

Improved Recursive HE Algorithms for Low Exposure Images

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Abstract—This paper proposes two methods for image enhancement based on exposure, which performs HE recursively. These methods are very effective for images captured in low light condition like underwater sequences or night vision images. The first method proposed is recursive exposure based sub-image histogram equalization (R-ESIHE) that performs ESIHE[20] method recursively till the exposure residue among successive iteration is less than a predefined threshold. The second method is recursively separated exposure based sub image histogram equalization (RS-ESIHE) which performs the separation of image histogram recursively; separate each new histogram further based on their respective exposure thresholds and equalize each sub histogram individually. In earlier case this problem was not fixed, we have performed experiments on HE based methods, low exposure images has been efficiently handled by these new methods. The performance evaluation of new methods is done in terms of mathematical calculation as well as by visualizing the Image quality.

Keywords —Recursive Exposure Based Sub-Image Histogram Equalization (R-ESIHE), Recursively Separated Exposure Based Sub Image Histogram Equalization (RS-ESIHE) Exposure Based Sub-Image Histogram Equalization (ESIHE)

INTRODUCTION

Although there is a tremendous advancement in image capturing devices, still natural images are often subject to low exposure problems under low light or under water conditions. Digital camera have a limited dynamic range as a result photographs acquired in high dynamic range scenes often exhibit underexposure artefacts in shadow regions [1]. An image captured in a dim light environment encounters low-exposure problem caused by non-ideal camera settings of aperture

and shutter speed. Exposure in an image determines the brightness or darkness of each element in the image [2]. In the low illumination scenario, post processing using image enhancement tools is needed to improve the quality of the acquired image. Many histogram equalization based image enhancement methods were proposed to cope with contrast related issues. Histogram equalization (HE) is most extensively utilized contrast enhancement technique due to its simplicity and ease of implementation [3]. Histogram equalization flattens the probability distribution and stretches the dynamic range of grey levels, which in result improves the overall contrast of the image[4]. Applying HE straight away on natural images is not suitable for most consumer electronics applications, such as TV, Cameras, etc., as it tends to change the mean brightness of the image to the middle level of the grey level range, which in turn produces annoying artefacts and intensity saturation effects.

LITERATURE SURVEY

Xu et al. (1996) have proposed a method for image enhancement via wavelet shrinkage and nonlinear adaptive gain. It addresses both de-noising and contrast enhancement issues. It is based on a multiscale wavelet analysis framework and takes both soft thresholding and hard thresholding wavelet shrinkage techniques to reduce noise. Non linear processing is carried out for contrast enhancement.

KIM et al. (1997) [1] has discussed that the intensity of scene can be altered following the histogram equalization, which is because of the flattening attribute of the histogram equalization. KIM (1997) [6] proposed

histogram equalization referred to as mean preserving bi-histogram equalization to conquer the disadvantage of the histogram equalization.

The essence of the proposed algorithm is to conserve the mean intensity of an image while the contrast is enhanced. Given method initially break an input image into two sub-images based on the mean of the input image. One of the sub-images is the set of specimens that are less than or equal to the mean whereas the other one is the set of specimens greater than the mean.

Tae et al. (1998) [2] has discussed a block-overlapped histogram equalization system for improving the contrast of an image sequences using numerous applications. The conventional histogram-based contrast enhancement technique is limited in real time application due to a large computational and storage requirements and it also exhibit quality degradation caused by possible loss of infrequently distributed pixel intensities, which may result in terrible loss of vital information.

J.C. Fu et al. (2000) have proposed a method to enhance magnetic resonance images. A wavelet based technique is applied to enhance left ventricular endocardial and epicardial profiles as the preprocessor for a dynamic programming based automatic border detection algorithm. Digitized MR images are taken as inputs in this method and histogram equalization is performed. The reason for performing histogram equalization is to arrange the grey level of the MR image in straight line.

Sa et al. (2000) have proposed a method for denoising MR images via wavelet analysis. The method is based on shrinking noisy discrete wavelet transform coefficients via thresholding. The first step in denoising the data is to transform them to wavelet space. From sorted wavelet spectrum (SWS), it is observed that coefficients presenting low amplitude are noise and high amplitude as data. Distinguishing between these two is very easy. Once the noisy dominated regime is identified, attempt is made to obtain an unbiased noise-free estimator of the signal in the wavelet domain. The result shows that denoising algorithm works fine for MR images.

Jia et al. (2004) have proposed a color image enhancement method based on histogram processing. Histogram equalization is a well known

method for grey scale image contrast enhancement. Given a grey scale image I with grey levels in the range $[0, L]$, its normalized histogram is a discrete function $H(l)=n_l / n$, where l is the l th grey level, n_l is the frequency of occurrence of the corresponding grey level and n is the total pixel population in the image.

Huang et al (2004) have proposed a method to denoise a color image using wavelet thresholding and HSV model. In this method, the RGB image is converted into CIELAB color space. Wavelet decomposition is done on this and the results are passed on to CSF compensation block. This block describes the pattern sensitivity of the HV as a function of contrast and spatial frequency. CSF plots visual sensitivity for detecting sinusoidal gratings as a function of their spatial frequency. Inverse DWT is applied to the image coefficients. The resultant is in CIELAB color space and they are converted back to RGB colorspace.

Blair et al. (2005) have proposed an image enhancement method using logarithmic transform of coefficient histogram shifting. In this method, first the image is transformed using DCT, Fourier or other transformations and the logarithmic value of magnitude coefficients are calculated. A coefficient histogram is constructed by using this value. The histogram is shifted by k bins, where 'k' is the constant and the mapping is done with transform data and shifted histogram. Then exponentiate the data and inverse transformation is applied to the resultant data.

Yue et al. (2005) [3] has discussed a non-linear image enhancement method based on Gabor filters, which allows selective enhancement based on the contrast sensitivity function of the human visual system. The image enhancement of the given approach is especially appropriate for digital applications to enhance the perceive visual feature of the images due to numerous reasons, including interpolation.

Sai et al. (2006) [4] has proposed an image enhancement algorithm for digital images captured under such tremendously non-uniform lighting conditions. The new technique constitutes three issues viz, adaptive intensity enhancement, contrast improvement and color restoration which were considered separately to make the algorithm more adaptable to the image characteristics. The adaptiveness of the

transfer function, depending on the mean of each pixel's neighborhood makes the algorithm more flexible and easier to control.

Chin et al. (2008) has proposed an image enhancement technique by adjusting the contrast of spatial frequencies. The idea is that a digital signal is represented in two bases, high frequency and low frequency. In this method, the grey levels of two neighbouring pixels are taken (I1 and I2). If the difference between I1 and I2 is greater than or equal to zero, new high frequency and low frequency information is calculated by using the below equation.

Steve et al. (2008) have proposed an image enhancement method based on wavelet least squares estimators. With the evolution of imaging technology, an increasing number of imaging modalities becomes available. The images which contain several image planes are said to be multi component images. Even RGB color images are said to be multi component images as they contain three components. In these types of images, trade-off exists between spectral resolution and signal to noise ratio.

Nya et al. (2008) [5] has proposed a new method called Brightness Preserving Weight Clustering Histogram Equalization that can simultaneously preserve the brightness of the original image and enhance visualization of the original image. Given method assigns each nonzero bit of the original image's histogram to a separate cluster, and computes each cluster's weight. To reduce the number of clusters, three criteria are used (cluster weight, weight ratio and widths of two neighboring clusters) to merge pairs of neighboring clusters. The clusters obtain the equal partition as the result image histogram. At last, transformation functions for each cluster's sub-histogram are calculated, and the sub-histogram's gray levels are mapped to the result image by the equivalent transformation functions.

Chit et al. (2009) have proposed a contrast enhancement method for minimum mean brightness error from histogram partitioning. First the image is separated by class by calculating the threshold value and each class histogram is then equalized. This method separates the input image's histogram into various groups based on input mean before equalizing them independently.

Fan et al. (2010) [6] has proposed a new method for image contrast enrichment which is especially suitable for multiple-peak images. The given method has been used to remove the two disadvantages of HE algorithm i.e. firstly the input image has been convolved by a Gaussian filter with optimum parameters. Then, the original histogram has been divided into various areas by the valley values of the image histogram. The given method outperforms others on the aspects of simplicity and adaptability. The result demonstrates that the proposed algorithm has good performance in the area of image enrichment. Due to its simplicity, it can be realized by simple hardware and consumer electronics.

Md. Faisal et al. (2010) [7] has proposed a method of medical image enhancement based upon non-linear technique and the logarithmic transform coefficient histogram equalization. Logarithmic transform histogram matching uses the truth that the relation between stimulus and perception is logarithmic. A measure of improvement based on the transform has been used as a tool for evaluating the performance contrast measure with respect of the proposed enhancement technique. This method improves visual quality of images that contain dark shadows due to limited dynamic range of imaging like x-ray images.

Kwok et al. (2010) [8] has proposed a strategy of local sector improvement by histogram equalization. In given strategy the image has been first divided into sectors and they are independently enhanced by histogram equalization, intermediate images are then provoked recursively by making use of this approach and a resultant image has been obtained by a weighted-sum aggregation on the basis of an intensity gradient measure. Local sectors with higher contrast dominate the others thus achieving overall global contrast improvement. An enhanced image is then produced where the intermediate images are repeatedly averaged using a weight-sum strategy.

Cheng et al. (2012) [9] has discussed a novel approach for the detection of over-enhancement. Firstly the causes for generating over - enhancement has been analyzed in

detail, then an objective and effective criterion has been presented. The experimental results demonstrate that the given approach can locate the over enhanced areas accurately and effectively, and provide a quantitative criterion to assess the over-improvement levels well. The given method will be useful for vigorously monitoring the quality of the improved image, and optimizing the parameter settings of the contrast improvement algorithms.

Deepak et al. (2012) [10] has proposed a method for improving the color images based on non-linear transfer function and pixel neighborhood by conserving details. In given method, the image improvement has been applied only on the V (luminance value) component of the HSV color image and H and S component are kept unaffected to prevent the degradation of color balance between HSV components. The V channel has been enhanced in two steps. First the V component image has been divided into little overlapping chunks and for each pixel inside the chunk the luminance improvement is carried out using non-linear transfer function. Secondly, each pixel has been further improved for the adjustment of the image contrast depending upon the center pixel value and its neighborhood pixel values. Finally, original H and S component image and enhanced V component image are converted back to ROB image.

Rajib et al. (2012) [11] has proposed a contrast enhancement technique using scaling of internal noise of a dark image in discrete cosine transform domain. The mechanism of improvement is attributed to noise-induced transition of discrete cosine transform coefficients from a poor state to an improved state. This transition is effected by the inner noise present due to lack of adequate illumination and can be modeled by a general bi-stable system exhibiting dynamic stochastic resonance. The given approach has adopted a limited adaptive processing and significantly improves the image contrast and color information while ascertaining good perceptual quality. When compared with the present improvement approaches such as adaptive histogram equalization, gamma correction etc. the given approach has shown extraordinary

performance in terms of relative contrast enhancement, colorfulness and visual quality of improved image.

P.Kanna et al (2012) have proposed a contrast enhancement method of sports images using two approaches viz. histogram equalization and enhancement based on fuzzy rule. Modified sigmoid function has also been used in their work to enhance contrast. Sigmoid is a continuous non-linear function obtained from the fact that the function is ‘S’ shaped.

PROPOSED METHOD

Both the proposed methods undergo two fundamental steps: exposure thresholds calculation and histogram clipping. The description of each step is presented in the following subsections.

4.1. Exposure threshold calculation

The categorization of an image into low or high exposure is done based on exposure threshold [20]. The normalized range of exposure value is [0–1]. The images containing majority of low exposure regions possess exposure values lesser than 0.5 tending towards zero however, the overexposed images have exposure values greater than 0.5 tending towards one. Image intensity exposure value can be calculated as Eq. (1)

$$Exposure = \frac{\sum_{k=0}^{L-1} h(k)k}{L \sum_{k=0}^{L-1} h(k)} \quad (1)$$

Where $h(k)$ is histogram of image and L is total number of grey levels. Parameter X_a (as calculated in Eq. (2)) is the grey level boundary value that divides the image into under exposed and overexposed sub images.

$$X_a = L (1 - Exposure) \quad (2)$$

4.2. Histogram clipping

The idea behind histogram clipping is to prevent over enhancement leading to the natural appearance of the image. For limiting the enhancement rate, we need to limit the first derivative of histogram or the histogram itself [12]. The histogram bins having values greater than the clipping threshold are limited to the thresh-old (Fig. 1)

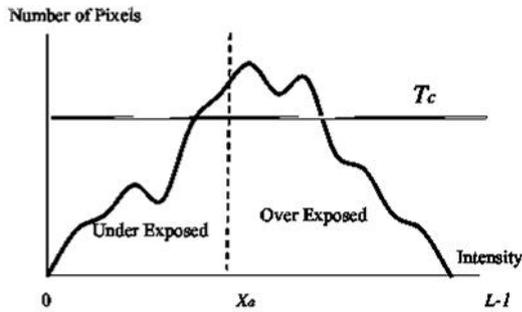


Fig. 1. Process of histogram sub division and clipping.

The clipping threshold is calculated as an average number of grey level occurrences. The formula for clipping threshold T_c is presented in (3) and (4) calculates the clipped histogram

$$T_c = \frac{1}{L} \sum_{k=0}^{L-1} h(k) \quad (3)$$

$$h_c(k) = \begin{cases} h(k), & h(k) < T_c \\ T_c, & h(k) \geq T_c \end{cases} \quad (4)$$

Where $h(k)$ and $h_c(k)$ are the original and clipped histogram, respectively. This method of histogram clipping is computationally efficient and consumes lesser time.

4.3. Recursive exposure based sub-image histogram equalization (R-ESIHE)

The proposed RESIHE method is a recursive variant of ESIHE. The number of recursions is dependent on the exposure difference between successive iterations. The fundamental step of the algorithm is histogram subdivision and equalization.

4.3.1. Histogram sub division and equalization

The original histogram is first bisected in two sub images W_L and W_U ranging from grey level 0 to X_a-1 and X_a to $L-1$ based on exposure threshold value X_a as calculated in (2). $P_L(k)$ and $P_U(k)$ are corresponding PDF of these sub images as defined in Eqs. (5) and (6)

$$P_L(k) = \frac{h_c(k)}{N_L} \quad \text{for } 0 \leq k \leq X_a - 1$$

$$P_U(k) = \frac{h_c(k)}{N_U} \quad \text{for } X_a \leq k \leq L-1 \quad (6)$$

Where N_L and N_U are total number of pixels in sub images W_L and W_U , respectively. $C_L(k)$ and $C_U(k)$ are corresponding CDF of individual sub images and CDFs can be defined

$$C_L(k) = \sum_{k=0}^{X_a-1} P_L(k) \quad (7)$$

$$C_U(k) = \sum_{k=X_a}^{L-1} P_U(k) \quad (8)$$

The next step is histogram equalization of both sub images individually applying the transfer functions

$$F_L = X_a C_L \quad (9)$$

$$F_U = (X_a + 1) + (L - X_a + 1) C_U \quad (10)$$

F_L and F_U are the transfer functions used for equalizing the sub histograms individually. The final step involves the integration of both sub images into one complete image. The number of iterations is decided based on a threshold ϵ whose value is normally taken very less (here 0.01).

4.3.2. Algorithm of RESIHE

- Step 1: Compute the histogram $h(k)$ of the image.
- Step 2: Compute the value of exposure and threshold parameter X_a .
- Step 3: Compute the clipping threshold T_c and clip the histogram $h_c(k)$.
- Step 4: Divide the clipped histogram into two sub histograms using the threshold parameter X_a .
- Step 5: Apply the histogram equalization on individual sub histograms.
- Step 6: Combine the sub images into one image.
- Step 7: Repeat step 1–6 until the exposure difference between successive iterations is less than threshold ϵ .

4.4. Recursively separated exposure based sub-image histogram equalization (RS-ESIHE)

Conceptually, RS-ESIHE is a recursive version of ESIHE, which performs recursive decomposition of the histogram. ESIHE decomposes the input histogram only once based on the exposure threshold while RS-ESIHE decomposes it recursively based on exposure thresholds of individual sub histograms up to a recursion level r , generating 2^r sub-histograms. The decomposed sub histograms are then equalized individually. For simplicity recursion level, r is taken as two. The RS-ESIHE method comprises of exposure threshold calculations, histogram clipping and histogram sub division and equalization.

4.4.1. Exposure threshold calculations

The exposure threshold X_a of complete histogram is calculated as per equation (1). Two more exposure

thresholds (X_{al} and X_{au}) are calculated for two individual sub histogram divided based on X_a

$$X_{al} = L \left[\frac{X_a}{L} - \frac{\sum_0^{X_a-1} h(k)k}{L \sum_0^{X_a-1} h(k)} \right] \quad (11)$$

$$X_{au} = L \left[1 + \frac{X_a}{L} - \frac{\sum_0^{X_a-1} h(k)k}{L \sum_0^{X_a-1} h(k)} \right]^{\frac{1}{2}} \quad (12)$$

4.4.2. Histogram sub division and equalization

The original histogram is first bisected based on exposure threshold value X_a as calculated in (2). These individual sub histograms are further decomposed into two smaller sub histograms where the individual exposure threshold X_{al} and X_{au} as calculated in (6) and (7) acts as separating point of sub histograms. The histogram sub division process results in four sub images W_{Ll} , W_{Lu} , W_{Ul} and W_{Uu} ranging from grey level 0 to $X_{al}-1$, X_{al} to X_a-1 , X_a to $X_{au}-1$, X_{au} to $L-1$. $P_{Ll}(k)$, $P_{Lu}(k)$, $P_{Ul}(k)$ and $P_{Uu}(k)$ are corresponding PDF of these sub images as defined in equations (13–16)

$$P_{Ll}(k) = \frac{h_c(k)}{N_{Ll}} \text{ for } 0 \leq k \leq X_{al} - 1 \quad (13)$$

$$P_{Lu}(k) = \frac{h_c(k)}{N_{Lu}} \text{ for } X_{al} \leq k \leq X_a - 1 \quad (14)$$

$$P_{Ul}(k) = \frac{h_c(k)}{N_{Ul}} \text{ for } X_a \leq k \leq X_{au} - 1 \quad (15)$$

$$P_{Uu}(k) = \frac{h_c(k)}{N_{Uu}} \text{ for } X_{au} \leq k \leq L - 1 \quad (16)$$

N_{Ll} , N_{Lu} , N_{Ul} and N_{Uu} are total number of pixels in sub images W_{Ll} , W_{Lu} , W_{Ul} and W_{Uu} , respectively. $C_{Ll}(k)$, $C_{Lu}(k)$, $C_{Ul}(k)$ and $C_{Uu}(k)$ are corresponding CDF of individual sub images and CDFs can be defined as equations (17–20)

$$C_{Ll} = \sum_{k=0}^{X_{al}-1} P_{Ll}(k) \quad (17)$$

$$C_{Lu} = \sum_{k=X_{al}}^{X_a-1} P_{Lu}(k) \quad (18)$$

$$C_{Ul} = \sum_{k=X_a}^{X_{au}-1} P_{Ul}(k) \quad (19)$$

$$C_{Uu} = \sum_{k=X_{au}}^{L-1} P_{Uu}(k) \quad (20)$$

The next step of RS-ESIHE is to equalize all the four sub histograms individually. The transfer functions for histogram equalization based on equations (13–20) can be defined as equations (21–24)

$$F_{Ll} = X_{al} C_{Ll} \quad (21)$$

$$F_{Lu} = (X_{al} + 1) + (X_a - X_{al} + 1) C_{Lu} \quad (22)$$

$$F_{Ul} = (X_a + 1) + (X_{au} - X_a + 1) C_{Ul} \quad (23)$$

$$F_{Uu} = (X_{au} + 1) + (L - X_{au} + 1) C_{Uu} \quad (24)$$

F_{Ll} , F_{Lu} , F_{Ul} and F_{Uu} are the transfer functions used for equalizing the sub histograms individually. The RS-ESIHE output image is produced by the combination of all four transfer functions.

4.4.3. Algorithm of RS-ESIHE for recursion level $r = 2$

Step 1: Compute the histogram $h(k)$ of the image.

Step 2: Compute the value of exposure and threshold parameter X_a .

Step 3: Compute the clipping threshold T_c and clip the histogram $h_c(k)$.

Step 4: Divide the clipped histogram into two sub histograms using the threshold parameter X_a

Step 5: Compute exposure thresholds X_{al} and X_{au} for lower and upper sub histograms, respectively and divide the sub histograms into further sub histograms using X_{al} and X_{au} as decomposing threshold, resulting in total four sub histograms.

Step 6: Apply the histogram equalization on individual sub histograms and combine the sub images into one image for analysis.

Experimental results

In this section, the simulation results of proposed methods are compared with existing histogram equalization based methods i.e. RMSHE, RSIHE, QDHE, RSWHE, BHEPL and ESIHE. In order to analyze and compare the existing methods four low exposure test images: fish1, fish2, mosque and couple are taken. Both fish images are underwater

sequence while other two images are captured in low light conditions.

5.1. Performance assessment based on visual quality

Images acquired in low light conditions including under water sequences are taken to test the robustness of the proposed method for low exposure imaging. The analysis of visual results from Figs. 2–5 shows the effective-ness of recursive methods especially on low light conditions. The concrete results in terms of contrast enhancement can be clearly observed in Fig. 2 of mosque image. RSIHE, RMSHE, RSWHE and BHEP-L methods are not able to increase the exposure, however,

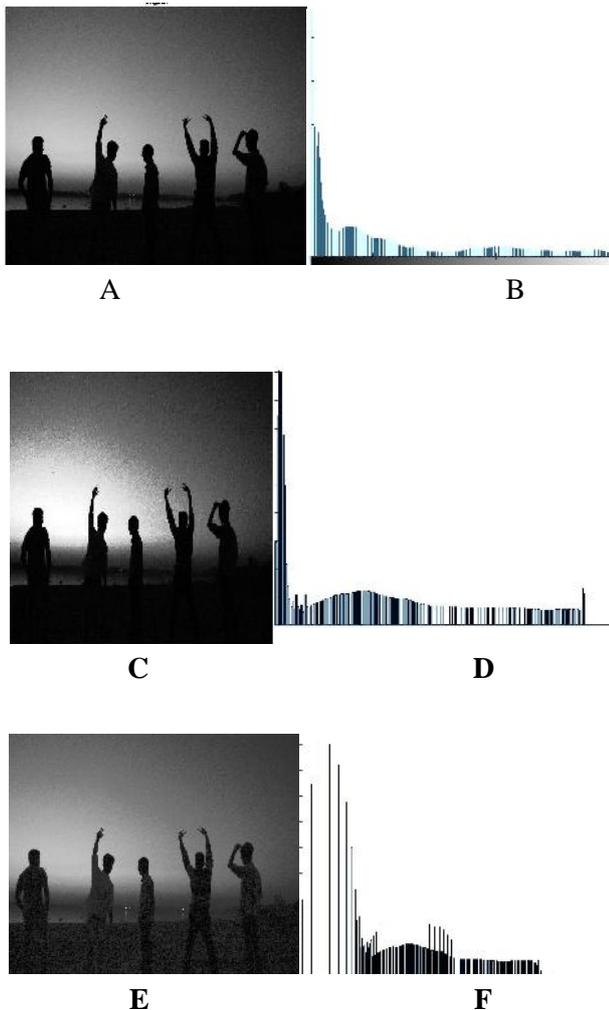


Fig. 1. Results of image1 (a) original, (b) Histogram of original image, (c) RESHIE, (d) Histogram of RESHIE image (e) RS-ESHIE Histogram of RS-ESHIE

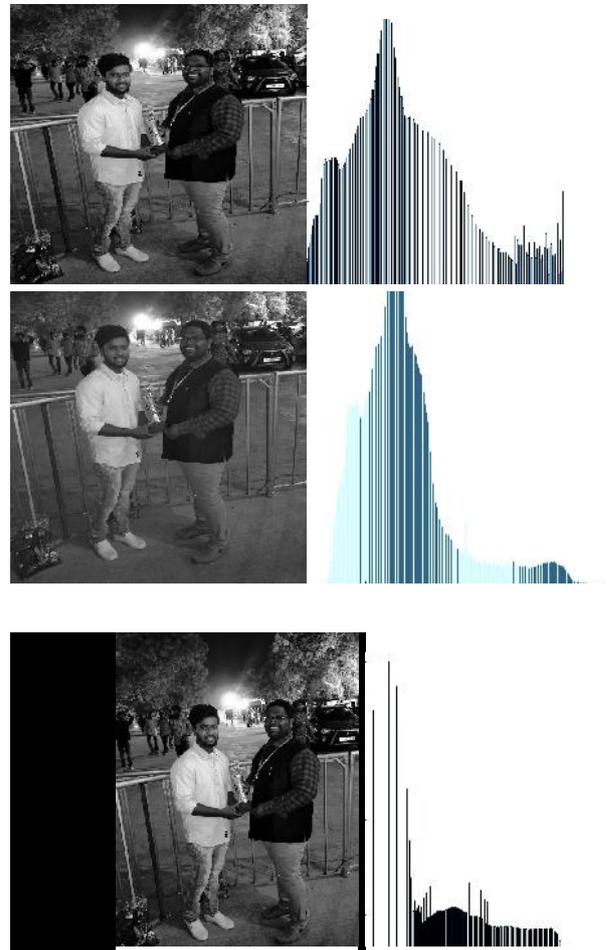


Fig. 2. Results of image2 (a) original, (b) Histogram of original image, (c) RESHIE, (d) Histogram of RESHIE image (e) RS-ESHIE Histogram of RS-ESHIE

both RESIHE and RS-ESIHE has improved the contrast and the objects are clearly visible. In both the underwater images i.e. fish1 and fish2 in Figs. 3 and 5, respectively the original images are acquired in very dim light condition and the object is not clearly visible. The resultant fish images of proposed methods have the objects clearly distinguishable. The proposed algorithms enhance the over-all image quality of couple image in Fig. 4. Low-intensity regions in the background are properly exposed resulting clear vision.

Conclusion

Effective recursive histogram equalization techniques are proposed here for enhancement of low exposure images. Decomposition of histogram

based on exposure based thresholds and individual sub histogram equalization provide very efficient results for low exposure imaging. The methods can be very effective for contrast enhancement of images acquired in dim light conditions i.e. under water or night vision images. Better results in terms of average information contents make these methods more suitable for bringing out information contents of the images. The histogram clipping technique is also combined with histogram equalization to provide control on over enhancement that leads to natural enhancement. The visual quality of resultant images of recursive methods shows the robustness of the methods on low light images.

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