

# Stabilization of Collisional Drift Waves in a Complex Plasma

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## ABSTRACT

Presence of highly charged dust grains exhibit charge fluctuations in plasma. The effect of these dust charge fluctuations (DCF) on non-linear coupling between collisional drift waves and a lower hybrid pump wave is studied in a magnetized plasma cylinder. A dispersion relation & growth rate expression for the collisional drift wave is derived incorporating both pondermotive and ohmic heating effects. It is shown that the unstable collisional drift mode frequency increases and the growth rate decreases sharply with the relative density of positively charged dust grains.

**Index Terms:** Dust grains, Drift waves, growth rate & Lower hybrid waves.

## 1. INTRODUCTION

Drift waves are low frequency spontaneously excited waves which are produced due to density or temperature gradient in plasma. They are well known source of microinstabilities in plasma devices. The suppression of drift waves by the application of lower hybrid pump wave have been reported by many investigators [1-4]. The parametric excitation and suppression of drift waves by electric field near the lower hybrid frequency was studied by Sundaram and Kaw [1]. Gore *et al* [2] and Liu & Tripathi [3] demonstrated the suppression of drift waves by injection of lower hybrid waves. Recently, there has been a great deal of interest in studying electrostatic waves in dusty plasmas [5-9]. Barkan *et al.* [6] found experimentally that the presence of negatively charged dust grains enhanced the growth rate of the instability of current driven electrostatic ion cyclotron (EIC) wave in a dusty plasma. Sharma and Ajay [9] have studied the effect of dust charge fluctuations on the excitation of upper hybrid wave in a magnetized plasma cylinder. The dust has also been noted to influence a three-wave parametric process in unmagnetised plasmas [10-12] and magnetized plasma [13]. Praburam and Jain [14] have studied the enhancement of collisional drift waves in a plasma cylinder without dust environment. In this paper, we study the effect of dust charge fluctuations on collisional drift waves in a magnetized plasma cylinder.

## II. INSTABILITY ANALYSIS

We consider a cylindrical dusty plasma column of radius  $a_1$  immersed in a uniform axial magnetic field  $B_s$  in the z direction with electron, ion and dust particle densities given as  $n_e(r)$ ,  $n_i(r)$  and  $n_d(r)$ . The charge, mass and temperature of the three species are denoted by  $(-e, m_e, T_e)$ ,  $(e, m_i, T_i)$  and  $(-Q_{d0}, m_d, T_d)$ , respectively. The density of a three species varies as

$n_e(r) = n_{e0} \exp\left(-\frac{r^2}{a_1^2}\right)$ ,  $n_i(r) = n_{i0} \exp\left(-\frac{r^2}{a_1^2}\right)$  and  $n_d(r) = n_{d0} \exp\left(-\frac{r^2}{a_1^2}\right)$  in the interior region of the dusty plasma column and falls off rapidly near the edge. At

equilibrium, the electrons acquire a diamagnetic drift velocity  $\vec{v}_d = -\frac{\hat{r} 2rv_{te}^2}{S_{ce} a_1^2}$ , where  $v_{te} = \sqrt{\frac{2T_e}{m_e}}$  is the electron thermal velocity and  $\xi_{ce} = \frac{eB_s}{m_e c}$  is the electron cyclotron frequency.

This equilibrium is perturbed by a low- frequency electrostatic perturbation (i.e., a drift wave)

$$W = W(r)\exp[-i(\check{S}t - l_r - k_z z)]. \quad (1)$$

The high amplitude lower hybrid pump wave  $w_0 = W_0(r)\exp[-i(\check{S}_0 t - l_{0r} - k_{z0} z)]$  couples with a drift mode  $W$  and two lower hybrid sidebands  $W_{1,2}$  as

$$w_{1,2} = W_{1,2}(r)\exp[-i(\check{S}_{1,2} t - l_{1,2r} - k_{z1,2} z)], \quad (2)$$

where  $\check{S}_1 = \check{S} - \check{S}_0$ ,  $\check{S}_2 = \check{S} + \check{S}_0$ ,  $k_{z1} = k_z - k_{z0}$ ,  $k_{z2} = k_z + k_{z0}$ ,  $l_1 = l - l_0$ ,  $l_2 = l + l_0$ , i.e., phase matching condition. Here we have considered only lowest order coupling. This is a four wave parametric interaction process.

The perturbed densities of electrons, ion and dust are given by

$$n_{ei} = \frac{n_{e0} e}{T_e} (w_p + w)(1 + i\tau), \quad (3)$$

where,  $\tau = \frac{\epsilon_{ei} (\check{S} - \check{S}^*)}{k_z^2 v_{te}^2}$ ,  $\check{S}^* = k_z |v_d| = \frac{2lv_{te}^2}{a_1^2 \check{S}_{ce}}$  is the adiabatic drift frequency,  $\epsilon_{ei}$  is the electron-ion collision frequency and

$$w_p = \frac{e}{2im_e \check{S}_{ce}^2 \check{S}_0} [(\nabla w_0 \times \check{S}_{ce}) \cdot \nabla w_1 - (\nabla w_0^* \times \check{S}_{ce}) \cdot \nabla w_2].$$

$$n_{i1} = \frac{n_{i0} e \nabla^2 w}{m_i \check{S}_{ci}^2} + \frac{n_{i0} e \check{S}^* w}{T_e \check{S}}, \quad (4)$$

where  $\check{S}_{ci} \left( = \frac{eB_s}{m_i c} \right)$  is the ion cyclotron frequency.

$$n_{d1} = -\frac{n_{d0} Q_{d0} k^2 w}{m_d \check{S}^2}. \quad (5)$$

We obtain dust charge fluctuations by following Jana *et al.* [5] as

$$Q_{d1} = \frac{|I_{e0}|}{i(\check{S} + iy_m)} \left[ \frac{e \nabla^2 w}{m_i \check{S}_{ci}^2} + \frac{e \check{S}^* w}{T_e \check{S}} - \frac{e}{T_e} (w_p + w)(1 + i\tau) \right] \text{ where} \quad (6)$$

$$y = 0.79a \left( \frac{\check{S}_{pi}}{J_{Di}} \right) \left( \frac{1}{u_m} \right) \left( \frac{m_i T_i}{m_e T_e} \right)^{\frac{1}{2}} \text{ is the dust charging rate and } u_m = n_{i0} / n_{e0}.$$

Substituting perturbed densities in the Poisson's equation  $\nabla^2 w = 4f(n_{e1} e - n_{i1} e + n_{d0} Q_{d1} + Q_{d0} n_{d1})$ ,

we obtain

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \left( p_1^2 - \frac{l^2}{r^2} \right) w = \frac{(1 + i\tau) w_p}{L} \left[ -\frac{\check{S}_{pe}^2}{v_{te}^2} + \frac{is}{(\check{S} + iy)} \frac{\check{S}_{pi}^2}{C_s^2} \frac{n_{e0}}{n_{i0}} \right], \quad (7)$$

where  $p_1^2 = \check{p}^2 - k_z^2$ ,  $p^2 = \frac{M}{L}$ ,

$$M = \left[ \frac{\check{S}_{pi}^2 \check{S}^*}{C_s^2 \check{S}} - \frac{\check{S}_{pe}^2}{v_{te}^2} (1 + i\tau) + \frac{is}{(\check{S} + iy)} \frac{\check{S}_{pi}^2 \check{S}^*}{C_s^2 \check{S}} \frac{n_{e0}}{n_{i0}} + \frac{is}{(\check{S} + iy)} \frac{\check{S}_{pi}^2}{C_s^2} \frac{n_{e0}}{n_{i0}} (1 + i\tau) + \frac{\check{S}_{pd}^2 k^2}{\check{S}^2} \right],$$

$$L = 1 + \frac{\check{S}_{pi}^2}{\check{S}_{ci}^2} + \frac{is}{(\check{S} + iy)} \frac{\check{S}_{pi}^2}{\check{S}_{ci}^2} \frac{n_{e0}}{n_{i0}},$$

$C_s^2 = \frac{T_e}{m_i} \cdot \tilde{S}_{pe} \left( = \sqrt{4f n_{e0} e^2 / m_e} \right), \tilde{S}_{pi} \left( = \sqrt{4f n_{i0} e^2 / m_i} \right)$  and  $\tilde{S}_{pd} \left( = \sqrt{4f n_{d0} Q_{do}^2 / m_d} \right)$  are the electron, ion and dust plasma frequencies, respectively and  $S = \frac{|I_{e0}|}{e} \left( \frac{n_{d0}}{n_{e0}} \right) = 0.397 \left( 1 - \frac{1}{u_m} \right) \left( \frac{a}{v_{te}} \right) \tilde{S}_{pi}^2 \left( \frac{m_i}{m_e} \right)$  is the coupling parameter.

In the absence of non-linear coupling terms, Eq. (7) has a well known solution  $w = A J_l(p_1 r), p_1 = p_{n1}$ . At  $r = a_1$ ,  $w$  must vanish, hence  $J_l(p_1 a_1) = 0$ , i.e.,  $p_1 a_1 = x_n$  [where  $x_n$  are the zeros of the Bessel function  $J_l(x), n=1, 2, 3, \dots$ ]. In the presence of a finite R.H.S. of Eq. (7), we express  $w$  in terms of a complete orthogonal sets of wave function:

$$w = \sum_m A_m J_l(p_{m1} r) \tag{8}$$

Substituting the value of  $w$  from Eq. (8) in Eq. (7), multiplying both sides of Eq. (7) by  $r J_l(p_{n1} r)$  and integrating over  $r$  from 0 to  $a_1$  (where  $a_1$  is the plasma radius), retaining only the dominant mode ( $m = n$ ), we obtain

$$\left\{ \frac{\tilde{S}_{ci}^*}{\tilde{S}} - (1+ir) \frac{n_{e0}}{n_{i0}} \frac{\tilde{S}_{ci}^2}{C_s^2} - k^2 + \frac{is}{(\tilde{S} + iy)} \frac{\tilde{S}_{ci}^2 \tilde{S}^* n_{e0}}{C_s^2 \tilde{S} n_{i0}} - \frac{is}{(\tilde{S} + iy)} \frac{\tilde{S}_{ci}^2 n_{e0} (1+ir)}{C_s^2 n_{i0}} - \frac{is}{(\tilde{S} + iy)} k^2 \frac{n_{e0}}{n_{i0}} + \frac{\tilde{S}_{pd}^2 k^2 \tilde{S}_{ci}^2}{\tilde{S}^2 \tilde{S}_{pi}^2} \right\} A_n = (1+ir) \frac{n_{e0}}{n_{i0}} \frac{\tilde{S}_{ci}^2}{C_s^2} \left( 1 + \frac{is}{(\tilde{S} + iy)} \right) \frac{\int_0^{a_1} J_l(p_{n1} r) w_p r dr}{\int_0^{a_1} J_l^2(p_{n1} r) r dr} \tag{9}$$

where  $k^2 = k_z^2 + p_{n1}^2$ .

Using  $C_s^2 = C_{so}^2 \left( 1 + \frac{\Delta T_e}{T_{eo}} \right)$ , where  $\Delta T_e$  is the increase of electron temperature under the influence of the pump,

$T_{eo}$  is the equilibrium electron temperature and  $C_{so}^2 = \frac{T_{eo}}{m_i}$ . Eq. (9) can be rewritten as

$$\left\{ \frac{\tilde{S}_{ci}^2}{C_{so}^2} \left[ \frac{\tilde{S}^*}{\tilde{S}} \left( 1 - \frac{\Delta T_e}{T_{eo}} \right) - \frac{n_{e0}}{n_{i0}} \left( 1 - \frac{\Delta T_e}{T_{eo}} \right) - \left( 1 - \frac{\Delta T_e}{T_{eo}} \right) ir \frac{n_{e0}}{n_{i0}} + \frac{is}{(\tilde{S} + iy)} \frac{n_{e0}}{n_{i0}} \left( \frac{\tilde{S}^*}{\tilde{S}} - \frac{\tilde{S}^* \Delta T_e}{\tilde{S} T_{eo}} + \frac{\Delta T_e}{T_{eo}} - 1 \right) + \frac{r S}{(\tilde{S} + iy)} \frac{n_{e0}}{n_{i0}} \left( 1 - \frac{\Delta T_e}{T_{eo}} \right) + \frac{\tilde{S}_{pd}^2 k^2}{\tilde{S}^2} \frac{C_{so}^2}{\tilde{S}_{pi}^2} \left( 1 - \frac{\Delta T_e}{T_{eo}} \right) \right] - k^2 - \frac{is}{(\tilde{S} + iy)} k^2 \frac{n_{e0}}{n_{i0}} \right\} A_n = \frac{n_{e0}}{n_{i0}} \frac{\tilde{S}_{ci}^2}{C_{so}^2} \left( 1 - \frac{\Delta T_e}{T_{eo}} \right) \left( 1 + \frac{is}{(\tilde{S} + iy)} \right) \frac{\int_0^{a_1} J_l(p_{n1} r) w_p r dr}{\int_0^{a_1} J_l^2(p_{n1} r) r dr} \tag{10}$$

Similarly following Praburam *et al.* [4], we obtain

$$\left( \frac{\tilde{S}_{p0} k_{z1} a_1}{(\tilde{S}_0 - \tilde{S})} - \right)_{n_1, l_1} A_{n_1} = - \frac{e \tilde{S}^*}{i \tilde{S}_1 \tilde{S}_{ce}^2 \tilde{S} T_e} \frac{\int_0^{a_1} r dr \tilde{S}_p^2(r) \mathcal{E}_{n_1, l_1}^{(1)*} (\nabla W_0^* \times \tilde{S}_{ce}) \cdot \nabla W}{\int_0^{a_1} J_l^2(p_{n1} r) r dr} \tag{11}$$

where  $A_{n_1}$  is the constant of wave function  $W_1$ .

$$\left(\frac{\check{S}_{p0}k_{z2}a_1}{(\check{S}_0 - \check{S})} - \right\}_{n_2, l_2} A_{n_2} = - \frac{e\check{S}^*}{i\check{S}_2\check{S}_{ce}^2\check{S}T_e} \frac{\int_0^{a_1} r dr \check{S}_p^2(r) \mathbb{E}_{n_2, l_2}^{(1)*} (\nabla W_0 \times \check{S}_{ce}). \nabla W}{\int_0^{a_1} J_l^2(p_{nl}r) r dr}, \quad (12)$$

where  $A_{n_2}$  is the constant of wave function  $W_2$ .

Using the value of  $W_p$ , assuming the pump to be azimuthally symmetric ( $l_0=0$ ), and considering the radial mode numbers of the two sidebands to be same, i.e.,  $n_1 = n_2$ , Eqs. (10), (11) and (12) gives a dispersion relation

$$\begin{aligned} & \frac{\check{S}^*}{\check{S}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \left(1 - \frac{\Delta T_e}{T_{eo}}\right) i r \frac{n_{e0}}{n_{i0}} + \\ & \frac{i s}{(\check{S} + iy)} \frac{n_{e0}}{n_{i0}} \left(\frac{\check{S}^*}{\check{S}} - \frac{\check{S}^* \Delta T_e}{\check{S} T_{eo}} + \frac{\Delta T_e}{T_{eo}} - 1\right) + \frac{\Gamma s}{(\check{S} + iy)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) \\ & + \frac{\check{S}_{pd}^2 k^2 C_{so}^2}{\check{S}^2 \check{S}_{pi}^2} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - k^2 \dots_{so}^2 - \frac{i s}{(\check{S} + iy)} k^2 \dots_{so}^2 \frac{n_{e0}}{n_{i0}} = \\ & \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) \left(1 + \frac{i s}{(\check{S} + iy)}\right) \frac{\check{S}^*}{\check{S}} \end{aligned} \quad (13)$$

where

$$\check{\gamma}_1 = 2 - \frac{\left(\frac{\check{S}_{p0}k_{z1}a_1}{\check{S}_0} - \right\}_1}{\left(\frac{\check{S}_{p0}k_{z2}a_1}{\check{S}_0} - \right\}_1} - \left(\frac{\check{S}_{p0}k_{z0}a_1}{\check{S}_0}\right)^2}, \quad \dots_s^2 = C_s^2 / \check{S}_{ci}^2.$$

and

$$\check{\gamma} = \left[ \frac{e^2 \check{S}^*{}^2}{2m_e \check{S}_{ce}^2 \check{S}_0^2 T_e} \int_0^{a_1} r dr J_l^2(p_{nl}r) \right] \left| \int_0^{a_1} \check{S}_p^2(r) r dr \mathbb{E}_{n_1, l_1} \frac{\partial W_0}{\partial r} J_l(p_{nl}r) \right|^2, \quad (14)$$

Equation (13) can be rewritten as

$$v_r(\check{S}, k) + i v_i(\check{S}, k) = 0, \quad (15)$$

where

$$\begin{aligned} v_r(\check{S}, k) = & \frac{\check{S}^*}{\check{S}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - k^2 \dots_{so}^2 + \frac{sy}{(\check{S}^2 + y^2)} \frac{\check{S}^*}{\check{S}} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) \\ & - \frac{sy}{(\check{S}^2 + y^2)} k^2 \dots_{so}^2 \frac{n_{e0}}{n_{i0}} - \frac{n_{e0}}{n_{i0}} \frac{\check{\gamma}_1}{\check{S}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{sy}{(\check{S}^2 + y^2)} \frac{\check{\gamma}_1}{\check{S}} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) \\ & - \frac{sy}{(\check{S}^2 + y^2)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) + \frac{sr\check{S}}{(\check{S}^2 + y^2)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) + \frac{\check{S}_{pd}^2 k^2 C_{so}^2}{\check{S}^2 \check{S}_{pi}^2} \left(1 - \frac{\Delta T_e}{T_{eo}}\right), \end{aligned} \quad (16)$$

$$\begin{aligned} v_i(\check{S}, k) = & -r \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) + \frac{s\check{S}}{(\check{S}^2 + y^2)} \frac{\check{S}^*}{\check{S}} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{s\check{S}}{(\check{S}^2 + y^2)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) \\ & - \frac{s\check{S}}{(\check{S}^2 + y^2)} \frac{n_{e0}}{n_{i0}} k^2 \dots_{so}^2 - \frac{s}{(\check{S}^2 + y^2)} \frac{n_{e0}}{n_{i0}} \frac{\check{\gamma}_1}{\check{S}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{sry}{(\check{S}^2 + y^2)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right). \end{aligned} \quad (17)$$

Let us write  $\check{S} = \check{S}_r + i\chi$  and assume that the wave is either weakly damped or growing (i.e.,  $|\chi| \ll \check{S}_r$ ). Then we may set

$$v_r(\check{S} = \check{S}_r, k) = 0 \quad \text{from Eq. (15).} \quad (18)$$

Equation (15) yields

Growth rate:

$$\chi = - \frac{v_i(\check{S}_r, k)}{\partial v_r(\check{S}_r, k) / \partial \check{S}_r}. \quad (19)$$

Now, we consider two cases of interest

Case I: In the presence of dust charge fluctuations, i.e., dust charging rate is finite.

Case II: In the absence of dust charge fluctuations, i.e.,  $Q_{d1} = 0$  when dust charging rate is finite.

In the absence of dust grains, i.e.,  $u_m = n_{i0}/n_{e0} = 1, \delta_m \rightarrow 0$ , we recover the expressions for the unstable drift mode frequency and growth rate of Ref.[14] (cf. page no. 470). Dust grain is negatively charged for  $u_m > 1$  and positively charged for  $u_m < 1$ .

### III. RESULTS AND DISCUSSIONS

To estimate the numerical values of the real frequency and growth rate of the drift wave instability, we use typical dusty plasma parameters:  $n_{i0} = 10^9 \text{ cm}^{-3}$ ,  $T_e = 2.0 \text{ eV}$ ,  $T_i = 0.2 \text{ eV}$ ,  $B_s = 0.55 \times 10^3 \text{ G}$ ,  $\omega_{ci} = 3.0 \times 10^5 \text{ rad/sec}$ ,  $\check{S}^* = 1.5 \times 10^5 \text{ rad/sec}$ ,  $a_1 = 1.0 \text{ cm}$ , length of plasma column  $L = 70 \text{ cm}$ ,  $m_i/m_e \approx 7.16 \times 10^4$  (Potassium), average dust grain size  $a = 1 \mu\text{m}$ , mode number  $n = 1$ , i.e., the first zero of the Bessel function,  $k_{\perp n} = 3.85 \text{ cm}^{-1}$ ,  $k_z = f/L$ , pump wave amplitude  $w_0 = 6.6 \times 10^{-4} \text{ esu}$ . We vary  $u_m$  from 0.4 to 0.95 for positively charged dust grains.

Using Eq. (18) we have plotted in Fig.1 the normalized real frequency  $\check{S}_r/\check{S}_{ci}$  of the unstable collisional drift waves as a function of  $u_m = n_{i0}/n_{e0}$ . In Fig.1 it is seen that the normalized wave frequency  $\check{S}_r/\check{S}_{ci}$  increases by a factor  $\sim 1.44$  when  $\delta_m$  changes from 0.4 to 0.8 if dust charge fluctuations are taken into account under the plasma parameters listed above. Barkan *et al.* [6] have found that the wave frequency was about 10-20% larger than the ion-cyclotron frequency in the presence of negatively charged dust grains. Chow and Rosenberg [7] have shown, in their kinetic analysis on the effect of negatively charged dust grains on the collisionless electrostatic ion cyclotron instability, the wave frequency  $\check{S}_r/\check{S}_{ci}$  increases about 11% when  $\delta_m$  is changed from 1 to 4 under similar conditions. Thus the increase is considerably more in case of collisional drift waves in presence of positively charged dust grains in a plasma cylinder.

In Fig.2, we have plotted the normalized growth rate  $\chi/\check{S}_{ci}$  obtained from Eq. (19) as a function of  $\delta_m$  for the same parameters as those used in Fig.1. From Fig.2 it can be seen that the normalized growth rate  $\chi/\check{S}_{ci}$  decreases with  $u_m$  in both cases. However, decrease is more drastic when the dust charge fluctuations are taken into consideration. Thus the dust charge fluctuations can play a significant role in suppression of the collisional drift waves in case of positively charged dust grains.

### CONCLUSION

The present work investigates the role of positively charged dust grains in suppression of collisional drift waves. The positively charged dust can play a major role in dusty plasma experiments in the earth ionosphere using space shuttle exhaust. The drift wave instability also plays a crucial role in international ITER and other fusion reactors.

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## FIGURE CAPTIONS

Fig.1: Normalized real frequency  $\tilde{\omega}_r/\tilde{\omega}_{ci}$  of the collisional drift wave as a function  $u_m (= n_{i0}/n_{e0})$  [with and without dust charge fluctuations].

Fig.2: Normalized growth rate  $\chi/\tilde{\omega}_{ci}$  of the collisional drift wave as a function  $u_m$  [with and without dust charge fluctuations].

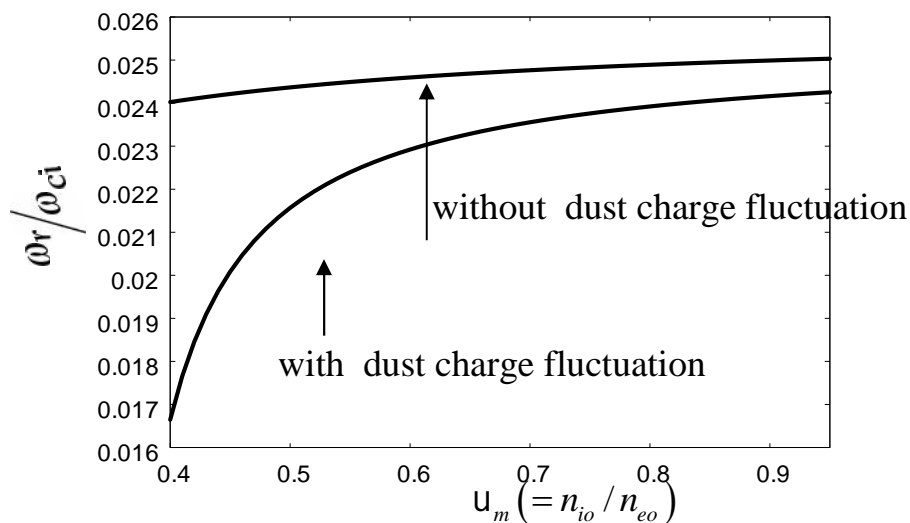


Fig.1

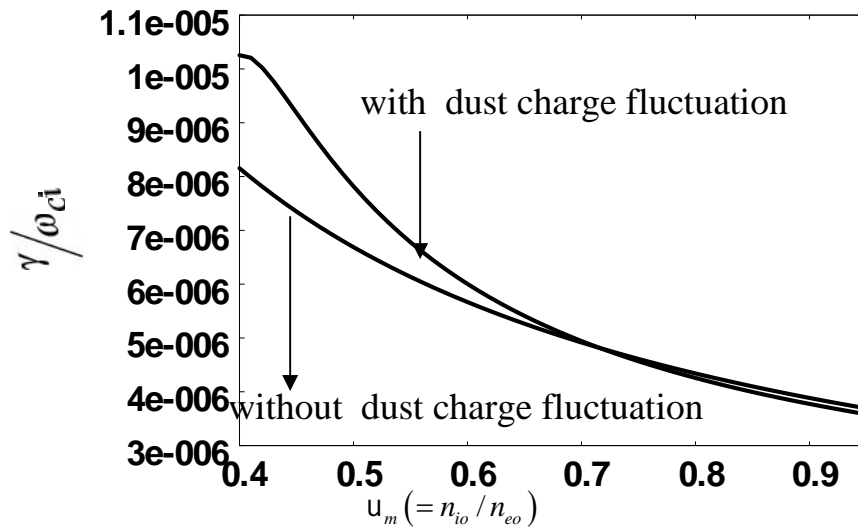


Fig.2