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Effects of Yield Stress and Flow Index Behavior on Non-Newtonian Flow of Blood by Overlapping Stenosed Arteries

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Abstract:
The present study pact with a speculative analysis of the characteristics of blood flow through a overlapping stenosed. Here we imagine non-Newtonian behavior of blood and rheology of blood is Herschel-Bulkley fluid.. Using proper boundary conditions, diagnostic expressions for the pressure gradient, wall shear stress and volumetric flow rate have been established. The results that we have got are shown graphically for different values of few sensitive parameters such yield stress and flow index behavior. The investigation informs that wall shear stress and volumetric flow rate shows reducing nature as flow index behavior and yield stress grow. It is also seen that pressure gradient amplifies and reduces with increased values of flow index behavior and yield stress respectively. Besides this variations of these flow characteristics with stenosis heights and axial distance have also illustrated.

Key words: Stenosis, Herschel-Bulkley fluid, Non-Newtonian Fluid, pressure gradient, wall shear stress, yield stress, flow index behavior, volumetric flow rate

1. Introduction:

Atherosclerosis is a one of the most health problem causes of uncured high blood pressure in developing and most effective in developed countries. When blockage due to cholesterol, fatty acid and plagues deposition in blood vessels occurs that interrupts the supply of blood to the heart muscles. That's why atherosclerosis is a annoyance to the human civilization. Many scientist, doctors and mathematicians have been made their efforts to overcome this severe condition to give relaxes for such patients. Many of the researchers have contributed their work some of them have been mentioned in this section of this paper.

Ismail et al. (2008) presented a non-Newtonian model of flowing of blood through a tapered stenosed artery by considering blood rheology is generalized power law fluid. They found in their work is that in the axial velocity, WSS and volumetric flow rate have minor significance where as resistance to flow has major importance in comparison with assumption of blood behaves like Newtonian fluid. In their exploration **Ikbal et al. (2009)** carried assuming blood a non-Newtonian fluid passing by a stenosed artery in the existence of transverse magnetic field. They assumed blood rheology as generalized power law fluid. They have showed the response of Hartmann number, flow index behavior and generalized Reynolds number on WSS, velocity of blood and volumetric flow rate. **Haque et al. (2014)** established a correspondence between pulsatile flow of blood and plague (obstruction in artery) in which he consider that blood behaves like Newtonian over and above non-Newtonian fluid.
Srivastava and Rastogi (2010) explored that flow resisatnce reduces with amplified the stenosis shape parameter but it demonstrated reverse behavior with the increased hematocrit percentage, stenosis height and catheter dimension.
Sankar and Lee (2011) invesigated that incresed pressure gradient, body acceleration and pulsatile Reynolds number lifts the plug flow velocity and volumetric flow rate. They also recorded that peripheral layer breadth shows diminishing nature with the raising stenosis height, yield stress and front angle. **Tian (2013)** imitate a pulsatile non-Newtonian flow of blood go by a various rigorous stenosed artery**.** They shows the effects of

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various flow parameter like plague size, flow pulsatilty and shear rate dependent viscosity on WSS, pressure gradient and vessels wall pressure and shear rate.

Akbar (2014) considered Prandtl flow model of blood passing through tapered stenosed arteries. She investigated that velocity profiles indicates decresing trends on enhancing Prandtl current factor, stenosis shape parameter and utmost height of stenosis.**Nadeem and Ljaz (2015)** found in their research that resistance to flow shows reducing effect on rising in nano particle capacity proportion.Further They found that curvature of blood vessel shows dominating nature with the grouping of curvature and stenosis.**Akbar (2016)** studied MHD flow of blood during a lessened stenosed artery by considering blood Rheology of Walter's B Fluid. She used perturbation technique to solve the ODE related to problem. **Zaman and Sajid (2017)** have discussed the effect of body acceleration on of pulsatile blood flow through overlapping stenosed artery in the porous medium. They used cross model to describe blood and show that Darcy' law is associated to Cross Model. **Bali and Gupta (2018)** discussed a model of flowing of blood passing through non-symmetrical stenosed arteries making use of K-L model under no slip condition on the arterial wall. To obtain flow characteristics for instance resistance to flow, wall shear stress and volumetric flow rate, they apply Gauss Quadrature formula.

2. Formulation of the problem:

The geometry of the stenosis, supposed to be marked in the arterial section is expressed

$$R(z) = R_0 \begin{cases} 1 - \dfrac{3\delta}{2R_0 l_0^{\,4}}\left\{11(z-d)l_0^{\,3} - 47(z-d)^2 l_0^{\,2} + 72(z-d)^3 l_0 - 36(z-d)^4\right\}; & d \le z \le d+l \\ 1 & Otherwise \end{cases} \tag{1}$$

where $R(z)$ and R_0 be the radii of blood vessels in stenotic and non-stenotic region, l_0 is the dimension of stenosis, d stands for its position and δ is the maximum depth of the stenosis into the lumen, shows at the two different places: $z = d + \frac{1}{6} l_0$ and $z = d + \frac{5}{6} l_0$.

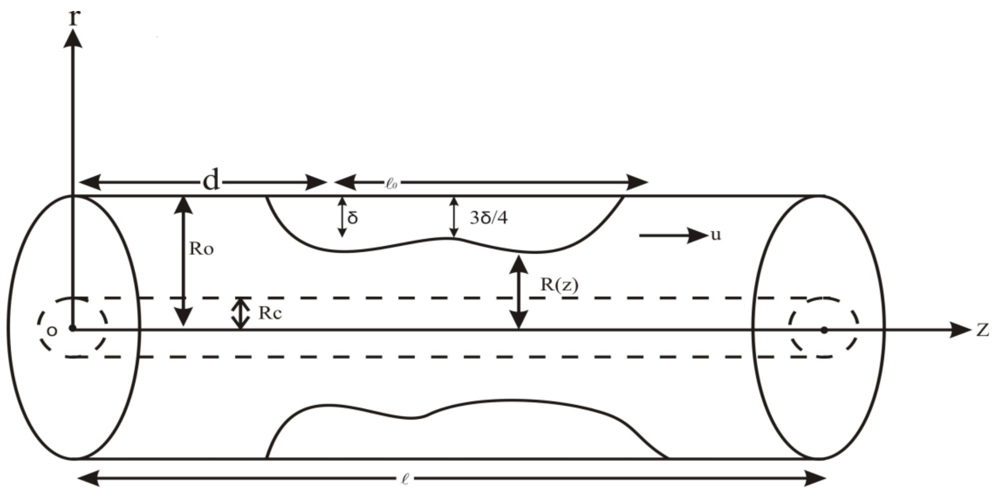


Figure1. Geometry of an overlapping stenosis in an arterial segment

The constitutive equation for Herschel-Bulkley fluid is given by

$$-\frac{\partial w}{\partial r} = g(\tau) = \frac{(\tau - \tau_0)^n}{K}; \tau \ge \tau_0 \qquad =0 \;; \qquad \tau \le \tau_0 \tag{2}$$

Where K and n are parameters which represent non-Newtonian effects, τ_0 is the yield stress, z and r are axial and radial co-ordinates and w is the axial velocity along z -direction.

The boundary conditions are

$w = 0$ at $r = R(z)$ (3)

τ is finite at $r = 0$ (4)

$$Q = 2\pi \int_0^R rw\,dr$$

$$= \pi \int_0^R \left(-\frac{\partial w}{\partial r} \right) r^2 dr \tag{5}$$

$$\tau = -\frac{r}{2}\frac{\partial p}{\partial z} \quad \text{And } \tau_R = -\frac{R}{2}\frac{\partial p}{\partial z} \Rightarrow r = \frac{\tau R}{\tau_R}$$

$$\Rightarrow dr = \frac{R}{\tau_R} d\tau \tag{6}$$

Substituting (6) in (5) to obtain

$$Q = \frac{\pi R^3}{\tau_{R^3}} \int_0^{\tau_R} \tau^2 g(\tau) d\tau \tag{7}$$

$$= \frac{\pi R^3}{\tau_{R^3}} \int_0^{\tau_R} \tau^2 \frac{(\tau - \tau_0)^n}{K} d\tau$$

$$Q = \frac{\pi R^3 \tau_R^{\ n}}{K(n+3)} \left(1 - \frac{\tau_0}{\tau_R} \right)^{n+1} \left[1 + \frac{2}{(n+2)} \left(\frac{\tau_0}{\tau_R} \right) + \frac{2}{(n+1)(n+2)} \left(\frac{\tau_0}{\tau_R} \right)^2 \right] \tag{8}$$

Using the condition $\frac{\tau_0}{\tau_R} << 1$ in equation (8), we get

$$-\frac{dp}{dz} = \left(\frac{2^n QK(n+3)}{\pi} \right)^{1/n} \frac{1}{R^{\frac{3}{n}+1}} + \frac{2(n+3)}{(n+2)} \frac{\tau_0}{R} \tag{9}$$

Integrating eq. (9) and using the condition $p = p_0$ at $z = 0$ and $p = p_1$ at $z = l$

$$p_1 - p_0 = \left(\frac{2^n QK(n+3)}{\pi R_0^{3+n}} \right)^{1/n} \int_0^l \frac{1}{\left(\frac{R}{R_0} \right)^{\frac{3}{n}+1}} dz + \frac{2(n+3)\tau_0}{(n+2)R_0} \int_0^l \frac{dz}{\left(\frac{R}{R_0} \right)} \tag{10}$$

Resistance to flow λ is defined as

$$\lambda = \frac{p_1 - p_0}{Q} \tag{11}$$

Which, on using eqs. (1) and (11), gives

$$\lambda = \left(\frac{2^n K(n+3)}{Q^{n-1}\pi R_0^{n+3}}\right)^{\frac{1}{n}} \left[\int_0^d dz + \int_d^{d+l_0} \frac{dz}{\left(\frac{R}{R_0}\right)^{\frac{3}{n}+1}} + \int_{d+l_0}^{l} dz\right] + \frac{2(n+3)\tau_0}{Q(n+2)R_0}\left[\int_0^d dz + \int_0^{d+l_0} \frac{dz}{\left(\frac{R}{R_0}\right)} + \int_{d+l_0}^{l} dz\right] \tag{12}$$

$$\lambda = \left(\frac{2^n K(n+3)}{Q^{n-1}\pi R_0^{n+3}}\right)^{\frac{1}{n}} \left[l - l_0 + \int_d^{d+l_0} \frac{dz}{\left(\frac{R}{R_0}\right)^{\frac{3}{n}+1}}\right] + \frac{2(n+3)}{(n+2)}\frac{\tau_0}{R_0} + \left[l - l_0 + \int_0^{d+l_0} \frac{dz}{\left(\frac{R}{R_0}\right)}\right] \tag{13}$$

$$\lambda = f_1[l - l_0 + I_1] + f_2[l - l_0 + I_2] \tag{14}$$

Where $f_1 = \left(\frac{2^n K(n+3)}{Q^{n-1}\pi R_0^{n+3}}\right)^{\frac{1}{n}}$, $f_2 = \frac{2(n+3)}{(n+2)}\frac{\tau_0}{R_0}$, $I_1 = \int_d^{d+l_0} \frac{dz}{\left(\frac{R}{R_0}\right)^{\frac{3}{n}+1}}$, $I_2 = \int_0^{d+l_0} \frac{dz}{\left(\frac{R}{R_0}\right)}$

In the nonappearance of any constriction, the resistance to flow λ_N is given by

$$\lambda_N = (f_1 + f_2)l \tag{15}$$

Resistance to flow ratio is

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = 1 - \frac{l_0}{l} + \frac{1}{l}\frac{f_1 I_1 + f_2 I_2}{(f_1 + f_2)} \tag{16}$$

For a general value of n, the definite integrals in eq. (14) can be evaluated numerically only.

Since $\tau_R = -\frac{R}{2}\frac{dp}{dz}$ (17)

From equation (9), the term for WSS ratio, τ_R can be written as

$$\tau_R = \left[\frac{QK(n+3)}{\pi R^3}\right]^{\frac{1}{n}} + \frac{(n+3)\tau_0}{(n+2)} \tag{18}$$

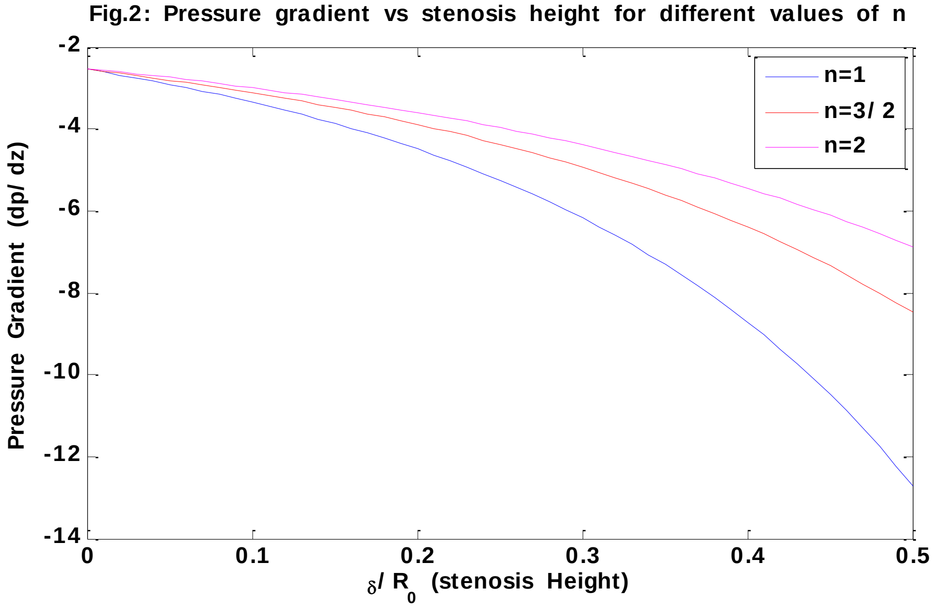
In the deficiency of any obstruction i.e. when $R = R_0$, the term for WSS is

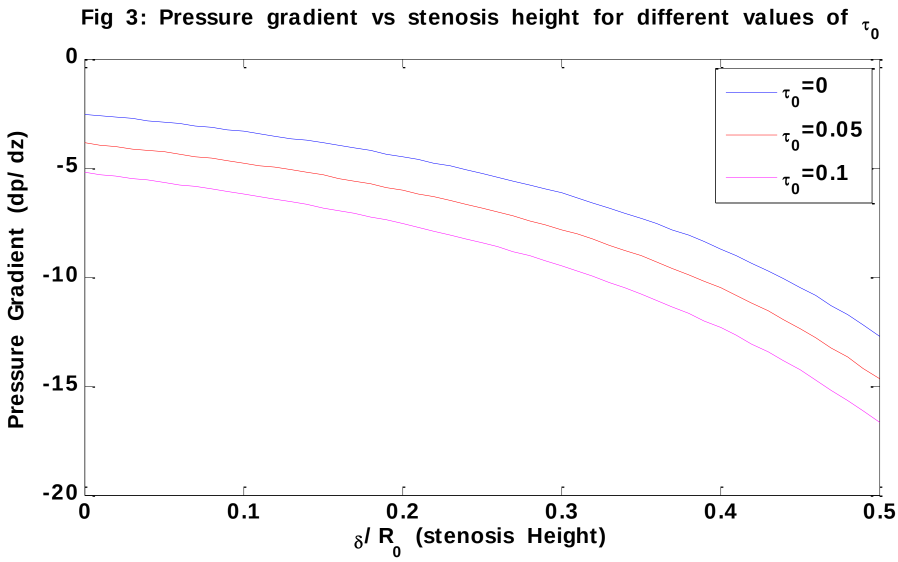
$$\tau_N = \left[\frac{QK(n+3)}{\pi R_0^{\ 3}}\right]^{\frac{1}{n}} + \frac{(n+3)\tau_0}{(n+2)} \quad (19)$$

A non-dimensional term for WSS fraction can be put as

$$\overline{\tau}_R = \frac{\tau_R}{\tau_N} = \frac{\{KQ(n+3)\}^{1/n} + \left(\frac{n+3}{n+2}\right)(\pi R_0^{\ 3})^{1/n}\left(\frac{R}{R_0}\right)^{3/n}\tau_0}{\{KQ(n+3)\}^{1/n} + \left(\frac{n+3}{n+2}\right)(\pi R_0^{\ 3})^{1/n}\tau_0}\left(\frac{R}{R_0}\right)^{-3/n} \quad (20)$$

3. Results and Discussion:





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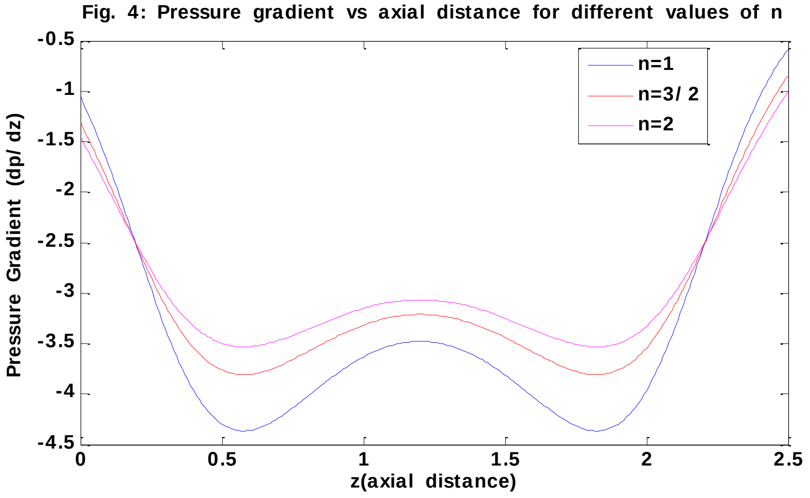
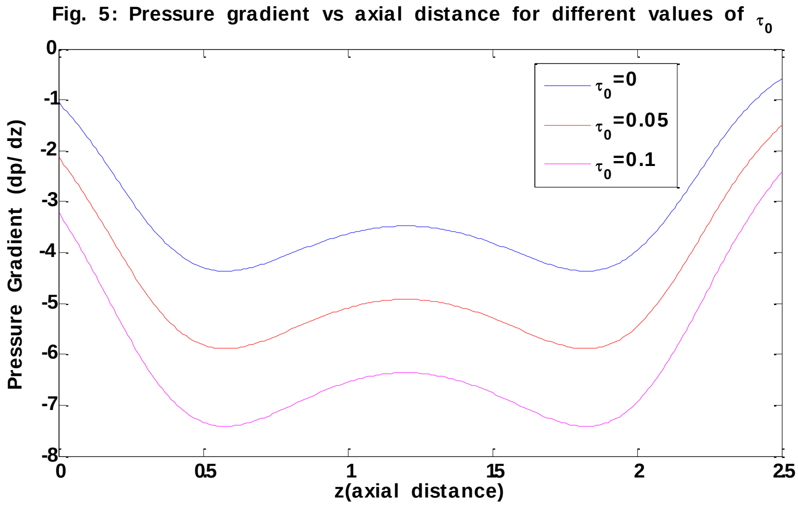




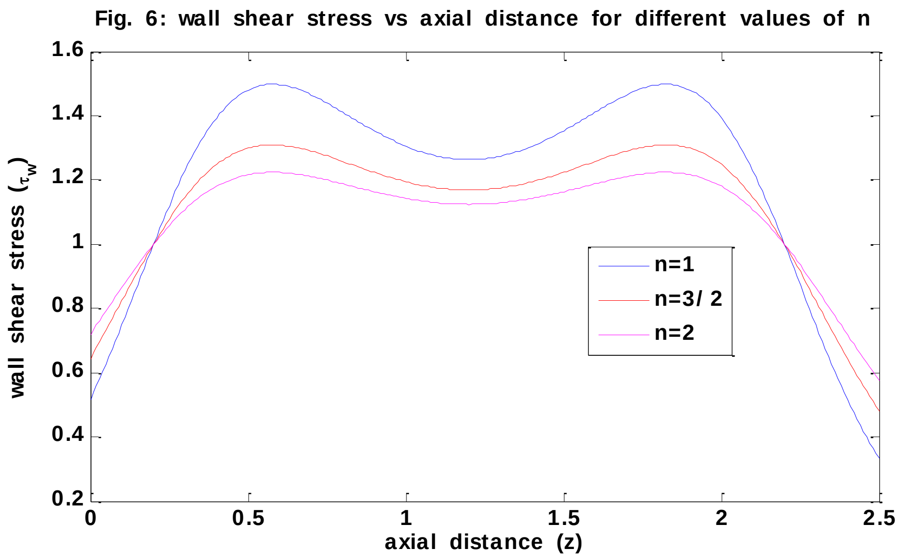


Fig. 7: wall shear stress vs axial distance for different values of τ_0

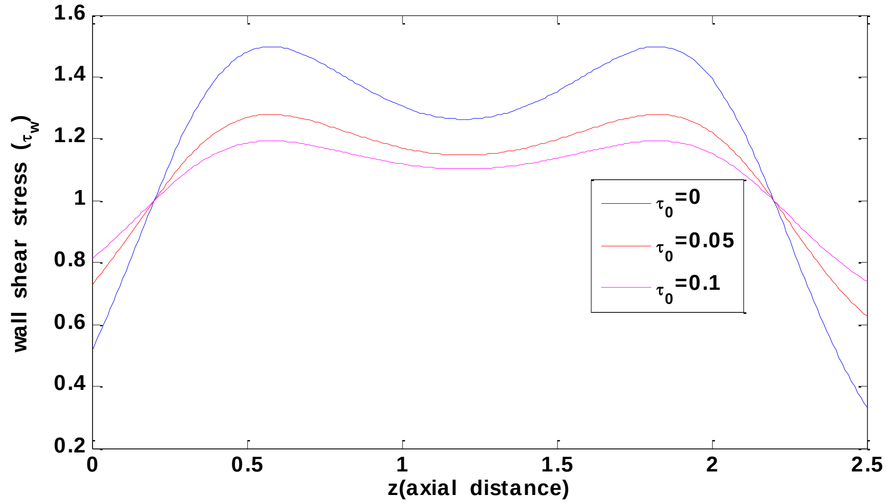


Fig. 8: wall shear stress vs stenosis height for different values of n

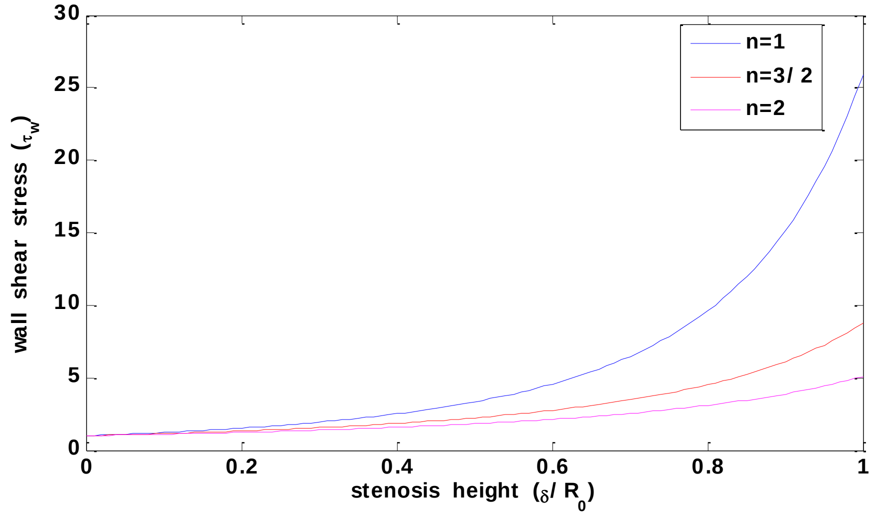
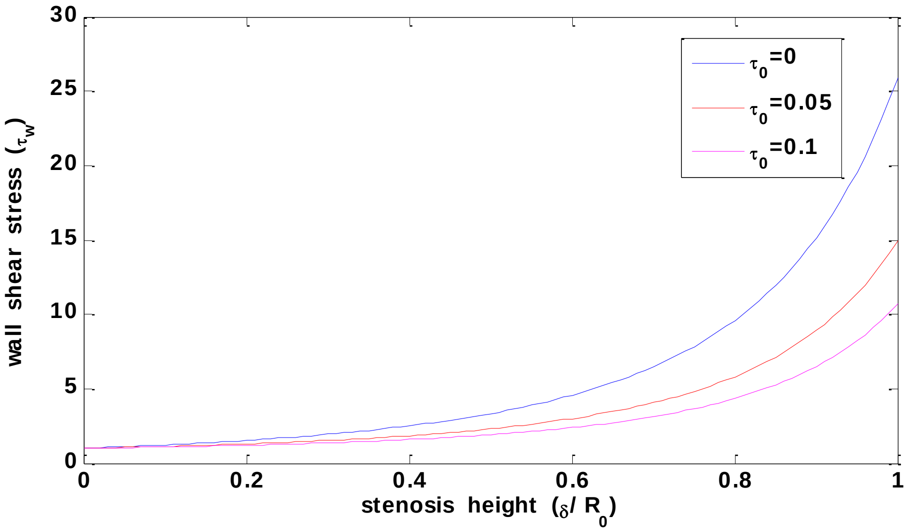


Fig. 9: wall shear stress vs stenosis height for different values of τ_0



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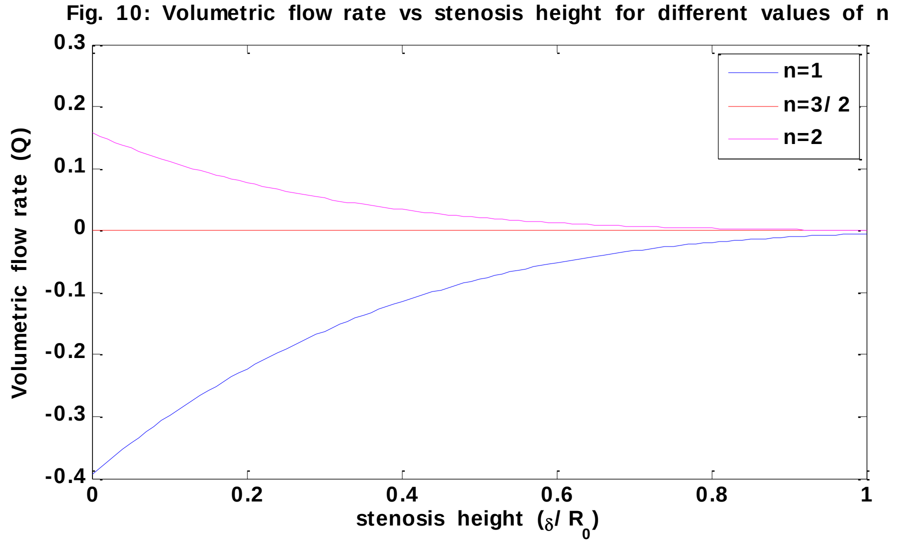

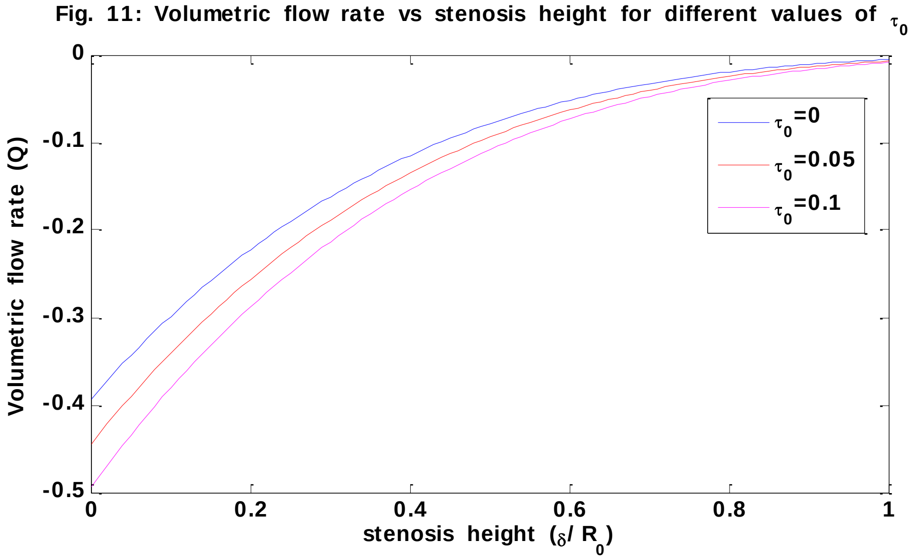

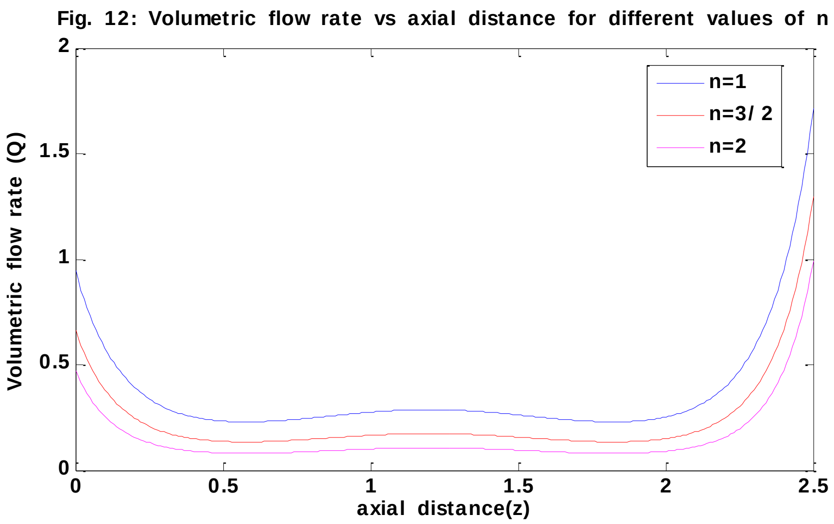

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Fig. 13: Volumetric flow rate vs axial distance for different values of τ_0

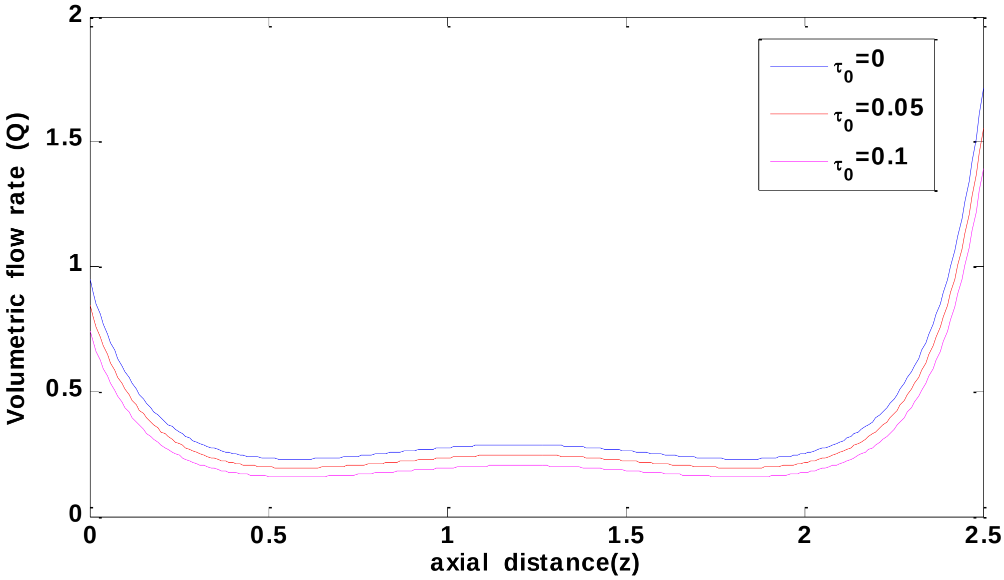
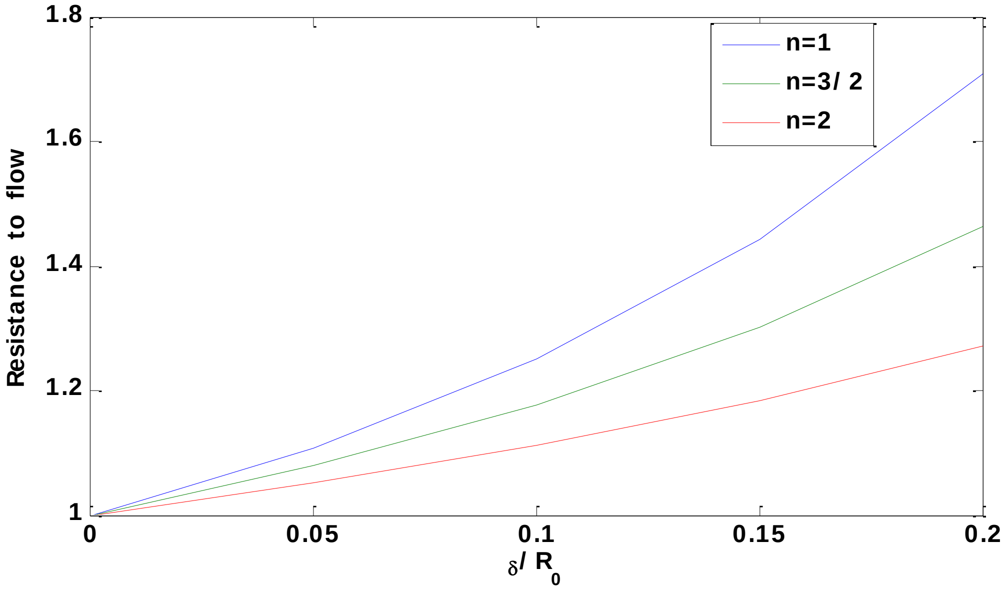


Fig. 14:Resiatnce to flow vs stenosis height for different value of n



To make the results more visible we have designed the graphs of various flow characteristics such as pressure gradient, wall shear stress (WSS), volumetric flow rate and resistance to flow for some sensitive parameter for instance flow index behavior and yield stress which can be responsible for different kind of cardiovascular diseases.

Graphs (2) and (3) display the variations of $\frac{dp}{dz}$ with $\frac{\delta}{R_0}$ for different values of n and τ_0 respectively. From these figures It is seen that pressure gradient shows increasing trend as flow index behavior raises while it diminishes with the lifting of yield stress. Graphs (4) and (5) illustrated variations of $\frac{dp}{dz}$ with Z for different values of n and τ_0 respectively. The variations in WSS with $\frac{\delta}{R_0}$ for different values of n and τ_0 has been provided in graphs (6) and (7) respectively. It is noted that lifting the values of n *and* τ_0 reduces WSS. The deviation of WSS with Z for different values of n and τ_0 has been depicted in graphs (8) and (9). Graphs (10) and (11) demonstrated the variations in volumetric flow rate with $\frac{\delta}{R_0}$ for different values of n and τ_0. The results of these graphs gives the information that volumetric flow rate have decreasing nature whenever n and τ_0 reduce. The graphs (12) and (13) are designed between volumetric flow rate and axial distance for different values of n and τ_0. The information from the graph (14) is that whenever stenosis height grows resistance to flow amplifies. To get lower value of resistance to flow we have to give an increment in flow index behavior.

Over all the information that we have get from these results is that to obtain lower wall shear stress that is necessary for reduce high blood pressure we have to maintain flow index behavior and yield stress at high level. If we can set up such environment in the human stenosed artery as discussed in this paper by means of medicine or by other medical tool that will help to control cardiovascular diseases.

4. Concluding Remarks

The proposed work deals with the Response flow index behavior and yield stress on the flowing of blood though an overlapping stenosis in which blood acts a Herschel-Bulkley fluid that shows the non-Newtonian effects. Assumption of non-Newtonian aspects provides more accurate results in comparison with Newtonian characteristic. It is the adviced to medical practitioners for give relief to patients who are suffering from atherosclerosis that they must made proper adjustment of flow parameters such as flow index behavior and yield stress to reduce the performance measures regarding to blood flow. At last I expected that this small effort to correlate blood flow to plagues may help them to improve existing diagnosis tool and develop new tools.

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