
A Critical Study on the Empirical Period - Height Relationship for Reinforced Concrete Shear Wall Buildings.

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ABSTRACT

Most of the building codes provide the empirical equations to estimate period of vibration of the building. It is shown that, even though the code formulas provide periods that are generally shorter than measured periods, these formulas can be improved to provide better correlation with the measured data. The present study makes an attempt to gather the possible empirical period- height relationships for RC buildings. It points out the new trends that have become apparent, reflects on the gaps that have been missing in our knowledge and speculates on probable future studies that will allow to enlarge knowledge of this relationship.

KEY WORDS: *fundamental period; height; empirical formula ;RC;MRF*

1. INTRODUCTION

The Fundamental natural period is an important parameter in earthquake-resistant design. Design horizontal acceleration A_h or design horizontal base shear coefficient V_B/W of a building is a function of its translational natural periods in the considered direction of design lateral force. In case of seismic design of building the fundamental natural period of the structure helps in finding out the base shear to be resisted by the structure and mode shape gives the distribution of base shear at every storey. An improved understanding of the global demands on a structure under a known seismic input can be obtained from this single characteristic. A reasonably accurate estimation of the fundamental period is necessary in earthquake analysis of structures. Fundamental period of vibration is dependent on the mass, strength and stiffness of the structure .

At initial stage in design of building, when the exact size of a structural member is not known, the fundamental time period can be calculated by the empirical expression suggested by the seismic code of a country. Simple equations are employed in the seismic design codes to relate the fundamental period to the height of the frame. These equations have conventionally been obtained by regression analysis on the periods of vibration measured throughout earthquakes. Most of the building codes provide the empirical equations to estimate period of vibration from height or number of floors of the building.

Many researchers have attempted to revise these code specified formulae. These formulae were evaluated using available periods measured during past earthquakes. Empirical equations were obtained from the regression analysis. Generally, the code formulae were calibrated to give shorter periods than that measured to produce conservative design seismic forces. However, some times the degree of conservatism may be excessive, while at the other times the period may be overestimated due to the omission of non structural elements in period calculations, leading to non-conservative design forces. Although some improvements have been introduced over the years, these equations still need to be verified and improved, as new data become available after earthquakes.

The present study makes an attempt to gather the possible empirical period-height relationships for RC buildings available in the literature. It points out the progress made and the new trends that have become

apparent, reflects on the gaps that have been missing in our knowledge and speculates on probable future studies that will allow to enlarge our knowledge of this relationship.

2. FUNDAMENTAL NATURAL PERIOD OF BUILDING

Each building has a number of natural frequencies, at which it offers minimum resistance to shaking induced by external effects and internal effects. All of these natural frequencies and the associated deformation shape of a building constitute a natural mode of oscillation. The mode of oscillation with the smallest natural frequency and largest natural period is called the Fundamental Mode; the associated natural period is called the Fundamental Natural Period.

The period of vibration (T) of a single degree of freedom oscillator can be obtained from equation (1):

$$T = 2\pi \sqrt{m/k} \quad (1)$$

Where m is the mass of the oscillator and k is the stiffness. The stiffness of the oscillator can be found from the base shear (V) divided by the lateral displacement (Δ)

2.1 Factors influencing Natural Period

(a) Effect of Stiffness

Increasing the column size increases both stiffness and mass of buildings. But, when the percentage increase in stiffness as a result of increase in column size is larger than the percentage increase in mass, the natural period reduces. Hence, the usual discussion that increase in column size reduces the natural period of buildings, does not consider the simultaneous increase in mass; in that context, buildings are said to have shorter natural periods with increase in column size. Stiffer buildings have smaller natural period.

Factors affecting stiffness

- Length : It is inversely proportional to stiffness.
- Material Properties: Material properties like modulus of elasticity are directly proportional to stiffness.
- Moment of Inertia :It is directly proportional to stiffness

(b) Effect of Mass

Mass of a building that is useful in lateral oscillation during earthquake shaking is called the seismic mass of the building. It is the addition of its seismic masses at different floor levels. Seismic mass at every floor level is equal to full dead load plus suitable fraction of live load. The fraction of live load depends on the intensity of the live load and connection with the floor slab. Seismic design codes of each country/region provide portion of live loads to be considered for design of buildings to be built in that country/region. Heavier buildings have larger natural period

(c) Effect of Building Height

As the height of building increases, its mass increases but its in general stiffness decreases. Thus, the natural period of a building increases with increase in height. Taller buildings have larger natural period

d) Effect of Column Orientation

Orientation of rectangular columns influences lateral stiffness of buildings along two horizontal directions. Hence, changing the orientation of columns changes the translational natural period of buildings. Hence, natural period of buildings along the longer direction of column cross-section is smaller than that along the shorter direction. Buildings with bigger column dimension oriented in the direction reduces the translational natural period of oscillation in that direction.

(e) Effect of Unreinforced Masonry Infill Walls in RC Frames

In many countries, the space among the beams and columns of building are filled with unreinforced masonry infills. These infills take part in the lateral response of buildings and as a consequence alter the lateral stiffness

of buildings. Therefore, natural periods of the building are affected in the presence of URM. Natural Period of building is lower when the stiffness contribution of URM infill is considered.

3. FUNDAMENTAL NATURAL PERIOD OF RC INFILL MASONRY AND SHEAR WALL BUILDINGS IN DESIGN CODES

The approximate formula proposed for RC MRF building with infill walls and concrete or masonry shear wall building had the form as presented in Eq. (2), where H is the height and D is the dimension of the building at its base, in feet, in the direction of the applied force. This formula is used in many seismic codes but the type of structures to which it is applied varies from country to country.

$$T = \frac{C_t H}{\sqrt{D}} \quad (2)$$

In ATC3-06 (ATC, 1978) C_t was proposed as 0.05, when H and D are in feet, this would become 0.09 when these dimensions measured in meters. As discussed by Pinho and Crowley (2010) formula comes from the equation of the frequency of vibration of a cantilever (considering shear deformation only), with the thickness of the wall considered to be more or less constant and thus only the width/length of the building is an input parameter, as presented in eq. (3).

$$T = \sqrt{\frac{m}{G}} \frac{H}{\sqrt{A}} = \frac{\alpha G H}{\sqrt{D} t_w} = \frac{\alpha_1 H}{D} \quad (3)$$

Where m is the mass per unit length, G is the shear modulus, α is the shape factor to account for non-uniform distribution of shear stresses, D is the length of the cantilever, t_w is the thickness. Some codes use this formula specifically for buildings with both frames and shear walls, some use the equation for reinforced concrete MRF with masonry infill panels, but many specify it for use with any building except moment resisting space frames. Many countries adopted such formula, including, but not limited to, are ATC3-06:1978 (ATC, 1978), IS 1893-1984 (IS 1893-84, 1984), KBC 1988 (KBC88, 1988), NBCC 1995 (NBCC-95, 1995), IS 1893: 2002 (IS 1893 (Part I), 2002), IS 1893 Proposed (Jain & Murty).

NEHRP-94 (NEHRP-94, 1994) has mentioned the eq. (2), for Shear wall buildings in U.S., where the numerical coefficient $C_t = 0.02$ has to be used when H is the building height measured in feet and $C_t = 0.05$ when height of the building measured in the metre above the base. Such formula is generally recommended for other structure category and can found in IRAN:1999 (2800, 1999), NZ 2004 (NZS 1170.5:2004 SUPP, 2004), Taiwan:2005 (Taiwan, 2005), NBCC:2005 (NBCC-05, 2005), Korea:2005 (KBC-05, 14 2005), AS 1170.4-2007 (AS 1170.4-07, 2007). Korean code (KBC-05, 2005) uses the exact value of $C_t = 0.049$ whereas rest of the codes rounding it up as 0.05.

SEAOC-96 (SEAOC-96, 1996) and UBC-97 (UBC-97, 1997) suggest an alternative value of C_t as described in eq.(2) to be used in eq.(4) when height is measured in feet.

$$C_t = \frac{0.1}{\sqrt{A_c}}; \text{feet} \quad (4)$$

Where A_c is the combined effective area (in square feet) of the shear walls, and is defined as shown in eq. (5) in which A_i is the horizontal cross-sectional area (in square feet); D_i is the dimension in the direction under consideration (in feet) of the i^{th} shear wall in the first storey of the structure; and NW is the number of shear walls.

$$A_c = \sum_{i=1}^N A_i \left[0.2 + \left(\frac{D_i}{H} \right)^2 \right]; \quad D_i/H \leq 0.9 \quad (5)$$

The numerator of eq. (2) becomes 0.075 when the dimensions of the structure are measured in metres. This equation found its way in Euro Code (EC8-2004, 2004) and Korean code (KBC-05, 2005).

In 1998, Goel and Chopra started working to improve the empirical formulae to estimate the fundamental vibration period of concrete shear wall (SW) buildings. Study shows the inadequacy of the code formulas, such as NEHRP-94 (NEHRP-94, 1994), SEAOC-96 (SEAOC-96, 1996), UBC-97 (UBC-97, 1997) and ATC3-06 (ATC, 1978), for estimating the fundamental period of concrete SW buildings. They discussed how there is little correlation between the H/D of eq. (2) and the period of vibration. This could be because the shear walls do not extend for the whole dimension D of the building, but for just a small proportion. Whereas eq. (3) was found to be more correlated to the period as it was including explicitly the dimensions of the walls, but this also found too conservative.

An improved formula Eq.(6) is developed by calibrating a theoretical formula, derived using Dunkerley's method (Inman, 1996), against the measured period data for the nine concrete SW buildings (17 data points) which responded to eight Californian earthquakes starting with the 1971 San Fernando earthquake and ending with the 1994 Northridge event. This equation has been included in ASCE 7-05 (ASCE/SEI 7-05, 2006) and ASCE 7-10 (ASCE/SEI 7-10, 2010).

$$T_L = \frac{0.0}{\sqrt{C_W}} H \quad (6)$$

Where H is the height of building in feet and C_W is given by eq.(7).

$$C_W = \frac{1}{A_B} \sum_{i=1}^N \left(\frac{h_i}{h_i} \right)^2 \frac{A_i}{\left[1 + 0.8 \left(\frac{H_i}{D_i} \right)^2 \right]} \quad (7)$$

Where A_B is the base area of the structure in square feet; A_i is the area of shear wall i in square feet; D_i is the length of shear wall i in feet; h_i is the height of shear wall i in feet; N is the number of shear walls in the building effective in resisting lateral forces in the direction under consideration.

4. STUDIES FOR ESTIMATION OF FUNDAMENTAL PERIOD OF VIBRATION

Housner and Brady (1963) concluded that no single, simple, empirical equation will give reasonably accurate estimates for the periods of buildings having shear wall characteristics. Theoretical analysis was carried out for an idealised building with shear walls with expressions derived from the Rayleigh method. They compared results with the expressions given by Californian Building Code of 1960 and measured periods on 77 steel and RC buildings during the 1993 Long Beach earthquake in California. Researchers concluded that good estimate of shear wall buildings could be obtained only if the actual wall stiffness was considered.

Goel and Chopra (1998) evaluated code formula for RC shear wall buildings. They collected available data on the fundamental vibration period of buildings measured from their motions recorded during several California earthquakes. They carried out the regression analysis of 16 RC Shear wall building measured data to develop formulas to estimate fundamental periods of the buildings. It was shown that current code formula for estimating the fundamental period of concrete shear wall buildings are grossly inadequate. Subsequently, an improved formula is developed by calibrating a theoretical formula, derived using Dunkerley's method, against the measured period data through regression analysis. Also recommended a factor to limit the period calculated by a rational analysis such as Rayleigh's method.

Yong et al.(2000) carried out full-scale measurements out on fifty RC apartment buildings in Korea, and the reliability of the current empirical code formula was evaluated. Results shows that the measured period in the longitudinal direction was longer than that in the transverse direction, which shows that this type of buildings are very different dynamic characteristics from those the current KBC would tend to imply.

Balkaya C. (2003) stated that the Shear-wall dominant multi-storey reinforced concrete structures constructed by using a special tunnel form technique had been built in countries facing a major seismic risk, such as Chile, Japan, Italy and Turkey. They investigated the uniformity of equations in those seismic codes. It was observed that the empirical equations for prediction of fundamental periods of this specific type of structures produce results. For that reason, a total of 80 different building configurations were analyzed by using three-dimensional finite element modelling, and a set of new equations was proposed. This study demonstrated that

current earthquake codes overestimated the results of finite-element analyses for rectangular shaped plans and most of the time underestimated the periods for square shaped plans. This observation was due to omission of torsional disturbance as a parameter in the code defined equations. In fact, torsion was an exceptionally important criteria appearing in the dynamic mode of those structures that should have been taken into account in the design phase. As a result of these intensive exercises and analyses, the fundamental period estimation equation (8 & 9) took the form given below:

$$T = Ch^{b1}\beta^{b2}\rho_a^{b3}\rho_a^{b4}\rho_m^{b5} J^{b6} \quad (8)$$

$$J = I_x + I_y \quad (9)$$

Here, T is the period in s; h is the total height of the building in m; β is the ratio of long side to short-side dimension; ρ_a is the ratio of short-side shear-wall area to total floor area; ρ_a is the ratio of long-side shear-wall area to total floor area; ρ_m is the ratio of minimum shear wall area to total floor area; C, b1, b2, b3, b4, b5 and b6 are the parameters to be investigated by regression analysis; J is the polar moment of inertia of the plan given in Equation.

Faouzi Ghrib et al (2004) investigated the fundamental period of shear wall buildings considering the flexibility of the. Experimental and analytical approaches were used to assess the effect of the base flexibility on the fundamental period of shear wall structures. In total, twenty buildings built on different types of soil are tested under ambient vibration. An analytical approach was used for improved the estimation of the fundamental period of shear wall buildings.. The analytical predictions improve the estimation of the fundamental period and keep the computation simple. The error between the measured period and the analytical results is, on average, less than 10%.

Oh-Sung Kwon et al (2010) evaluated building period formulas in seismic design code over 800 apparent building periods from 191 building stations and 67 earthquake events. The evaluation was carried out with the formulas in ASCE 7-05 RC shear wall buildings. Qualitative comparison of measured periods and periods calculated from the code formulas was conducted. The formula for shear wall buildings overestimates periods for all building heights. Based on these observations, it is suggested to use C_r factor of 0.015 for shear wall buildings.

Damien Gilles et al (2012) studied ambient motions recorded in 27 reinforced concrete shear wall (RCSW) buildings in Montréal to inspect how various empirical models to predict the natural periods of RCSW buildings compare to the periods measured in actual buildings under ambient loading conditions. They concluded that a model in which the fundamental period of RCSW buildings varies linearly with building height would be a significant improvement over the period equation suggested in the 2010 National Building Code of Canada. Models to predict the natural periods of the first two torsion modes and second sway modes are also presented, along with their uncertainty.

George and Kanapitsas (2013) recommended an empirical formula for estimating the fundamental period of RC structures on the basis of the analysis of the available data for the fundamental periods of 20 real 3-D buildings, evaluated by vibration analysis. This formula simultaneously considers the height and the length of the building under consideration, the soil flexibility, and the contribution of shear walls and external and internal infill walls.

Farid Chalah et al (2014) proposed a simplified formula for assessing the fundamental vibration period of SW buildings which takes into account the geometrical and the mechanical characteristics of the structure. In this work, the Dunkerley formula and hypothesis on constant height and mass of the floor was used. It allows the period assessment by a direct and simplified relationship expressed in terms of mechanical and geometrical characteristics and the total floors number of the SW building as given in equation (10).

$$T_1 = 1.8n(n + 1)[mh^3/(E)]^{1/2} \quad (10)$$

where:

E : Young modulus, I : level inertia in m^4 , h : floor height, m: floor mass.

Magdy I. Salama(2015) used the available data for the fundamental vibration period of reinforced concrete shear wall buildings measured from their motions recorded during eight California earthquakes. Improved formulas for estimating the fundamental period of vibration (T) of concrete shear wall buildings were developed by regression analysis of the measured period data. The results indicate that the value of coefficient Ct in the current US and Egyptian building codes formula should be decreased from its present value 0.02 to 0.014. Also, factors to limit the period calculated by rational analysis, such as Rayleigh's method, are recommended in this paper. Comparisons between the periods determined using the proposed formula and the measured values show good agreement.

Marijana(2015) performed a parametric study on a RC structure models with shear walls with varied building height, number of bays and ratio of shear wall area to floor area. The differences between obtained periods for different models with equal percentage of walls and different layouts with the same percentage of walls led to conclusion that the number of bays should also be taken into account in expressions for the fundamental period of RC SW structures.

Pulkit and Ramancharla (2017) carried out ambient vibration tests for 21 RC shear wall buildings, located in Mumbai and Hyderabad cities, by placing vibration sensor on topmost accessible floor. The measured periods have been compared with the code provisions. It is found in the study that as the height of the building increases, natural period is not linearly proportional to height; rather it is becoming flexible. The following empirical Eq.(11) for estimating the fundamental natural periods was suggested.

$$T = 0.01 H^{1.1} \quad (11)$$

Where, H is height of building from the base (in m)

5. CONCLUSIONS

The fundamental time period is an primary parameter in seismic design methodologies Hence, accurate estimation of this parameter is very necessary as it affects the seismic design of structural members. The review of previous literature shows that the code proposed expressions were based on regression analysis performed on the data set consisting of experimentally determined period of few buildings located in a certain region. Extensive literature review suggested that code limits on the period are too conservative. The database must had been expanded to include the results from the new earthquake data. Therefore, there is a need of improved empirical expressions to estimate the fundamental time period for buildings.

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