neural net with the above settings the simulation block has been generated using the command Gensim (network_name, desired_sampletime) in the MATLAB command prompt. The simulation model of the trained neural network controller is shown.

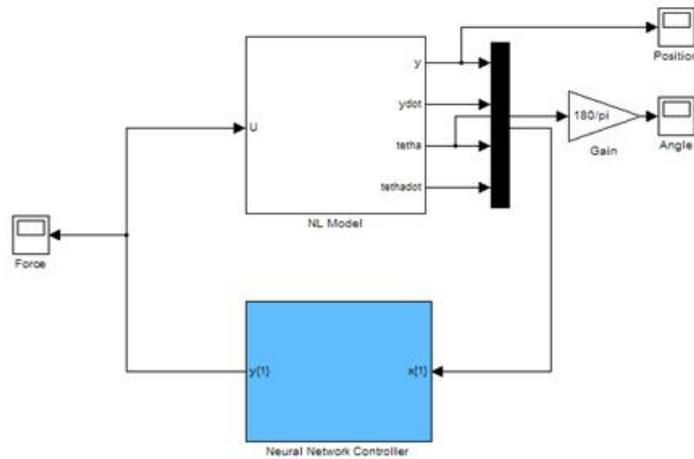


Fig 7: Simulation model of IP and ANNcontroller model

RESULT AND DISCUSSION

The simulation results show the closed loop response of inverted pendulum with and controller systems. First we have done dynamic modeling of inverted pendulum system by the help of Lagrange equation that gives four equations, two equations for the linear system and two for nonlinear inverted pendulum system. Then to reduce the non-linearity with the help of the control law and ANN intelligent controller which are used in feedback after their modeling. Then we take the four parameters of the inverted pendulum system i.e., cart mass $M= 1.2$ Kg, pole mass $m=0.3$ Kg, pole length $l=0.2$ m and gravitational force $g= 9.8$. In this experiment the initial parameters of the inverted pendulum system are $x(0)=0.0$ m, $\dot{x}(0) = 0.0$ m/s, $\theta(0) = 0$ rad, $\dot{\theta} = 0$ rad. These values are putted in the initial condition column of the integrator blocks of nonlinear inverted-pendulum model corresponding to cart position, cart-velocity, pendulum angle and pendulum angular velocity respectively. The simulation time is 10 seconds for all simulations & shows the result in fig 8. Second result of nonlinear inverted pendulum system with ANN intelligent controller and result shown in figure 9.

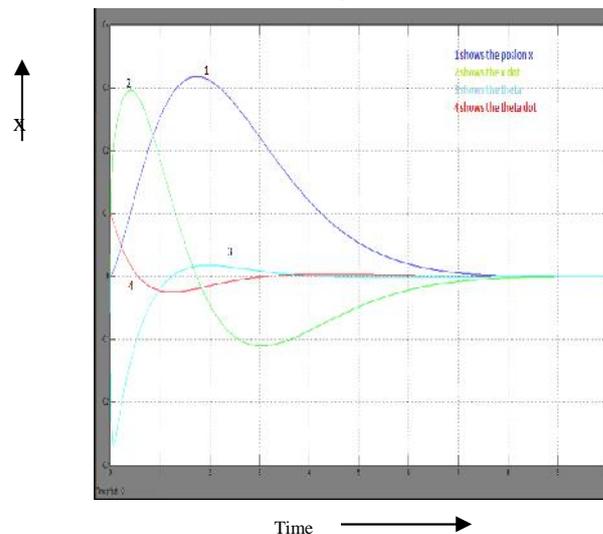


Fig.8: Plot of IP system with control law

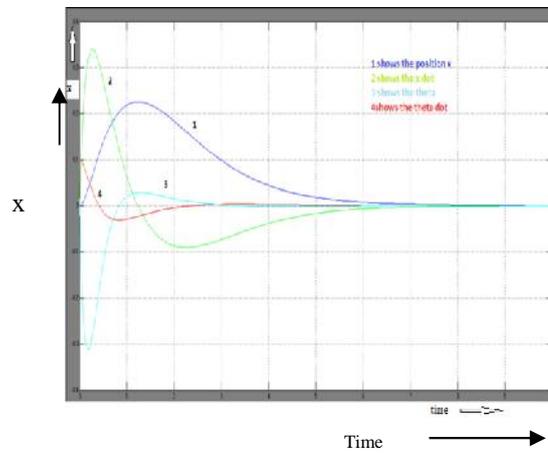


Fig.9: Plot of IP system with intelligent neural network controller

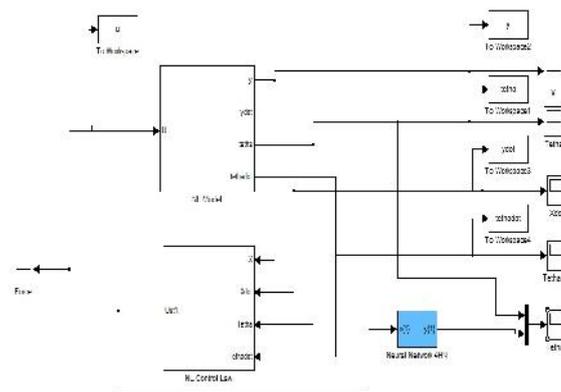


Fig.10: simulation modal of IP modal of conventional controller with hidden layer ANN based intelligent controller

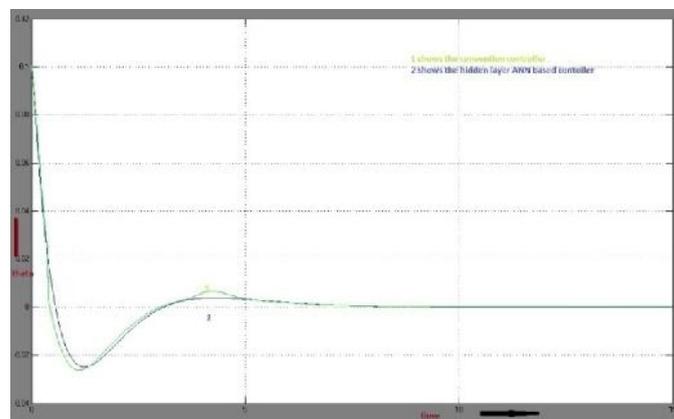


Fig11. Plot of IP system conventional controller with hidden layer ANN based intelligent controller

CONCLUSION

The paper introduced artificial neural networks to control the inverted pendulum system. The advantage of neural network is its nonlinear nature to predict the nonlinearity in the system. To generate supervised data to train the ANN controller, the IP system is stabilized by the control law. The data generated is used to train both type of neural network controllers so that they can mimic the output of control law by optimizing the linking weights and bias values. These trained neural network controllers are tested in MATLAB Simulink with nonlinear inverted pendulum model. ANN proves to be very fast and accurate in online prediction and control. Based on the simulation results proposed the ANN controllers can be used to control the real time IP system. By comparing both results we can say artificial neural network intelligent controller more stable, fast and accurate in comparison to the conventional controller.

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