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Numerical Study of 1-D Heat Conduction in Non-Uniform Grid by Finite Volume Method

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ABSTRACT

Among the three Finite Volume based methods (Explicit, Implicit and Crank-Nicolson) to solve the simple diffusion problems, here explicit method has been used where new temperature of any grid point is calculated using the previous time step value of the concerned grid and its adjacent neighbours and used in the discretised unsteady 1D heat conduction equation. The time step size to maintain the stability of the method is taken into account as well. The code has been developed in Scilab, which is an open source programming language. The temperature distributions have been plotted with and without heat generation. The results can be varied by varying different thermo physical properties of the material and boundary conditions. The result verification has been done by comparing with analytical results obtained by the relation of plane wall with uniform heat generation with different wall temperature.

Keywords

Physical law based FVM, Explicit method, Non-uniform grid, Cell center, Face center, Heat flux, CV, Unsteady, 1-D heat conduction.

INTRODUCTION

Although few papers on conduction problem [1] can be found by finite Difference Method, but no paper has been found on pure conduction by FVM [2]. In FVM, the problem domain is subdivided into certain no. of control volume or cell and each grid point is at the centre of each cell, also called cell centre. As the volume of the cell is very small, the volumetric property of each cell is considered as the property of cell centre. Same approximation is considered for the face or surface of the control volume, in which the value of incoming or outgoing flux at face centre is taken as the value of entire surface perpendicular to the flow direction. This type of approach is called physical law based FVM [3]. For the present study, in explicit method the temperature of any grid point is calculated using the temperature of previous time step value. Using the approximation for the volumetric term and flux term, the algebraic formulation of energy conservation equation in 1-D case, for a representative CV as shown in figure 1 with the previous and present time step as n and n+1 is;

$$\rho \ \frac{T_P^{n+1}-T_P^n}{\Delta t}\Delta x_p = (q_w^n - q_e^n) + q_{g\ \ ,p}\Delta x_p \qquad (1)$$

A second order central difference based approximation for heat flux at west and east boundary of CV is given as respectively

$$q_w^n = -k\frac{T_P^n-T_W^n}{\delta x_w}\text{And}q_e^n = -k\frac{T_E^n-T_P^n}{\delta x_e} \qquad (2)$$

The limiting interval of time step for two consecutive picture of temperature field for non-uniform grid is given as,

$$\Delta t = \frac{\Delta x_p \delta x_e \delta x_w}{\alpha(\delta x_e+\delta x_w)} \qquad (3)$$

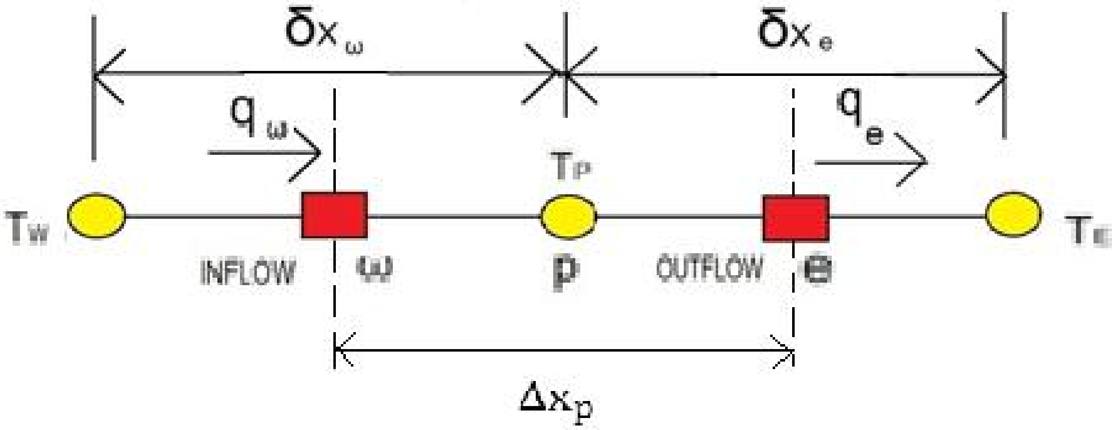

Figure 1: Details of A Control Volume

IMPLEMENTATION DETAILS:

2.1.**Problem Description:** In this study, a 1 cm thick steel sheet has been taken of 7750 kg/m^3 density , 16.2 W/m K thermal conductivity and 500 J/kg K specific heat which has an initial uniform temperature of 30^0C and is suddenly subjected to a constant temperature of 0^0C and 100^0C on two sides of the sheet. Assuming 1D heat conduction along the thickness of the sheet, the domain is subdivided into 12 grid points including 2 boundary of constant temperature for numerical study. Each grid point or cell center is at the centre of each CV or cell except at boundary where cell center and face center of side of a CV merge with each other. A heat generation of magnitude 100 MW/m^3 is also taken into consideration.

2.2.**Non Uniform Grid Generation:** Algebraic method is used to generate a non-uniform 1-D Cartesian grid. This method involves a transformation of a uniform grid in a ζ-coordinate based 1-D computational domain of unit length, to a non-uniform grid in a x-coordinate based physical domain of length L; using an algebraic equation [4] given as

$$x = L\frac{(\beta+1)[(\beta+1)/(\beta-1)]^{(2\zeta-1)}-(\beta-1)}{2[1+\{(\beta+1)/(\beta-1)\}^{(2\zeta-1)}]} \qquad (4)$$

Where, β is a clustering parameter which controls the fineness of grid size at two boundaries and gradually coarser grid size at the middle. For this study the value of β has been taken as 1.2.

2.3. **Geometrical Parameters:** The coordinates of the face center of the CVs x_i are computed by a non-uniform Cartesian grid generation method. The x_i'sare used to compute the other geometrical parameters as follows:

Face center: $x_i \;\; f \quad i = 1\; t\; i\eta \qquad -1$

Cell center: $xc_i = \frac{x_i + x_{i-1}}{2} \;\; fo \;\; i = 2\; t\; i\eta \qquad -1$

$xc_1 = x_1 \;\; a \qquad xc_{i\eta} \qquad = x_{i\eta} \quad {}_{-1}$

Width of CVs: $\Delta x_i = x_i - x_{i-1} \;\; f \quad i = 2\; t\; i\eta \qquad -1$

Distance between cell centers: $\delta x_i = xc_{i+1} - xc_i \;\; f \quad i = 1\; t\; i\eta \qquad -1$

(5)

E, P, W, e, and w of figure 1 are replaced by the running indices *i*+1, *i*, *i*-1, *i*, and *i*-1, respectively. This method is adopted for easier coding in computer as only one parameter *i*is required for calculating all other parameter s and simultaneously taking less computational time.

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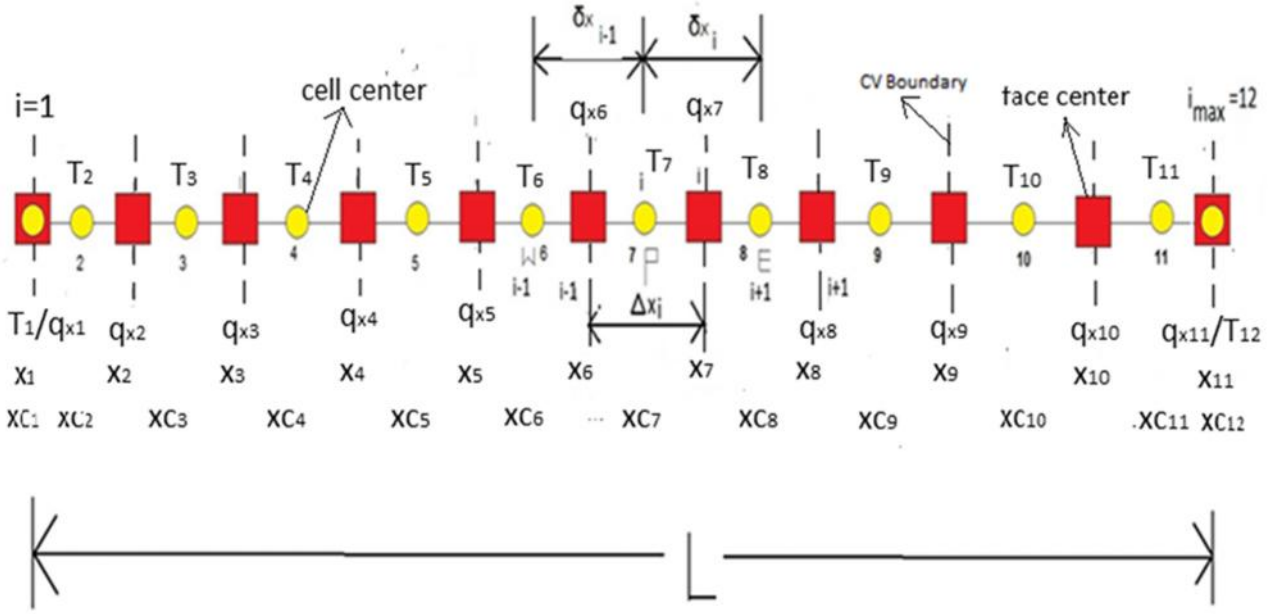

Figure 2: Details of problem Domain

RESUTS AND DISCUSSIONS:

The coding has been done in Scilab which is an open source programming language and generates the following plots. In Figure 3 the problem domain of thickness 0.01 m with finer grid in two boundary regions and coarser grid in the middle is demonstrated. By changing the value of β near to unity finer grid can be generated.

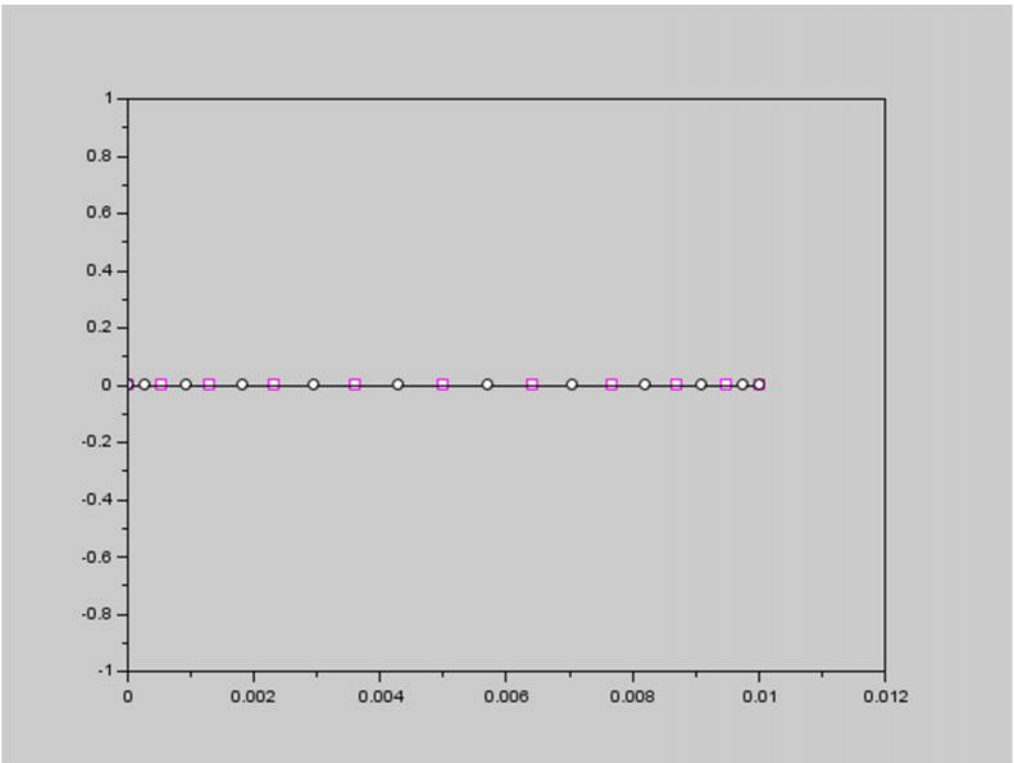

Figure 3: Non-Uniform Grid Structure

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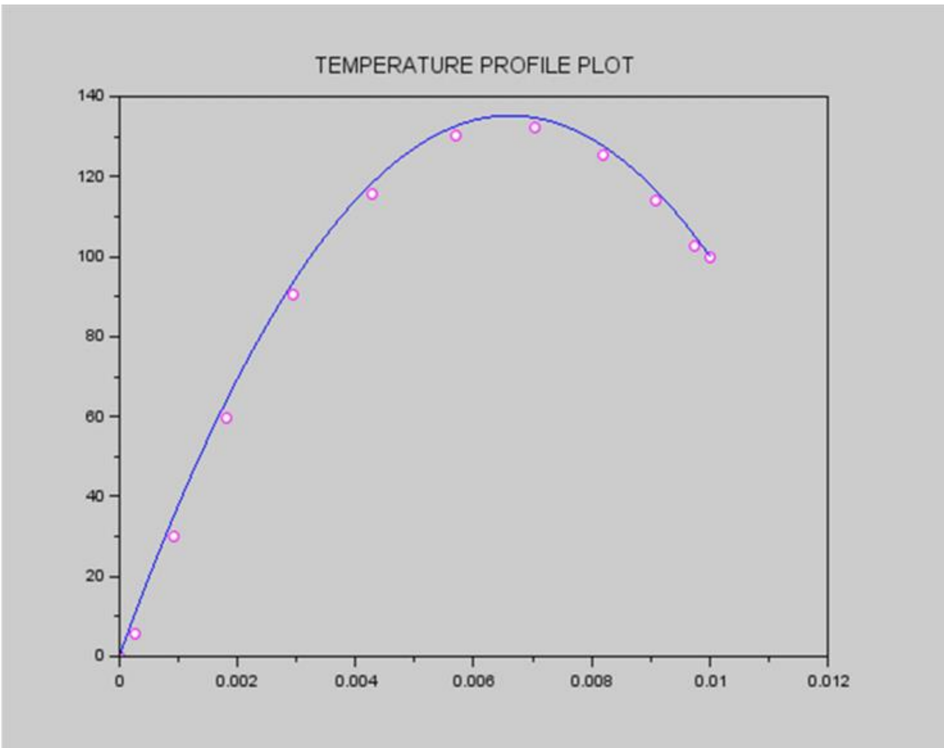

Figure 4: Temperature Profile with Uniform Heat Generation

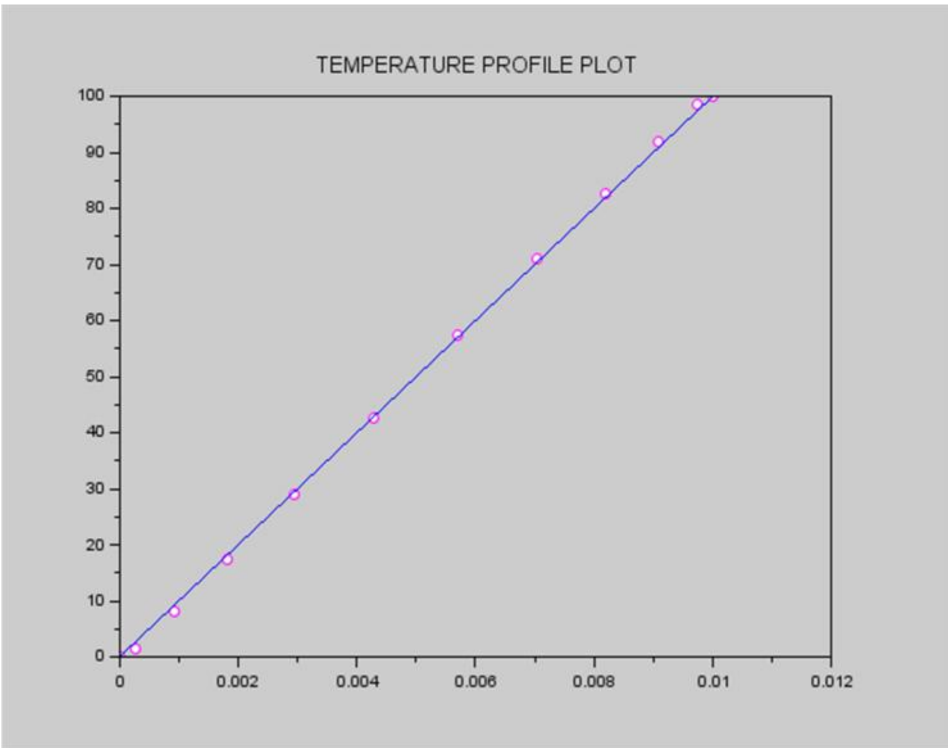

Figure 5: Temperature Profile without Heat Generation

Figure 4 & 5 plotted the temperature distribution along the thickness of sheet with and without heat generation respectively. The position of maximum temperature and the value of maximum temperature can be found out from the following two analytical relations. The numerical value obeys the analytical values.

The maximum temperature occurs at,

$$x = \frac{k(T_1 \ -T_1)}{Lq_g} + \frac{L}{2} \quad (6)$$

The value of maximum temperature [5],

$$t_m \quad = \left[\frac{q_g}{2k}\left\{L - \frac{k(T_1 \ \ - T_1)}{Lq_g} - \frac{L}{2}\right\} + \frac{T_1 \ \ - T_1}{L}\right]\left[\frac{k(T_1 \ \ - T_1)}{Lq_g} + \frac{L}{2}\right] + T_1 \quad (7)$$

CONCLUSION:

This study will help to understand the basic of physical law based FVM in non-uniform grid, which can be implemented to different heat transfer problems with complex geometry. The verification is also done with analytical solution which is nearly accurate. This also authenticates the usefulness of Scilab coding.

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