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Capital Investment Analysis for Reparable Aviation Assets : Use of Reduced Gradient Algorithm .

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ABSTRACT:
Investment analysis of Capital assets are carried out using traditional time discounted appraisal methodologies like Net present value , discounted cash flow and Internal rate of return methodologies. When the Capital item is maintenance intensive, recurring cash outflow due to maintenance requirements have to be factored in along with revenue generated by the asset. The amount of recurring maintenance expenditure depends on the initial investment in reliability and maintainability efforts. Accordingly, there are alternate investment options which needs to be evaluated in terms of investment in reliability and maintainability in terms of life cycle costs. Traditional algorithms take care of evaluating discrete investment options. When the investment options are continuous , there is a need to find the optimal mix of investment package. As long as the relationships are linear, Simplex method can be used to find the optimal solution. However reliability and maintainability functions are usually nonlinear following the law of diminishing returns. This paper explores the use of Calculus based algorithm like the gradient descent algorithm for evaluating capital investment options. A computer program is developed to carry out Capital investment analysis of reparable assets using gradient descent algorithm. The algorithm is illustrated by a case study on reparable helicopter asset.

Key words: Aviation asset maintenance, Continuous function, Design investments, Maintainability, Multi-objective optimisation, Reduced Gradient, Reliability.

1. BACKGROUND

Core industrial sector like mining, refining, infrastructure, Aerospace and ship building employ capital intensive assets which need to be operated at high levels of availability to ensure business continuity and profitability. Adequate attention needs to be placed at the time of new asset procurement and upgradation to ensure that the investment yields steady and continuous returns. Decision making on investment in capital assets forms a major managerial activity of Project Managers, Investment planners and Entrepreneurs. Capital equipment investment decisions have long standing impact on business performance and need to be carefully analyzed.

Aviation Capital Assets like aircrafts and helicopters involve huge investments with long pay back periods. They need a high level of safety and reliability for sustained operations. These assets need high levels of maintenance during the life span of the equipment to maintain the required operational availabilities.

2. INTRODUCTION

2.1 Aviation asset reliability

Flying assets like Aircrafts and helicopters need to have a high level of reliability for it to take to the skies. Military and civil regulatory agencies place a lot of importance on high reliability and safety of the equipment.

Reliability is the probability of an asset to successfully achieving its mission. It can also be defined as the inverse of the Mean Time Between Failure (MTBF). This feature is built in during the design and manufacture of the asset. Some methods of improving reliability include design robustness, systems engineering approach, redundancy in the system, process control, certification and training, etc., Design robustness is achieved by building in factor of safety beyond 1.5 in the specifications to account for noise factors in the operating environment. Systems Engineering approach looks at arriving at aircraft configuration taking into account all interfacing systems and not designi9ng in isolation. Certain systems are mission critical and endanger the safety of the crew. In such cases, a standby system is built into the equipment. Examples are additional engine in an aircraft, dual control systems in a helicopter, parallel pumping systems in a process industry etc., Systems engineering approach, process controls and certifications are built into the organizational systems and templates and do not involve additional costs. However, Design robustness and redundancy involve additional upfront investments. They are technology dependent and prove to be cost prohibitive beyond a particular level. Reliability investments should be consistent with performance requirements and revenues thereof.

2.2. Aviation asset maintainability

An Aircraft is a high value capital investment with advanced technologies and system complexities. It calls for high levels of safety, reliability and quality assurance requirements. This is achieved through a series of planned maintenance interventions. Airborne assets are highly maintenance intensive. Other than the scheduled maintenance, a lot of unscheduled maintenance, checks and repair activities are required to be carried out to maintain the high levels of safety and reliability.

The operator holds adequate amount of spares as maintenance inventory to replace unserviceable units and put the unserviceable unit back into the repair loop. The time it takes for the replacement and recycling depends on the depth of repair facility available at the operating bases and the availability of intermediate repair depots. Maintainability is measured as the Mean Time To Restore System and is a combination of investment in insurance spares and base repair infrastructure.

A lot of research has been carried out on optimal spares provisioning by the operator to minimise his operating costs yet maintain the performance desired, while research is sparse on impact of reliability, reparability and maintainability on the life cycle cost. With the increasing complexity of the technology used on the platforms and the increased investment and planning associated with maintenance, operators have realised the need to evolve optimum levels of maintainability to ensure availability of the Aircraft.

2.3. Life cycle costs of aviation assets

The cost of maintenance and operations over the life cycle is quite significant compared to the acquisition costs. Table 1 shows the relative costs of various typical aerospace products which reveals the significant elements of costs that need to be considered while making an investment appraisal for an Aviation asset.

TABLE 1: LCC OF TYPICAL AEROSPACE ASSETS

In Rs. Crores

Type of asset	Life cycle cost		Acquisition cost		Operation Cost		Maintenance cost		Salvage cost
Luxury Car	0.22	=	0.12	+	0.09	+	0.04	-	0.03
Helicopter	120	=	40	+	40	+	60	-	20
Aircraft: Military	520	=	300	+	200	+	100	-	80
Aircraft : Civil	680	=	350	+	300	+	150	-	120

Source : Conklin & Decker 2013

One method of reducing maintenance costs is investment in product reliability, but this entails an upfront investment. As investment in reliability is increased, the Mean Time Between Failure (MTBF) increases. This means lesser number of repair withdrawals over the life cycle and hence lesser maintenance costs. There is a particular value of reliability investment beyond which the benefits of lesser maintenance costs is more than offset by the investment there-of. Any investment in reliability beyond the threshold value will increase the total Life Cycle Cost as reflected in the Graph 1.

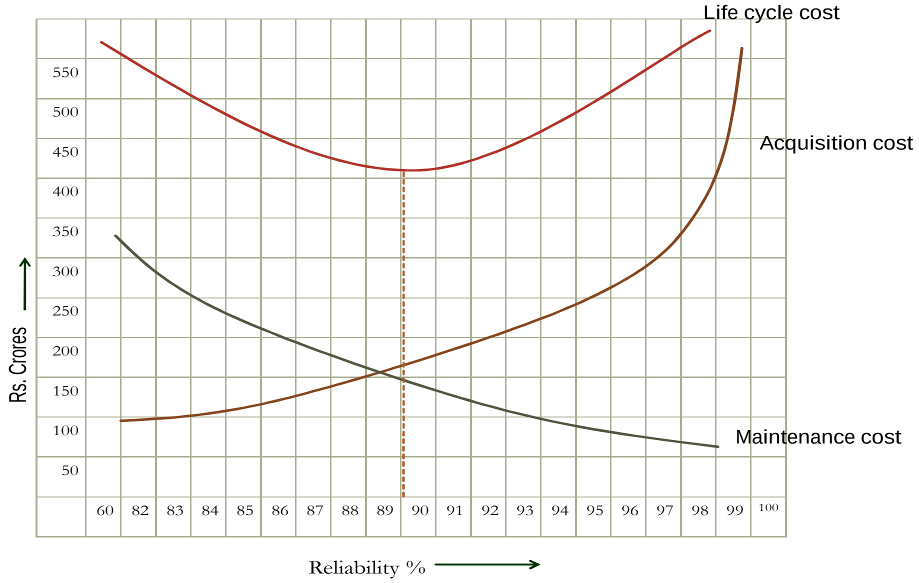

FIGURE 1 : Reliability Investment and its impact on Life cycle cost

2.4. Multi objective optimisation function

While the primary objective of Capital investment appraisal is to reduce the Life Cycle Cost (LCC) of the asset, there are other objectives and constraints that need to be considered during Capital investment appraisal. Even though Least life cycle cost is achieved at a particular level of investment in Design (Reliability) and maintenance infrastructure (Maintainability), the asset procuring organisation may not have adequate capital resources to invest in the asset. The limited initial investment has to be optimally invested in a combination of design investment, insurance spares investment and base infrastructure investment.

Having invested in a Capital asset, the operator will want to obtain maximum revenues from utilisation of the asset. For this, the asset should be available for productive deployment for a maximum amount of time. The availability of an asset is given by

Ao=MTBF/(MTBF+MTTRS)

MTTRS= f(MLDT, MTTR)

where, Ao = Availability

MTTRS = Mean Time To Restore System

MTBF = Mean Time Between Failure

MLDT = Mean Logistics Down Time

MTTR = Mean Time ToRepair

A more reliable asset has more available time for productive use during its life span than a less reliable asset. A less reliable asset may have a low life cycle cost but may be inoperative for a significant period of its life. These assets are classified as Non Productive Assets (NPAs) of the organization. As the NPAs of an organization keep increasing, more number of assets are procured to build in redundancy to maintain the required performance requirements of the organization.

If reliability can be improved (even beyond the LCC ordained value) to ensure more number of available days in the asset’s life cycle, the productivity of the organization can increase. Revenue generation for the customer is a direct function of availability. Another method to ensure that the asset is available for more number of days is to turn around the asset faster when it becomes unserviceable. The Mean Time To Restore System

(MTTRS) is a function of Mean Time To Repair (MTTR) and Mean Logistics Down Time (MLDT). MTTR depends on the level of insurance spares stocked at the repair base so that the asset does not need to wait for inventory availability. MLDT depends on the depth of repair possible at the repair base and the number of repair bases available so that the time spent in moving the unserviceable asset to repair depot and back is reduced.

Hence the multiple objectives of Capital investment analysis are to choose an optimum combination of investment in reliability, maintainability and serviceability to obtain targeted availability at not to exceed initial investment and targeted life cycle costs. Such Multi Objective Optimisation Problems (MOOP) can be solved using enumerative methodologies or can be converted into linear inequalities and solved using Simplex methodology. Where the constraints take on continuous values, enumerative method will fail. Further, if the objective function is a nonlinear function of the constraint variables, traditional linear programming methods like the Simplex method will not yield the result.

2.5 Gradient descent algorithm

Calculus based approaches can be used to solve nonlinear multi objective optimisation functions. Calculus based algorithms work on the principle of identifying that point on the objective function map which has the minimum slope in any direction. Such a point is an inflexion point. There are likely to be many inflexion points for a given surface representing the objective function. The goal is to identify the global minima from the local minima. The partial differential of the objective function with respect to one independent variable is the slope of the surface in that direction. Hence by evaluating the partial differentials with respect to each of the independent variables the slope in each direction can be found out. The objective would be to minimise the slope in all directions.

Calculus based algorithms which depend on identifying the least gradient are called gradient descent algorithms or reduced gradient algorithms. These are first order optimization problems in the sense that they consider first derivative of the objective function. To find the local minimum of a function using gradient descent, it is required to take steps opposite to the gradient of the approximate function at the current point. The equation is converted into a linear approximation at the point of computation and the first order derivative is computed. Then the point is moved by an incremental step opposite to the direction of the gradient so that the function slips down to a smaller value than the initialized value. By iterative incremental slipping down the gradient, the function descends to the minima around the initial value.

3. LITERATURE SURVEY

Life cycle costing (LCC) models are gaining importance in initial investment analysis of high cost, long life assets requiring maintenance intervention through their life cycle. The type and depth of maintenance required for such reparable assets depends on the nature of initial investments carried out in design, spares management and repair philosophy. These are decided in the Capital acquisition stage itself. Hence appropriate investment model needs to be evolved in the initial stage of equipment procurement so as to optimize life cycle costs.

LCC was originally designed for procurement purposes in the US department of Defence and is mostly used predominantly in the Military and Construction Industry. The Life cycle costing guidelines issued by the New South Wales treasury [2004] sets the guidelines for various stages of life cycle cost analysis. The ISO standards for life cycle costing in the construction industry is researched by Davis Langdon Management Consulting [2006] as part of a EU funded project.The study provides an analytical hierarchical approach to arriving at elements of life cycle costs and whole life costs. Though it deals primarily with the construction industry, the framework can be used in any costly asset needing periodic maintenance.

Use of Life cycle costing approach has been limited predominantly in military and construction industry. A total of 205 published articles on use of LCC approach in decision making has been reviewed by Eric Korpi et al [2008] of Helsinki University of Technology. The analysis revealed that many of the LCC analysis was done from the buyer point of view with a fewer LCC analysis from Manufacturer point of view. There was no case of LCC involving manufacturer maintaining the product also. Design trade off requirements triggered LCC analysis in about 25% of the cases which justifies design efforts being evaluated from an LCC point of view.

Cryut & Ghobbar discuss the impact of supportability in terms of spares inventory on incremental availability for a given level of reliability of Aircraft. The paper focuses on entry into service period of aircraft and utilizes Monte Carlo simulation for stochastic arrival patterns. It brings out that reliability has a greater impact on availability compared to supportability in terms of spares stocking.

Linking of the Life cycle cost to availability of the asset has been carried out by Dinesh Kumar et al in which the significance of total cost of ownership has been brought out. A case study on railway wagons is carried out to bring out relevant costs of ownership. A mathematical model for determining availability and total life cycle costs is made to evolve a criterion for evaluation of alternate decision parameters. The concept of arriving at availability from MTBF has been used in this subject paper.

Analysis of Life Cycle costs and capital investments to optimise the same involves multiple and sometimes conflicting objectives involving other than life cycle costs, initial investments, revenues from operations and the corresponding levels of reliability, maintainability and availability. These objectives are converted mathematically into inequalities with the above variables. The Multi Objective Optimisation Problem (MOOP) is worked out with the objective to find good trade-offs to find an optimum solution.

In MOOP, optimum is what is defined by Francis YsidroEdgeworth an Irish economist in 1881 and later generalised by Vilfredo Pareto in 1896. In a Multi Objective Optimisation problem, optimum is defined as that set vector “x” such that there exists no other feasible solution which would decrease some criterion without simultaneously increasing one or more of the other criteria. Hence the Pareto solution is not one single solution but a set of solutions referred to as the Pareto front. The model developed has to reflect the influence of the various decision variables on the objectives of the optimisation. Once the model is formulated, various methodologies are explored to solve the model.

The earliest and popular method of solving multi objective decision making problems has been the Goal programming approach based on George Dantzig’s Simplex algorithm. Dantzig [1987] initially evolved the Simplex procedures for standardising the processes of decision making in the US Army. It was initially used to solve unconstrained equations and later on updated to include linear multi objective constrained optimisation. Dantzig’s method was expanded to solve inequalities by Charnes and Cooper [1984] who brought in the concept of positive and negative deviational variables and modified the optimisation function to minimise the net deviations. However this model was limited to single objective priority. When conflicting objectives were introduced, The model could not handle it simultaneously.

Ijiri [1965] brought in the concept of pre-emptive priority levels to handle conflicting goals in order of their importance. Simultaneous handling of pre-emptive goals was presented by Lee [2016] by introducing separate columns for positive and negative decision variables. However this increased the computation complexity. All the above research provided flexibility to solve multi objective optimisation models but it necessitated linearity of the constraint functions. Graphical methods of resolving multi-objective optimisation also provide a set of intuitive points of optimisation even for nonlinear functions but this is limited to maximum of three variables.

Dinesh Kumar, David Nowicki, Marquez et all [2007] have utilised linear goal programming to optimise reliability, maintainability and supportability under Performance Based Contracting. Robert A Vraciu [1980] has modelled investment and financing decisions in Hospital management using Goal programming approach for choosing among alternate services options. Nijhoff [1981] has similarly utilised this approach for financial planning and Capital budgeting . All the above applications convert constraints into linear inequalities and optimise the corner feasible points.

One of the limitations of linear optimisation models is the necessity of linearity of the constraint functions. When the functions become nonlinear, Goal programming methods donot provide the solutions. A calculus based Reduced gradient algorithm was first proposed by Marguerite Frank and Philip Wolfein [1956] and is popularly known as the Frank-Wolfe algorithm. This method has been generalised to consider all types of non linear objective functions (both convex and concave) by Abadie and Carpentier [1969]. This method called the Generalised Reduced Gradient (GRG) method has found wide applications in solving nonlinear multi objective optimisation. The computation efforts required for the GRG methodology cannot be carried out manually as the number of variables increase. It is necessary to develop computer based algorithm which can be solved on the computer. This increases the speed of solving as well as the number of variables that can be factored in.

Calculus based algorithms are used where one or more of the functions are nonlinear in nature. Engineering applications include aerofoil design optimisation, optimising parameters in rapid prototyping, fluid flow dynamics etc.,. Managerial applications include financial auditing by sampling to reduce audit errors , river basin quality management, multi objective scheduling etc.,

This paper explores the use of Reduced Gradient algorithm to optimise capital investment for high value reparable asset constrained by limits of initial investment while attempting to maximise revenues and minimise life cycle costs. For using such an algorithm it is first required to convert the problem statement into a set of mathematical inequalities. Once the problem has been defined and modelled as a mathematical formulation, various methodologies can be applied to obtain a Pareto set of solutions.

4. PROBLEM DEFINITION

This paper addresses the problem of selecting an optimal mix of decision functions to meet the multiple objectives of the Capital investment for reparable aviation assets used in revenue generating activities. The asset being considered is a medium tonnage helicopter used for revenue generation by civil operators. The operator is funding the design and development of the aircraft to meet his exacting standards of operational availability. The operator needs to choose the amount of investment he needs to do in the initial design and fabrication of the helicopter. The amount of money invested in design and realisation of the equipment depends on the levels of reliability that needs to be incorporated to achieve the availability levels. Further, the operator also determines the investment required in insurance spares and the depth of repair capability at his operating base that he needs to invest in so that the helicopter is airborne at the earliest after it is withdrawn for scheduled or unscheduled maintenance activity. The number of flying hours available during the life cycle of the helicopter determines the revenue earning capability of the asset.

The operator would like to invest more upfront in design of the product as well as the maintenance support systems so as to accrue reduced costs during maintenance as well as reduction in down time. However, he has a cap on the maximum funds that he can invest in the asset.

The investment appraisal comprises of identifying the quantum of investment to be carried out in design of the product and the quantum to be invested in base infrastructure necessary to obtain the targeted revenues. A model for such an investment analysis is sought to be developed in the paper. A mathematical model will be developed relating the various variables involved. The paper will explore alternate methodologies to solve the multi-variate resource optimisation problem and illustrate it with a case study of a helicopter platform.

5. MODEL FORMULATION FOR AVIATION ASSET MAINTENANCE

A mathematical model for investment strategy involves identification of the objective functions and the decision variables that impact the objective function. The intermediate variables that relate the two also need to be identified, defined and relationship established.

As discussed earlier, in the capital investment of helicopters the objective functions are two:

1. Minimising initial investment (i) and
2. Maximising operating revenues (y)

The decision maker, while seeking to attain the above objectives has two decisions to make:

1. Amount to be invested in Design to achieve the levels of reliability (d)
2. Amount to be invested in maintenance infrastructure to attain the levels of maintainability (b).

The first objective function can be written as

$$i = d + b \tag{1}$$

The second objective function is to maximise operating revenues.

Operating revenues (y) = Revenue from operation (R) – Cost of Maintenance (m)

Revenues from the operation is a function of hours flown per year (h)

$$\text{Revenues}(R) = (h * K_1 * p)*(1-r)^p$$

Where K_1 represents the revenues per hour of flying , "p" represents the number of years of operation of the asset and "r" is the discounting factor. The hours flown per year is given by

$$h = \frac{\text{Uptime}}{(\text{Uptime} + \text{Downtime})}$$

$$= \frac{\text{MTBF}}{(\text{MTBF} + \text{MTTRS})}$$

Where MTBF = Mean Time Between Failure
MTTRS = Mean Time To Restore System
$\text{MTBF} = K_3/\boldsymbol{\lambda}$

Where
K_3= Number of flying days in a year
$\boldsymbol{\lambda}$ = Failure rate in a year

The failure rate λ is given by

$$\lambda = f(d)$$

$$\lambda = \frac{K_3}{(K_6 * d + K_{10})} \qquad (2)$$

The constants K_6 and K_{10} represent the impact of design investment on failure rate, with K_{10} representing the base reliability and K_6 representing the reduction in annual failure rate with increase in design investment.

Further,

MTTRS= MTTR + MLDT

Where
MTTR (μ) = Repairability :Mean Time To Repair
MLDT (L) = Maintainability : Mean Logistics Down Time

$$\text{Hence, } h = \frac{K_3/\lambda}{[(K_3/\lambda)+ (\mu+L)]} \qquad (3)$$

The Mean Time To Repair μ depends on the DFMA (Design For Maintenance and Assembly) investment in design and can be represented as

$$\mu= K_{11} + K_7/ d \qquad (4)$$

Here, K_{11} represents the standard time required to position the helicopter into the maintenance facility, open up the cowlings and fairings and remove the unserviceable subsystem from the helicopter. This duration is fairly constant for a class of helicopters. The term "K_7/ d" the additional investment in design for assembly and maintenance which affects the duration required to repair the subsystem and put it back into service.

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The Mean Logistics Down Time (L) depends on the depth of repairs possible at the operating base and the level of insurance spares stocked in the repair base. When the helicopter operates from more than one base an additional decision on whether to have repair facilities at only one base or to duplicate the facilities at either of the bases has to be taken,. This will convert the effect of repair base investment on Mean Logistic Down Time into a stepped function and is represented as

$$L = K_8/s + K_9/b \qquad (5)$$

Here K_8/s represents the logistics downtime waiting for spares due to stock out position and K_9/b represents the logistic downtime waiting for facility to be available including time required to tranship the unserviceable system to the next available repair facility.

The impact of design investment (d) on MTTR and the relationship on base investment (b) on MLDT can be represented graphically as in Fig 2 below:

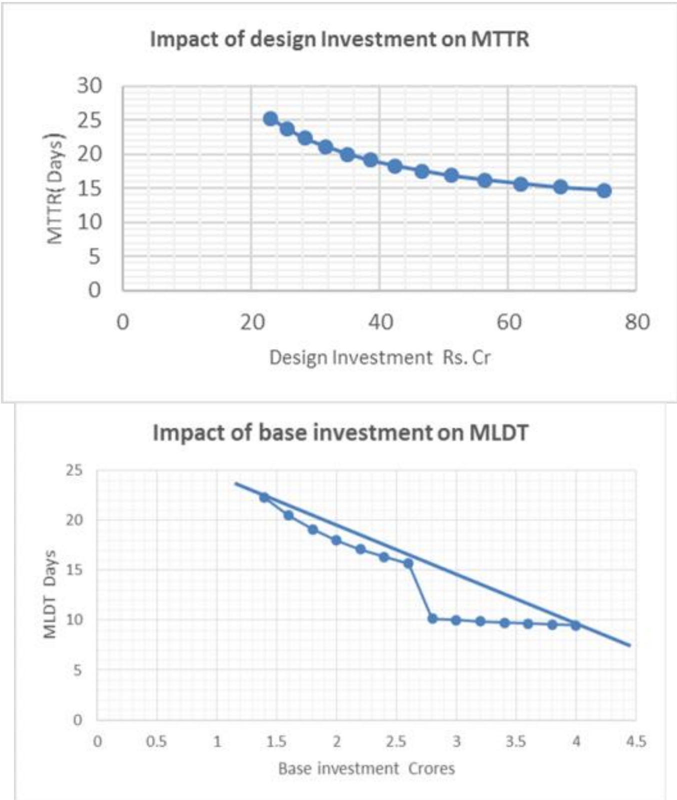

FIGURE 2 : RELATION BETWEEN INVESTMENT AND MTTR / MLDT

Replacing the values of , μ and L from Equation (4) and (5) in Eqn (3) ,

We get

$$h = \frac{(K_3/\lambda)}{(K_3/\lambda) + (K_{11} + K_7/d) + (K_8/s + K_9/b)} \qquad (6)$$

Cost of maintenance of an Aircraft has three components: snag rectification maintenance to rectify defect before turning around the aircraft for next flight, periodic maintenance mandated by design to maintain continued airworthiness and major overhaul of the aircraft after specified flying hours.

Accordingly, Maintenance cost (m)

$m = (\lambda * C_i * p + p/p_{pm} *C_{pm} + p/p_o * C_o) *1/(1-r)^p$

Substituting these values, we get the second objective function as

Maximise operating revenues given by

$$y = (h*K_1 – ((K_3/(K_6* d + K_{10}) *C_i * p + p/p_{pm} * C_{pm} + p/p_o * C_o) * p * 1/(1-r)^p \quad (7)$$

Where,

h	= hours flown in a year
d	= investment in design
λ	= failure rate in a year
p	= period of operations
p_{pm}	= Periodicity of preventive maintenance
p_o	= Periodicity of overhaul
C_i	= Cost of repairs
C_{pm}	= Cost of periodic maintenance
C_o	= Cost of major overhaul
r	= Rate of interest (Discount factor)

Introducing slack and surplus variables to convert the functions into a Goal programming model, we get the objective function as

Minimise $W_i * D_i^+ + 0 * D_i^- + 0 * D_y^+ + W_y * D_y^{--}$

Subject to constraints

a) $i = d + b – W_i * D_i^+ + 0 * D_i^-$ (8)

b) $y = (h*K_1 – ((K_3/(K_6* d + K_{10})* C_i * p + p/p_{pm} * C_{pm} + p/p_o * C_o) * p * 1/(1-r)^p - 0 * Dy^+ +W_y* Dy^-$ (9)

c) $d, b, p, s, D_i^-, D_i^+, D_h^-, D_h^+ >_= 0$ (10)

(Non negativity constraints)

The weights for negative deviation on investment and the positive deviation in operating revenues is given as zero since the objective is to minimise investment and maximise operating revenues. Different methodologies are adopted to optimise the multi-variate non-linear inequalities of the Goal programming model.

6. MULTI VARIATE RESOURCE OPTIMISATION

6.1 Enumerative algorithm

This is the most rudimentary method of inequalities and can be utilized when the decision variables take on a limited number of discrete values and the number of decision variables are small. When the variables take on Integer values only they are said to be discrete. When they can take on decimal values also between the bounds they are called continuous variables. Suppose there are "i" number of decision variables each of which take on "j" number of values, then it would be required to enumerate i*j combinations of the values of the decision variables and evaluate the values of the constraint variables for each such combination.

In our present model we have two decision variables design investment(d) and base infrastructure investments. Assume that each of them can take on 4 possible values at four states 1,2,3 and 4. That is, design investment has four options d_1, d_2, d_3 and d_4 each of which provides different values of reliability and

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maintainability. Base investment "b" has four levels of investment depending on the depth of maintenance planned b_1, b_2 , b_3 and b_4. Then, the decision envelope can be represented by the matrix

$$\text{Decision matrix} = \begin{bmatrix} d_1 & d_2 & d_3 & d_4 \\ b_1 & b_2 & b_3 & b_4 \end{bmatrix}$$

Now, for each set of decision, the objective function has to be evaluated. Accordingly, for this set, there will be $4^2 = 16$ combinations of decisions. Each decision will result in two values, one for initial investment and revenues per year. There is a trade off between minimising initial investment and maximisung revenues which needs to be carried out. Each of the sixteen decision sets have to be evaluated to identify the optimum mix of constraint variables in terms of minimum deviation from the constraint set. To compute this manually would take a long time. Computer simulation programs like the Linear Discrete Optimisation software (LINDO) can be used to evaluate and find the optimized value.

When the number of decision variables is less and the number of discrete values they adopt is also small, a complete enumeration method can be adopted. However, when each of these increase, the number of solutions to be enumerated increases exponentially. Even computer assisted computations may become tedious. In our present model, both design investment and base infrastructure investment take on continuous values. Hence, to use enumeration method, it is required to identify approximate nearness to discrete values, identify the optimal solution and fine tune it with incremental improvements from the base value so computed.

6.2 Simplex approach

When the decision variables in a goal programming model take on continuous values, linear optimisation using Simplex algorithm is the standard method to solve multi-variate inequalities. The Simplex algorithm was invented by George Dantzig in 1947 and is used to solve multiple objective optimization problems using the Gauss-Jordanian computation methodology. By this process, the algorithm tests adjacent vertices of a feasible set in sequence so that in each new vertex, the objective function improves till no further improvement is possible. The iterations proceed along the line of one objective function upto its vertex where-after it shifts to another line representing another constraint equation as represented in Fig. 3.

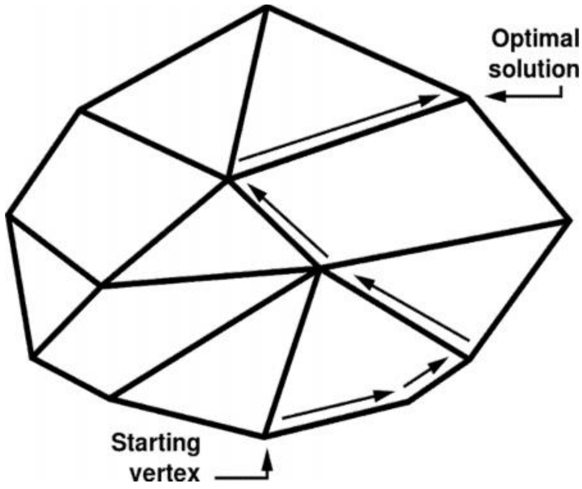

FIGURE 3 : FLOW OF ITERATIONS IN SIMPLEX ALGORITHM

The Simplex process requires a straight line approach from one vertex to another. If the function is non-linear, Simplex Algorithm fails to arrive at the optimum values. Let us analyse the use of Simplex algorithm for the model formulated for aviation asset maintenance.

There are three intermediate variables used in the model which needs to be examined for linearity :

i) Reliability expressed as Failure rate per year (λ)

Eqn (2) defines Failure rate per year (λ) as

$$\lambda = K_3 / (K_6 * d + K_{10})$$

This equation is nonlinear.

ii) Reparability expressed as Meantime to repair (μ)

Eqn (4) defines the mean Time To repair (μ) as

$$\mu = K_{11} + K_7 / d$$

This equation is nonlinear.

iii) Maintainability (Mean Logistics Down Time (L)

Eqn (5) defines Mean Logistics Down Time (L) as

$$L = K_8/s + K_9/b$$

This equation is also nonlinear.

The nonlinearities can be seen from Fig 2. When there are nonlinear intermediate functions, Simplex method will not be able to optimise the Goal Programming model.

6.3 Calculus based approach

When the relationship between the dependent and the independent variable is nonlinear and curved caclulus based approaches can be utilised. In this approach, the curve is broken down into a set of infinitesimally small linear segments the slope of which is determined by the first order differential at that point. By incrementally increasing the value of the independent variable, the slope at the succeeding point is computed iteratively till we arrive at the minimal slope point which determines the point of inflexion.

The most frequent calculus based nonlinear algorithm is the Generalised Reduced Gradient (GRG) algorithm. Calculus based algorithms work on the principle of identifying that point on the objective function map which has the minimum slope in any direction. Such a point is an inflection point. There are likely to be many inflection points for a given surface representing the objective function. The goal is to identify the global minima from the local minima.

Calculus based algorithms which depend on identifying the least gradient are called gradient descent algorithms or Reduced Gradient Algorithms. These are first order optimization problems in the sense that they consider first derivative of the objective function. Based on the sign of the derivative, the point is moved by an incremental step opposite to the direction of the gradient so that the function slips down to a smaller value than the initialized value. By iterative incremental slipping down the gradient, the function descends to the minima around the initial value. Appropriate choice of the initialisation values have to be taken or the process carried out at multiple initialisation points so that the global optimum value is obtained. This is true if the variable has multiple inflections.

Let us evolve the algorithm for application and computerisation of the Reduced Gradient algorithm. The steps involved are as under :

a) Let Minimise y = f (x) be the objective function. Here ,“x” is the independent variable and “y” is the constraint variable which we are trying to optimize. Let “x_i” be the initial value of “x”. The corresponding value of “y” computed using the function is “y_i”. Hence (xi, yi) is the initial point.

b) Next we find the first order partial differential of “y” with respect to “x” i.e., δy/δx. This gives the slope of the graph. If the value of δy/δx is negative , it means the graph at the initial point when approximated to a straight line has an increasing trend (Refer Figure 4) . Accordingly, “x_i” has to be reduced by a small amount in the opposite direction. Let the increment multiplier be Δ.

$x_1 = xi - \Delta * \delta y/\delta x$

c) Check whether the new value x_1 is within the bounds. If so, continue with the next iteration. If the new value of "x" has reached the bound, then the solution is reached.

d) This example is a non linear inequality in one variable. If the inequality has more than one variable, then the partial derivatives of each of the variables with respect to the objective function is computed and the variables are released in the direction of the steepest gradient. If one of the variable has reached the bounds, that variable is kept constant and the other variables are improved. The objective function moves along the bound line till all the constraints are achieved. The steps are repeated till a convergence is achieved. As the convergence nears, it will be required to reduce the value of the increment so that the function does not enter into continuous flip flop on either sides of the minima leading to instability.

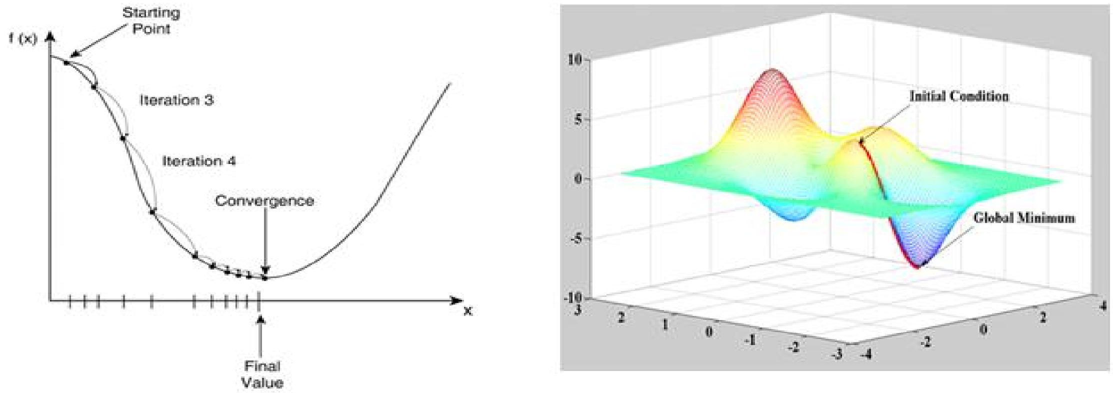

FIGURE 4 : GRAPH OF REDUCED GRADIENT METHOD FOR SINGLE AND MULTIPLE INDEPENDENT VARIABLES

The graphs of single variable inequality and multivariable inequality is shown in Fig 4 above. By incorporating non linear constraint equations and arbitrary bounds, the scope of the algorithm can be expanded. This algorithm is called the Generalised Reduced Gradient (GRG) algorithm. The Algorithm for the Generalised Reduced Gradient method is as below :

ALGORITHM

Step 1 : Initialisation

Initialise$(x_n)_i$

Step 2 : Identifying direction

Compute $f(x_n)_i$

Compute $f'(x_n)$

where $f'(x_n)$ is the first order partial derivative w.r.t. x_n

Identify steepest descent direction x_s

Step 3 : Step size determination

Compute $\Delta = f''(x_n)$

Step 4 :Updation

New value of $x_s = x_s - \Delta * \delta y/\delta x$

If new value of x_s reaches bounds

Move in next steep gradient

Else

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Repeat Step 3

Step 5 : Convergence

If (x_n) reaches bounds for all values of n

Reduce step size

Continue to Step 3

Step 6 : Stop function

If |f'(xn) | <= ε where ε is a convergence limit

Stop

Print (x_n)final

End

The Flow chart illustrating the algorithm is placed in Fig 5 below.

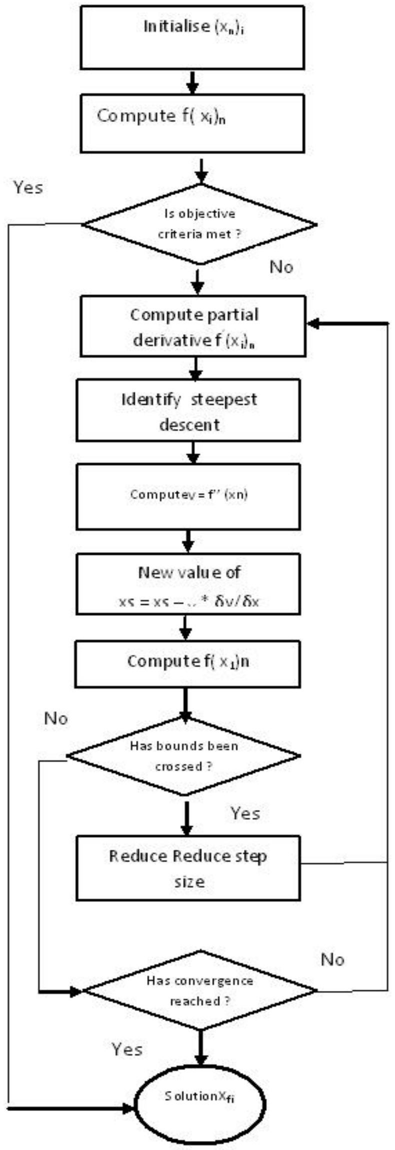

FIGURE 5 : FLOW CHART OF REDUCED GRADIENT ALGORITHM

7. USE OF REDUCED GRADIENT ALGORITHM FOR CAPITAL INVESTMENT APPRAISAL

Formulation of the model for capital investment analysis and solving the same using Generalised Reduced Gradient algorithm is illustrated by the case study on helicopter procurement. An investment appraisal exercise is instituted by an organization intending to fund a design and manufacture of a custom built helicopter along with its support infrastructure for operation as a Heli-Taxi for transport and tourism. An evaluation period of 10 years was taken and a discount factor of 10% over future revenues. The organization wants to maximize the revenues from the operations while minimizing the initial investment.

7.1 Evolving the capital appraisal model

A robust model has to be built for capital appraisal using Goal programming method. For this, first it is essential to arrive at the targeted goals, evolve the constraint inequalities and convert the inequalities into equations using weighted deviation variables as outlined in para (5). Let us first arrive at the goals of the model.

Goal 1**:** The Company has limitations on investable cash based on the position of its reserves and surplus as well as running and upcoming projects which need investment. Hence, it has to maintain a target value of initial investment for the helicopter. A market survey of helicopters world-wide existing in the category is placed in Table 2

TABLE 2 : COMPARATIVE COST OF HELICOPTERS

Rs. Crores

AW 109	AW 109a	AW 109C	AW 109GN	AW 109 GRAND	AW 109K2	AW109 POWER	AW109 Trekker	BELL205a
34.63	**34.63**	**36.87**	**57.54**	**51.99**	**42.02**	**44.08**	**57.54**	**43.13**

Accordingly, the organization has set of Rs. 40 Crores for a custom built indigenous helicopter including base infrastructure and insurance spares. The targeted estimates are as below :

a) Design , Development & Testing : 1500 Lakhs
b) Product realization & certification : 2000 Lakhs
c) Base infrastructure : 300 Lakhs
d) Insurance spares : 150 Lakhs
e) Contingencies : 50 Lakhs
TOTAL : 4000 Lakhs

Accordingly, the first goal is

Minimize $i \leq$ 40,00,00,000 (11)

Goal 2: The operator expects a net annualized returns of atleast 25% on the investment by operations. Aircraft operations business is risky, cost sensitive as well as capital intensive. The gestation period and payback periods are large. The pricing policies should enable attracting adequate market at the same time provide long term profit. A time period of 10 years is considered for appraisal purposes.

Revenue target = $25/100*4000000*10 / (1- 0.13)^{10}$

= 4025385462, say Rs. 40,00,000

Accordingly, the second goal is

Maximize $y \geq$ 40,00,00,000 (12)

Converting the goals into the Goal programming format and introducing the deviation variables and penalty weights, we get the objective function as

$i = f(d, b,) - W_i * D_i^+ + 0 * D_i^- = 40,00,00,000$

$y = f(h, c,) - 0 * D_y^+ + W_y * D_y^- = 40,00,00,000$

By substituting the functions from eqns (1) and (7), the Goal programming equation set becomes

Minimise $Wi * D_i^+ + 0 * D_i^- + 0 * D_y^+ + W_y * D_y^-$

Subject to

a) $i = d + b - W_i * D_i^+ + 0 * D_i^- = 40,00,00,000$

b) $y = (h*K_1 - ((K_3/(K_6 * d + K_{10}) \lambda * C_i * p + p/p_{pm} * C_{pm} + p/p_o * C_o)* p * 1/(1-r)^p$
$- 0 * Dy^+ + W_y * Dy^- = 40,00,00,000$

c) $d, b, p, s, D_i^-, D_i^+, D_y^-, D_y^+ >= 0$

The values of the constants K_1 to K_{10} and the costs for various levels of maintenance depends on the type of helicopter and is estimated as part of investment appraisal.

7.2. Solution using reduced gradient algorithm

The Goal programming model evolved above contains constraint variables, objective variables and intermediate variables. It also contains constants representing the interaction between the variables. The Reduced Gradient algorithm involves incremental iterative partial derivative with respect to each independent variable and computation of the objective function set at each point till the minima is reached. The algorithm is defined in Fig 4 of Para (6.2).

It is not possible to carry out the iterations manually. A computer software program is written to code the steps involved. The custom program is written using Python language on Microsoft platform. The initial values of constants and the independent variable are fed into the software. The iterative steps are computed sequentially until there is no change in the values of the objective function. This determines that the inflection point is reached. It is possible that the nonlinear equation has multiple inflection point. It is required to ensure that the program does not gravitate towards a local minima but determines the global minima. For this purpose, the program is run repeatedly using different initialized values for the independent variables till the lowest deviation is obtained.

The formulation of the computer model is placed at Table 3. The problem is attempted to be solved using traditional Simplex algorithm. For this, the EXCELSOLVER package is used. The software fails to compute a feasible solution set. The error message encountered during processing is because one or more of the functions is non linear. The model is then subjected to the custom software developed for reduced gradient algorithm.

The algorithm brings out a least gradient at a design investment of Rs. 36.59 Cr. and base investment of Rs. 2.88 Cr. While the revenue target of Rs. 40 Cr. is achieved, it will require an additional outlay of Rs. 1.48 Cr. in initial investment. The initial investment increases from the target value of Rs.40 Cr. to Rs. 41.48 Cr.

In order to hold the initial investment limit of Rs. 40 Cr. a second iteration is carried out by relaxing the revenue generation limit. While maintaining the initial investment limit of Rs. 40 Cr,., the maximum revenues that can be generated is Rs. 37.73 Cr.. The corresponding values of investment in design and base infrastructure are Rs. 35.36 Cr. and 2.63 Cr. respectively. The helicopter operates at 278.20 hours per year of flight. The Reliability achieved is 2.98 failures per year. The Mean Time To Repair MTTR is 19.89 days and the Mean Logistics Down Time is 15.60 days. The decision maker can using these iterations decide on an optimal value of investments by suitable trade offs between the two objectives A sensitivity analysis can be carried out to understand the impact of variation in actual investments. The results of the Reduced Gradient algorithm is placed in Table 4. The gradient slope is depicted in Fig 6.

TABLE 3 :FORMULATION OF THE COMPUTER MODEL

MULTI OBJECTIVE OPTIMISATION : CAPITAL INVESTMENT APPRAISAL FOR HELICOPTER PROCUREMENT

FORMULATION OF COMPUTER MODEL

Decision variables		Deviation variables				Constraint variables			
d	b	D^+_i	D^-_i	D^+_y	D^-_y		Description	Soln	Limits
Design Inv.	Maint.Inv.	1	-1	1	-1	I	Initial Investment	$_i + D^+_i + D^-_i$	= Target value
di	bi	D^+_i	D^-_i	D^+_y	D^-_y	Y I	Revenue from opn	$y_i + D^+_y + D^-_y$	= Target value
Lower bounds							Goal programming function		
0	0	0	0	0	0		Minimise Z= $(1 * D^+_i * W_i) - (1 * D^-_y * W_y)$		
	Penalty weights	W_i	0	0	W_y				

OBJECTIVE FUNCTIONS

1	Minimise intial investment	
	i = d + b	i
2	Maximise Revenue from opn.	
	y=(K1*h - c*λ)*p/(1-r)p	yi

INTERMEDIATE FUNCTIONS

1	Bandwidth hours flown in a year	
	h= (K3/λ)/[(K3/λ) + μ + l]*K3	h_i
2	Failure rate	
	λ=K3/(K6*d + K10)	$λ_i$
3	Mean Time to repair	
	μ=K7/d + K11	$μ_i$
4	Mean Logistics down time	
	L= K8/s + K9/b	l_i

CONSTANTS

K1–assured revenues per hour	350000
K2–Reward or penalty factor	1
K3=No. of working days in a year	360
K6– Design reliability relationship	0.0000002
K7=design reparability relationship	3500000000
K8= MLDT spares relationship	160000000
K9= Base investment MLDT relationship	200000000
K10=Design reliability constant	50
K11=design reparability constant	10
c=Unit cost of repair	20000000
p– period of usage	10
s= spares investment	20000000
r– Discount factor	10%

TABLE 4 : SOLUTION OF THE GOAL PROGRAMMING MODEL USING REDUCED GRADIENT ALGORITHM

Reduced Gradient Algorithm to minimise deviation

Decision variables		Deviation variables				Constraint variables	Solution values	
d	b	D_i^+	D_i^-	D_y^+	D_y^-		soln	limits
Design inv.	Maintenance inv.	-1	+1			Initial investment "i"	400000000	400000000
d	b			-1	+1	Net revenues from operations "y"	400000000	400000000
365995017	28878819.92	14873837	0	0	0			
	Lower bound	0	0	0	0			
						Objective function		
	Penalty weights	1	0	0	1	value 14873837		

Minimised objective function

Reduced gradient algorithm with relaxed constraint on revenues

Decision variables		Deviation variables				Constraint variables		
d	b	D_i^+	D_i^-	D_y^+	D_y^-	Description	soln	limits
Design inv.	Maintenance inv.	-1	+1			Initial investment "i"	400000000	400000000
d	b			-1	+1	Net revenues from operations "y"	377388844	350000000
353694290.5	26305709.54	0	0	0	0			
	Lower bound	0	0	0	0			
						Objective function		
	Penalty weights	1	0	0	1	value	0	

Relaxed constraint

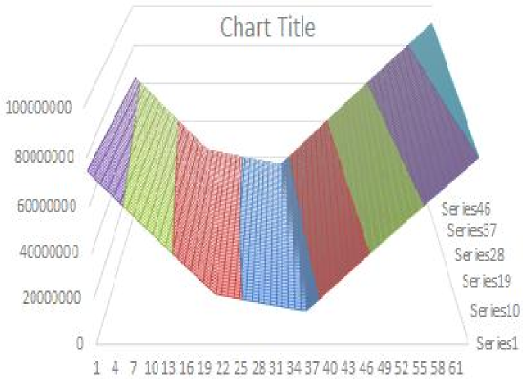

FIGURE 6 : SURFACE PLOT OF REDUCED GRADIENT SOLUTION OF MULTI OBJECTIVE PROBLEM

8. DISCUSSIONS

Many of the Capital investment appraisal processes involve nonlinear functions. This is because many investments follow the law of diminishing returns. For instance, as the investment in design increases, the corresponding improvement in reliability is proportional at the lower levels of investment and flattens out as the investment increases. Beyond a particular level of investment, further investment in reliability gives only marginal improvement in reliability. Similar is the case with impact of investment in base infrastructure facilities on Mean Time To Repair and Mean Logistics Down Time. The graphs at Fig 7 below represent the nonlinearity of impact of investments on the intermediate variables.

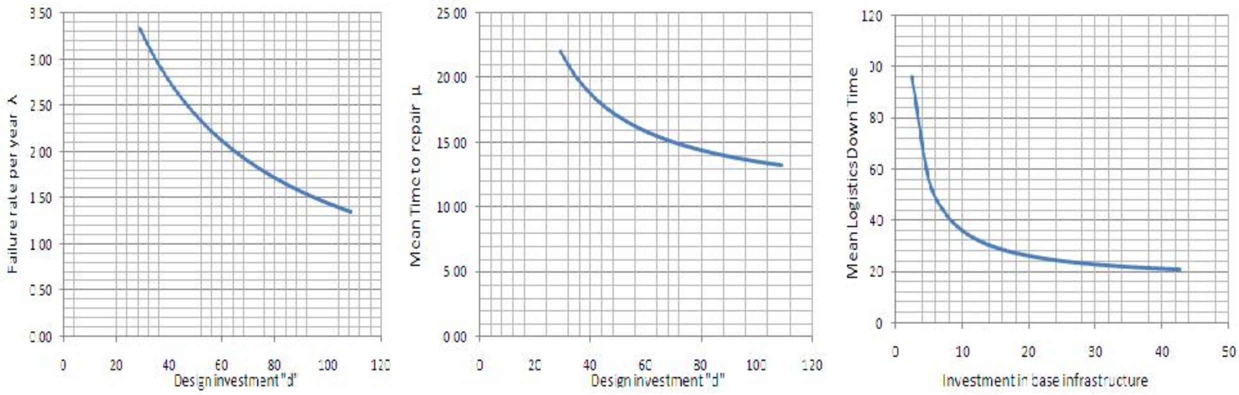

FIGURE 7 : NONLINEAR IMPACT OF INVESTMENT ON INTERMEDIATE VARIABLES

Under such conditions of nonlinearity, it will not be possible to solve the Multi objective optimisation problems using Simplex methods. Quadratic methods or graphical methods will need to be adopted. But as the number of variables increase, it will not be possible to obtain feasible solution set. The reduced gradient algorithm is very useful for investment appraisal involving nonlinear relationships as illustrated by this case study.

The Generalised Reduced Gradient (GRG) has many advantages over the Simplex and enumerative methods. It is logical, systematic and fast. It does not use much of computing resources or storage as the heuristic iterative processes. It is a deterministic method and for same input gives repeatedly the same output unlike the stochastic methods. There is no need to capture second order derivatives as required by other calculus based approaches. There are some precautions to be adopted while using the Reduced Gradient algorithm.

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a) Convexity : The GRG algorithm is efficient when there is only one minima (or maxima); that is the function is strictly convex. If the function has many peaks (or valleys), the program tends to get stuck with the local minima . This depends on the selection of the initial points. Different selection of initial point would result in different local optima being converged as seen from the figure 6 below. To avoid the convergence to local minima, the initialisation has to be carried out across the domain range of the independent variable to ensure gravitating to the global optimum.

b) Speed of Convergence : The critical aspect in GRG methodology is determining the step function. If the step is too small, it will take a lot of time to converge to the solution. If the step function is too large, the optimum point may be jumped over leading to a ping – pong effect, wherein the problem oscillates about the optima for a long time till it settles to the solution. The level of step function depends on the accuracy of the result sought. A sensitivity analysis can reveal whether the step is optimal in nature.

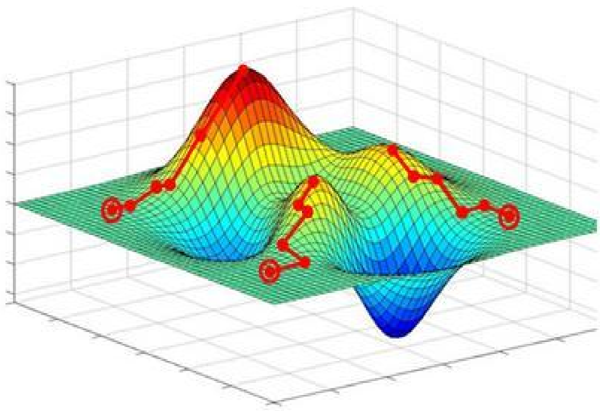

FIGURE 8: REDUCED GRADIENT ALGORITHM FOR MULTI MODAL FUNCTION

c) Discontinuity : The Gradient descent algorithm moves incrementally over the edges of the line / surface. If the edge of the surface is discontinuous or noisy, then the algorithm will abort abruptly. This can happen when the variable follows a step function or some points on the edge are untenable or the function cannot take up a narrow band of values. Under such conditions, it will be required to carry out the GRG algorithm ignoring the discontinuity assuming that the functions are continuous and then eliminate those solutions which are untenable because of the discontinuities.

9. SCOPE FOR FURTHER WORK

The key to utilising Gradient Descent algorithm (or for that matter any algorithm) to optimise decision making is in modelling the mathematical function to represent the effects of the various constraint parameters on the decision variables. Many of such constraints do not lend themselves amenable to mathematical formulation because the functions are complex and sometimes discontinuous. The success of application of search algorithms depends on the ability to develop the system of curves, or rather surfaces which represent the search space for the system. As the number of variables increases, the function generation becomes more complex. Use of past data and incomplete information to generate the first order equations and inequalities and refine the same as more information gets available requires the formulation of self-learning algorithms which needs further research.

As the size of the data becomes huge, gradient descent algorithm will become too cumbersome or reach local minima. It will be required to use stochastic gradient descent or batch gradient descent modifications. Reduction in iterations can be possible by optimising the step sizes. This can be achieved using advanced methods like Hessian approach and Armijo-Wolfe conditions which need further analysis for application to Capital appraisal situations.

Alternatives to gradient descent are the population based rather than boundary based algorithms like the evolutionary algorithms, Monte Carlo approaches and other stochastic approaches which need further study for application in capital investment analysis.

10. Additional Resources

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