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Estimation of Thermal Contact Conductance at the Interface of Two Bodies using Inverse Method

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Abstract

Heat transfer at the interface of two materials has an important role in different engineering systems ranging from cooling of gas turbine blades, Nuclear reactors, electronic circuits to the spacecraft structures and cryogenic transports. Inverse problem is highly useful in the application where direct estimation of a property is difficult or inaccurate such as at the interface of two materials. Hence, in this work, the inverse heat conduction problem has been utilized for the estimation of thermal contact conductance at the interface of two metallic bodies in continuous contact assuming one-dimensional heat conduction. The technique of Conjugate Gradient Method with Adjoint problem has been employed to estimate the functional form of Thermal contact conductance. The input to the technique is transient temperature data at fixed axial locations in the contacting bodies. Simulated measured data with three different random errors has been derived for the four test cases of thermal contact conductance with time, namely, Periodic function, triangular function, polynomial function and constant value function. One end of contacting solids was placed at constant heat flux while other at constant temperature. Estimated thermal contact conductance was observed to be dependent on the measurement errors. The results establish the usefulness of inverse heat conduction method using Conjugate Gradient Method for the estimation of the thermal contact conductance at the interface of two bodies.

1. Introduction

Engineering surfaces are never absolutely smooth and the surface irregularities become apparent when observed under a microscope. As a result, when two solids are in contact, actual contact is made only at a few discrete points separated by relatively large gaps. Due to the reduction in heat transfer area at the interface, there exists an extra resistance to heat flow, known as thermal contact resistance. The inverse of thermal contact resistance is called as the thermal contact conductance (TCC). Heat transfer across the interface can take place by means of conduction through solid-to-solid contact spots and conduction through the gaps. Heat transfer through the interfacial gap is thermal gap conductance which together with the conductance of contact spots makes the overall joint conductance of a thermal contact. In vacuum environment, the thermal gap conductance is negligible; therefore the joint conductance is predominantly the conductance of contact spots only. Radiative heat transfer across the gaps is negligible at moderate temperatures, and needs to be considered only if interface temperatures are above 300 ^{0}C [1].

Thermal contact conductance is an important factor in numerous applications, mainly because many physical systems contain a mechanical combination of two materials. Particularly in a vacuum environment, which is essential to spacecraft and cryogenic applications, accurate evaluation of thermal contact conductance is necessary for the design of efficient compact heat exchangers [2]. The development of spacecraft structures to withstand re-entry heating, the thermal insulation of cryogenic storage compartments and the optimum packaging and improvement of heat dissipation systems of electronic equipments all require a thorough knowledge of thermal contact conductance for their analyses and design [3]. Heat transfer through interfaces is also of considerable interest in microelectronic-chip cooling, heat exchangers and in nuclear reactors. Due to the importance of thermal contact conductance, it had been a topic of research for several decades. Many researchers have carried out their study on thermal contact conductance by using experimental methods as well as theoretical methods. The theoretical methods used in the literatures are mainly of two types as follows:

- by using deformation analysis of the contact surfaces and considering constriction of heat flow[4-9].
- by using inverse heat conduction method

The inverse method implies that the effect is utilized to estimate the cause. Inverse heat conduction method is useful in the fields where classical methods for property estimation can not be applied directly or do not provide the desired accuracy. In our field of interest that is for the estimation of thermal contact conductance at the metallic interface, it is not possible to obtain the real contact area of contacting bodies for a given pressure. To find the real contact area different numerical models such as CMY model, GW model have been developed but no model is perfect. As far as the experimental method is concerned to find the thermal contact conductance at the interface, it is very difficult to measure the temperature and heat flux at the interface by using conventional technique and extrapolation method is not perfect. In contrary to this, the inverse problem utilizes the temperature measurement at some fixed spatial locations in the contacting bodies to estimate the thermal contact conductance without paying attention to the real contact area and heat flux at the interface. But the inverse heat transfer problem is ill-posed problem so a regularization method should be used to make it well-posed and also it is very sensitive to random errors so special technique, should be used for its solution.

There are two general classes of inverse problem called as parameter and function estimation. Beck [10] discussed the problems that have aspects of both types of estimation related to the thermal contact conductance. He illustrated various types of estimation and analyzed two experiments for thermal contact conductance. One of the experiments was by Antonetti and Eid [11], a transient experiment that approached steady state. Their steady-state data were analyzed using parameter estimation. The other transient experiment by Moses and Johnson [12] was analyzed using both parameter and function estimation techniques. Huang et al. [13] developed an inverse solution methodology based on the conjugate gradient method to determine the time wise variation of the contact conductance between the mould and casting Specimen, from the transient temperature measurements taken with thermocouples inside the casting Specimen and at the outer mould surface. To illustrate the accuracy of approach in predicting $h(t)$ with inverse analysis, two test cases involving a triangular and a step contact conductance function were examined. Further, Huang et al. [14] worked to solve a nonlinear inverse problem for estimating the local two dimensional thermal contact conductance of a plate finned tube heat exchanger using multiple circumferential and temporal simulated transient temperature measurements. For the nonlinear inverse problem the Conjugate Gradient Method was used for minimization. Fieberg and Kneer [15] presented an experimental approach to derive the thermal contact resistance in terms of contact heat transfer coefficients for high temperature and high pressure conditions based on transient infrared temperature measurements and the contact heat flux was calculated by solving the related inverse problem. From the contact heat flux and from the measured temperature jump at the interface the contact heat transfer coefficient was calculated. Shojaeefard et al. [16] applied the technique of inverse heat conduction problem to solve the problem for estimating the periodic thermal contact conductance between one-dimensional, constant property contacting solids. The Conjugate Gradient Method of minimization with adjoint problem was utilized to solve inverse heat transfer problems of function estimation.

From the above literatures it is very clear that very few of the researchers had been employing inverse technique to estimate thermal contact conductance (TCC) and they too had been utilizing the inverse problem for a particular field of interest, like mould and casting specimen, see [13], heat exchanger, see [14], I.C engine, see [15], periodic contacts, see [16], and likewise. In this paper we have applied the technique of inverse heat conduction with function estimation to work out the problem of estimating thermal contact conductance for a generalized case, i.e., two metallic flat surfaces in contact. Actually, inverse technique with function estimation is better approach when no prior information is available regarding the variation of unknown function i.e. thermal contact conductance. One of the most stable algorithms for IHCP is the iterative method of optimization by conjugate gradient technique, even without a Tikhonov regularization term in the objective functional [17]. Therefore conjugate gradient method (CGM) has been employed to

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estimate the unknown thermal contact at the interface of two metallic solids conductance as a function of time. The input to the inverse problem is transient temperature distribution, which has been provided by simulation.

2. Problem formulation

The technique of Conjugate Gradient Method (CGM) of minimization with adjoint problem for function estimation has been utilized to the estimation of the time-wise variation of Thermal contact conductance (TCC) between two solid bodies in contact assuming one dimensional heat flow. In this approach, no priori information on the functional form of TCC is required, except for the functional space which it belongs to. The basic steps of this technique for the solution of function estimation problem include:

- Direct problem
- Inverse Problem
- Sensitivity Problem
- Adjoint Problem
- Gradient Equation
- Iterative Procedure
- Stopping Criterion
- Computational Algorithm

2.1 Direct Problem

The direct problem involves the determination of temperature distribution in the Specimens when the thermophysical properties, contact conductance $h(t)$ and the boundary conditions at the outer ends are known. Figure 1 shows the geometry and the computational grid generation for the one-dimensional heat transfer problem considered.

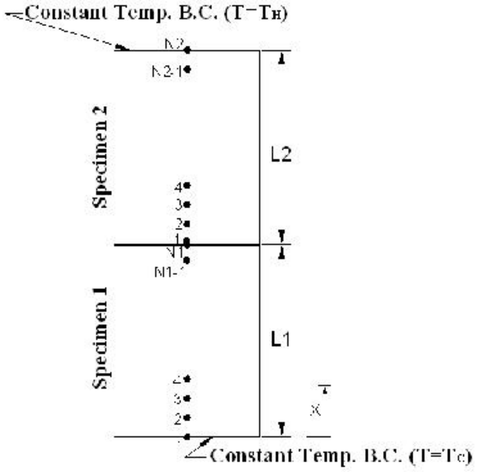

Figure 1: Geometry and computational grid generation

Two metallic solid bodies referred to as specimens 1 and 2 having length L_1 and L_2, thermal conductivity k_1 and k_2, thermal diffusivity α_1 and α_2 are kept in contact for time t_f, with contact conductance $h(t)$ at the interface. Both the bodies are having initial temperatures as T_i. The non-contacting ends are set at boundary conditions: Lower end at constant low temperature $(T = T_c)$ and upper end at constant heat Flux $(Q = Q_F)$. The mathematical formulation of this one dimensional heat conduction problem is given as:

Specimen 1 $(0 \le x \le L_1)$:

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$$\frac{\partial^2 T_1}{\partial x^2} = \frac{1}{\alpha_1}\frac{\partial T_1}{\partial t}; \quad 0 < x < L_1 \text{ and } 0 < t \le t_f \tag{1a}$$

$$T_1 = T_c; \quad x = 0 \text{ and } 0 < t \le t_f \tag{1b}$$

$$-k_1 \frac{\partial T_1}{\partial x} = h(t)(T_1 - T_2'); \quad x = L_1 \text{ and } 0 < t \le t_f \tag{1c}$$

$$T_1 = T_i; \quad t = 0 \text{ and } 0 \le x \le L_1 \tag{1d}$$

Specimen 2: $(L_1 \le x \le L_1 + L_2)$

$$\frac{\partial^2 T_2}{\partial x^2} = \frac{1}{\alpha_2}\frac{\partial T_2}{\partial t}; \quad L_1 < x < L_1 + L_2 \text{ and } 0 < t \le t_f \tag{2a}$$

$$-k_2 \frac{\partial T_2}{\partial x} = h(t)(T_1 - T_2'); \quad x = L_1 \text{ and } 0 < t \le t_f \tag{2b}$$

$$-k_2 \frac{\partial T_2}{\partial x} = Q_F; \quad 0 < t \le t_f \tag{2c}$$

$$T_2 = T_i; \quad t = 0 \text{ and } L_1 \le x \le L_1 + L_2 \tag{2d}$$

2.2 Inverse problem

The inverse problem is constructed by considering the contact conductance, $h(t)$ as unknown function of time but everything else in the system of equations (1) and (2) as known and temperature data are taken at some suitable locations within the Specimens1 and 2.

Referring to the nomenclature shown in Figure 1, we have divided the Specimen 1 into N_1 grid points and Specimen 2 into N_2 grid points and thermocouples located in Specimen 1 are from grid points 2 to $N_1 - 1$ and in Specimen 2 from grid point 2 to $N_2 - 1$. All the thermocouples are located at uniform distances of Δx_1 in Specimen 1 and Δx_2 in Specimen 2. And, the temperature data are taken with these thermocouples up to the time, $t = t_f$ and denoted by; $Y_1(t) \equiv Y_{1i}, i = 1, 2, \ldots N_1$ in Specimen 1 and $Y_2(t) \equiv Y_{2j}, j = 1, 2, \ldots N_2$ in Specimen 2. Now, the inverse problem is formed as: By utilizing the above mentioned measured temperature readings Y_{1i} and Y_{2j} and known thermophysical properties, estimate the unknown contact conductance $h(t)$ over the total time period t_f. It is assumed that no priori information is available on the functional form of $h(t)$, except that the total time of contact t_f. We have to find out the $h(t)$ over the whole time domain, $(0, t_f)$ with the assumption that $h(t)$ belongs to the Hilbert space of square-integrable functions in this domain, i.e.

$$\int_0^{t_f} [h(t)]^2 dt < \infty$$

The solution of the present inverse problem has to be obtained in such a way that the following functional is minimized:

$$S[h(t)] = \int_0^{t_f} \sum_{i=2}^{N_1-1} (T_{1i} - Y_{1i})^2 \, dt + \int_0^{t_f} \sum_{j=2}^{N_2-1} (T_{2j} - Y_{2j})^2 \, dt \tag{3}$$

where $T_{1i} \equiv T_1(t)$ and $T_{2j} \equiv T_2(t)$ are the estimated temperatures at the measurement locations in Specimens 1

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and 2, respectively.

2.3 Sensitivity problem

The sensitivity function $\Delta T(x,t)$ solution of sensitivity problem is defined as the directional derivative of the temperature $T(x,t)$ in the direction of the perturbation of the unknown function. The sensitivity function is required for the computation of the search step size β^k, required in iterative solution [18]. The sensitivity problem is obtained from the direct problem defined by equations (1) and (2). It is assumed that when $h(t)$ undergoes a variation $\Delta h(t)$, $T_1(x,t)$ is perturbed by $\Delta T_1(x,t)$ and $T_2(x,t)$ by $\Delta T_2(x,t)$. Then, by replacing in the direct problem $h(t)$ by $h(t)+\Delta h(t)$, $T_1(x,t)$ by $T_1(x,t)+\Delta T_1(x,t)$ $T_2(x,t)$ by $T_2(x,t)+\Delta T_2(x,t)$, subtracting from the resulting expressions the original direct problem and neglecting second order terms, the Sensitivity Problem for the sensitivity functions $\Delta T_1(x,t)$ and $\Delta T_2(x,t)$ is obtained.

2.4 Adjoint problem

In the present inverse problem, the estimated temperatures should satisfy two constraints, which are the heat conduction problems for Specimens 1 and 2, respectively. Therefore, two Lagrange multipliers come into picture here. Actually these Lagrange multipliers are needed for the computation of the gradient equation (after-mentioned). The following *Adjoint problem* has to be solved for the determination of the Lagrange multipliers $\lambda_1(x,t)$ and $\lambda_2(x,t)$:

Specimen 1 $(0 \le x \le L_1)$:

$$\frac{\partial^2 \lambda_1}{\partial x^2} + \frac{1}{\alpha_1}\frac{\partial \lambda_1}{\partial t} + 2\sum_{i=2}^{(N_1-1)} (T_{1i} - Y_{1i})\,\delta(x - x_i) = 0; \quad 0 < x < L_1 \text{ and } 0 < t < t_f \tag{4a}$$

$$\lambda_1 = 0; \quad x = 0 \text{ and } 0 < t < t_f \tag{4b}$$

$$\frac{\partial \lambda_1}{\partial x} = h(t)\left(\frac{\lambda_2}{k_2} - \frac{\lambda_1}{k_1}\right); \quad x = x_1 \text{ and } 0 < t < t_f \tag{4c}$$

$$\lambda_1 = 0; \quad t = t_f \text{ and } 0 \le x \le L_1 \tag{4d}$$

Specimen 2 $(L_1 \le x \le L_1 + L_2)$:

$$\frac{\partial^2 \lambda_2}{\partial x^2} + \frac{1}{\alpha_2}\frac{\partial \lambda_2}{\partial t} + 2\sum_{j=2}^{(N_2-1)} (T_{2j} - Y_{2j})\,\delta(x - x_j) = 0; \quad L_1 < x < L_1 + L_2 \text{ and } 0 < t < t_f \tag{5a}$$

$$\frac{\partial \lambda_2}{\partial x} = h(t)\left(\frac{\lambda_2}{k_2} - \frac{\lambda_1}{k_1}\right); \quad x = x_1 \text{ and } 0 < t < t_f \tag{5b}$$

$$\frac{\partial \lambda_2}{\partial x} = 0; \quad x = L_1 + L_2 \text{ and } 0 < t < t_f \tag{5c}$$

$$\lambda_2 = 0; \quad t = t_f \text{ and } L_1 \le x \le L_1 + L_2 \tag{5d}$$

where $\delta(\cdot)$ is the Dirac delta function.

2.5 Gradient equation

The following expression has been obtained for the gradient $S'[h(t)]$ of the functional $S[h(t)]$:

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$$S'[h(t)] = \left[\frac{\lambda_2(L_1,t)}{k_2} - \frac{\lambda_1(L_1,t)}{k_1}\right][T_1(L_1,t) - T_2(L_1,t)] \qquad (6)$$

2.6 Iterative procedure

For the estimation of thermal contact conductance $h(t)$ an iterative procedure is adopted through the minimization of the functional $\Delta S[h(t)]$. The iterative procedure for the conjugate gradient method is as follows:

$$h^{k+1}(t) = h^k(t) - \beta^k d^k(t) \qquad (7)$$

where β^k is the *search step size* in going from iteration k to iteration $k+1$ and $d^k(t)$ is the *Direction of Descent* given by:

$$d^k(t) = \nabla S[h^k(t)] + \gamma^k d^{k-1}(t) \qquad (8)$$

where γ^k is the *Conjugation Coefficient* and is determined from the Fletcher-Reeves expression as:

$$\gamma^k = \frac{\int_0^{t_f} \{\nabla S[h^k(t)]\}^2 dt}{\int_0^{t_f} \{\nabla S[h^{k-1}(t)]\}^2 dt} \quad \text{for } k = 1, 2, \ldots \text{ with } \gamma^k = 0 \text{ for } k = 0 \qquad (9)$$

The search step size β^k has been determined by minimizing the functional $S[h(t)]$ given by (3), which leads the following expression for β^k :

$$\beta^k = \frac{\int_0^{t_f} \left\{ \sum_{i=2}^{N_1-1} (T_{1i} - Y_{1i})\Delta T_{1i} + \sum_{j=2}^{N_2-1} (T_{2j} - Y_{2j})\Delta T_{2j} \right\} dt}{\int_0^{t_f} \left\{ \sum_{i=2}^{N_1-1} (\Delta T_{1i})^2 + \sum_{j=2}^{N_2-1} (\Delta T_{2j})^2 \right\} dt} \qquad (10)$$

where T_{1i} and T_{2j} are the solutions of the direct problem (1) and (2), obtained by using the current estimate for $h(t)$; while the sensitivity functions ΔT_{1i} and ΔT_{2j} are the solutions of the sensitivity problem (4) and (5), obtained by setting $\Delta h(t) = d^k(t)$

2.7 Stopping criterion

The problem has been converted into well-posed problem by employing the *Discrepancy Principle* in the iterative procedure. The stopping criterion founded on the discrepancy principle provides the conjugate gradient method of function estimation an *iterative regularization character*. The stopping criterion is given by:

$$S[h(t)] < \varepsilon \qquad (11)$$

where $S[h(t)]$ is computed with equation (3). The tolerance ε is chosen such that smooth solutions are obtained with measured temperatures containing random errors. The assumption is that the solution is sufficiently accurate when

$$|Y(t) - T[x_{meas}, t; h(t)]| \approx \sigma \qquad (12)$$

where σ is the standard deviation of measurement errors.

Thus ε is obtained from equation (3) using equation (18) as:

$$\varepsilon = N t_f \sigma^2 \qquad (13)$$

where *N is* the total number of thermocouples.

The above assumption for the temperature residuals in the discrepancy principle had also been adopted by Tikhonov [19] to find the optimal regularization parameter. If the measured temperatures are regarded as errorless, the tolerance $\aleph$ can be selected as a sufficiently small number, since the estimated minimum value for the objective function is zero. But for the actual problems the above stopping criterion method, employing the discrepancy principle demands *a priori* knowledge of the standard deviation of the measurement errors [18].

3. Numerical Implementation

The thermal contact conductance for two metallic solids in contact has been estimated by utilizing the function estimation inverse technique with conjugate gradient method. In the problem, one of the boundary condition, i.e. thermal contact conductance at the interface of two solids as a function of time is unknown while everything else, i.e., initial condition, thermophysical properties, other boundary condition is known and the transient temperatures at some fixed axial locations of the specimens are provided as an input. The transient temperature distribution in the specimens has been provided by simulated measured temperature data.

Simulated measurements

Simulated temperature measurements at fixed axial thermocouple locations of specimens have been obtained from the solution of one dimensional transient heat conduction equation by using a priori prescribed values for the unknown thermal contact conductance function $h(t)$. The solution of the direct problem (1-2) at the measurement locations by using various test function of $h(t)$, provides the exact measurements $Y_{ex}(x_{meas},t)$. The measurements containing random errors are simulated by adding an error term to $Y_{ex}(x_{meas},t)$ in the following form:

$$Y(x_{meas},t) = Y_{ex}(x_{meas},t) + \omega\sigma \tag{14}$$

where, $Y(x_{meas},t)$: simulated measurements containing random errors

$Y_{ex}(x_{meas},t)$: exact (errorless) simulated measurements

σ : standard deviation of the measurement error

μ : random variable with normal distribution, zero mean and unitary standard deviation. The random variable μ is generated using MATLAB function *normrnd*, which generates the variable at 95% confidence level.

Now, the inverse problem has been solved using conjugate gradient method as explained above providing the simulated measurements as an input to estimate the unknown thermal contact conductance with time. The stability of the inverse problem solution has been examined for various levels of measurement errors by generating measurements with different standard deviations. Both exact and inexact simulated temperature measurements have been considered but all the other parameters used in the analysis are supposed to be errorless. The physical problem based on the actual experiments is considered as follows:

Two identical cylindrical specimens each of length $L_1 = L_2 = 0.03$ m, diameter $d_1 = d_2 = 0.025$ m, made of brass, initially at a uniform temperature of $20\ ^{o}C$ are kept in contact. The properties of the Specimen 1 and Specimen 2 are: $k_1 = 107.6$ W/m-K, $\alpha_1 = 3.36 \times 10^{-5}\ m^2/s$, $k_2 = 136.6$ W/m-K, $\alpha_2 = 3.95 \times 10^{-5}\ m^2/s$ [20]. The specimens are kept in contact with boundary conditions as: Top end at constant heat flux, $Q_F = 150$ W and bottom end at constant low temperature T_C =10 ^{O}C. The Specimen 1 and Specimen 2 are divided into eleven grid points ($N_1 = N_2$ =11) with space steps, $\Delta x_1 = \Delta x_2 = 0.003$ m and the time step Δt is chosen as 1 sec.

The direct, adjoint and sensitivity problem for all the test cases with simulated measurements have been solved as explained above. The *Implicit form* of Finite Difference Method has been used for discretizing the *intermediate boundary value problems* as it is *unconditionally stable*. All the problems have been programmed with FORTAN according to the algorithm explained above. Four test cases analyzed are as follows:

Case A: Step function in $h(t)$. When the contacting solids are in the periodic contact, interface contact conductance is assumed to vary in the step function with time of contact and time of non-contact as 40 sec each. This function may test the algorithm and method of solution as the function containing discontinuities, and sharp corners are the most difficult to be recovered by an inverse analysis [18]. Thus,

$$h(t) = \begin{cases} 2381.1 \quad \text{(contact)} & 0 \le t \le 50 \\ 0 \quad \text{(non-contact)} & 50 < t \le 100 \end{cases} \tag{15}$$

Case B: Triangular jump in $h(t)$. The thermal contact conductance is assumed to vary in the form of triangular jump given by

$$h(t) = \begin{cases} 435 + 15(t+1) & 0 \le t \le 30 \\ 400 - \dfrac{100}{7}(t-65) & 30 \le t \le 65 \\ 400 & 65 \le t \le 100 \end{cases} \tag{16}$$

The triangular function is also chosen as it contains sharp corners.

Case C: Polynomial function in $h(t)$. Function estimation approach while utilizing the conjugate gradient method estimates the unknowns depending upon the number of time steps in order to establish the unknown function, whereas least square method works on few parameters depending upon the degree of polynomial to estimate the unknown function. Thus, when no *a priori* information is available for the unknown function, function estimation technique provides more accurate results than the least square method with lesser computation time and measurement errors (Huang *et al*., 1992). Hence, the function estimation technique of conjugate gradient method has been adopted, while the thermal contact conductance is assumed to be a cubic polynomial in time given by:

$$h(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 \quad \text{for } 0 \le t \le 64 \tag{17}$$

$$h(t) = 285 \quad \text{for } 64 < t < 100 \tag{18}$$

where constants $C_0 = 30,\ C_1 = 65.4542,\ C_2 = -2.2314,\ C_3 = 0.019835$

Case D: Constant value of $h(t)$. For one of the test cases an experimental value reported elsewhere [21] has been chosen to obtain the simulated measurements with or without noises.

$$h(t) = 17104.8 \tag{19}$$

The accuracy of the results obtained on the basis of simulated measurement has been evaluated by root mean square (RMS) error defined as [18]

$$e_{rms} = \left[\frac{1}{I} \sum_{i=1}^{I} \left(\bar{h}_{ex}(t) - \bar{h}_{est}(t) \right)^2 \right]^{1/2} \tag{20}$$

In addition, on the basis of percentage relative error (ε) of the results for the whole time domain t_f, which is defined as

$$e_{rms} = \left| \frac{1}{I} \sum_{i=1}^{I} \left(\bar{h}_{ex}(t) - \bar{h}_{est}(t) \right) / \bar{h}_{ex}(t) \right| \times 100 \quad (21)$$

where I is the number of transient measurement per thermocouple, $\bar{h}_{ex}(t)$ and $\bar{h}_{est}(t)$ refers to the exact and estimated dimensionless thermal contact conductance, respectively. In the above expression, the overbar quantity represent the non-dimensional function based on the factor L/k, i.e. $\bar{h}_{ex}(t) = h_{ex}(t)L/k$ and $\bar{h}_{est}(t) = h_{est}(t)L/k$.

4. Results and Discussion

Four test cases on the basis of their significance have been selected to predict the thermal contact conductance with time. A 10% overrun in final time has been used in performing the inverse analysis, as against the domain of interest, i.e $0 \le t \le 90$; which is based on the recommendation to get the reduction on the errors for both errorless measurements and measurements with random errors due to the effects of the initial guess on the solution, because of the null gradient at the final time [18].

For the Figures [2-6], h_{ex} refers to prescribed value of thermal contact conductance, used to predict the simulated temperature measurements and h_{est} refers to estimated thermal contact conductance resulted by solving inverse problem for different values of sigma (standard deviation of measurement errors).

Case A: Step function in $h(t)$

The two bodies are kept in contact with constant heat flux at the upper end and low constant temperature at lower end. The technique of conjugate gradient method has been employed to estimate the thermal contact conductance (TCC) with three set of standard deviation in measurement, i.e. $\pm \sigma = 0$ (no measurement error) $\pm = 0.5$ and $\pm = 1.0$. These values of standard deviations have been selected in order to accommodate the maximum possible scattering/measurement errors of the data acquired during the actual experimental run. From Figure 2, it is apparent that the estimated thermal contact conductance agrees well with the exact value but deviates at the starting time steps and at its sharp corners. Further, it has been confirmed that the inclusion of measurement errors lead to the deviation from the exact values. Furthermore, the rms error has been calculated as: $e_{rms} = 0.061$ for errorless measurements while including random errors ($\sigma = 1.0$), it goes to 0.071.

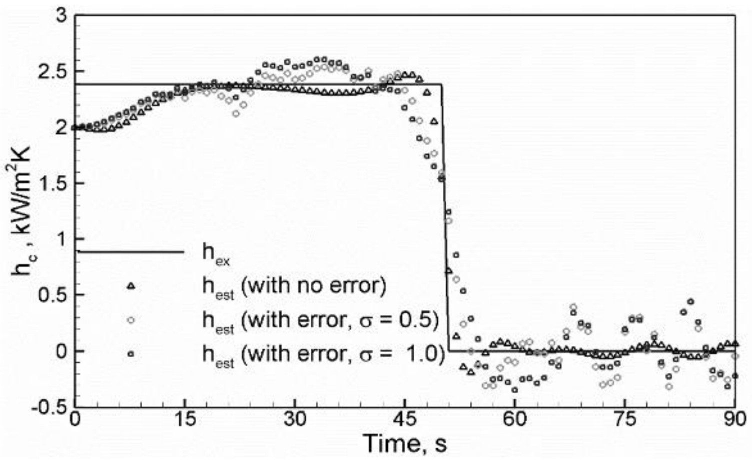

Figure 2: Variation of estimated thermal contact conductance for case A with different random errors

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Case B: Triangular jump in h ftA

For this case, the results of thermal contact conductance are shown in Figure 3. Here also, the results are pretty good for errorless measurements, while agreement is not that good for measurements with random errors. In addition, Figure 3 shows that the accuracy of the results reduces with the increase of measurement errors. Again, the differences between the estimated and exact TCC are mostly in the start and at the corners. To evaluate the results further, rms error and relative percentage deviation has been calculated with random errors. It has been found that e_{rms} X0.007 and ℵ = 2.4 % for errorless measurements while e_{rms} X0.012 and ℵ = 7.66 % for simulated measurements with ∃ X1.0.

The results from Figures 2 and 3 confirm the accuracy and effectiveness of the inverse algorithm even for the functions containing sharp corners and discontinuities.

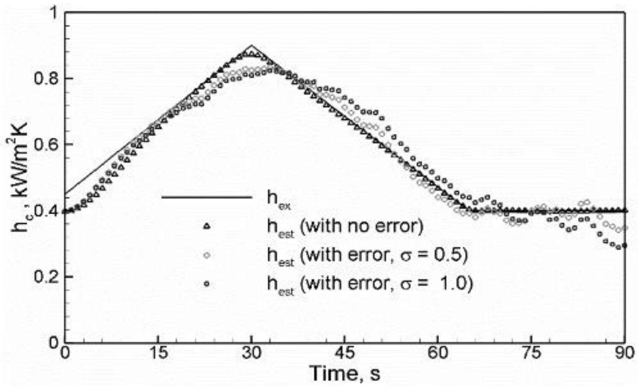

Figure 3: Variation of estimated thermal contact conductance for case B with different random errors

Case C: Polynomial function in h ftA

The estimated and the exact TCC with time are shown in Figure 4 for different standard deviation in simulated measurements. From Figure 4, it is observed that the results of estimated TCC are matching well with the exact value of TCC for both the cases with deviations at the start and peak. Finally, the agreement is affected adversely by the increase in random errors.

Further, in order to show the results quantitatively, rms errors and percentage relative deviations have been calculated for different random errors. For instance, e_{rms} X0.006 and ℵ = 5.45 % with errorless measurements while e_{rms} X0.019 and ℵ = 22.1 % including random errors with ∃ X1.0.

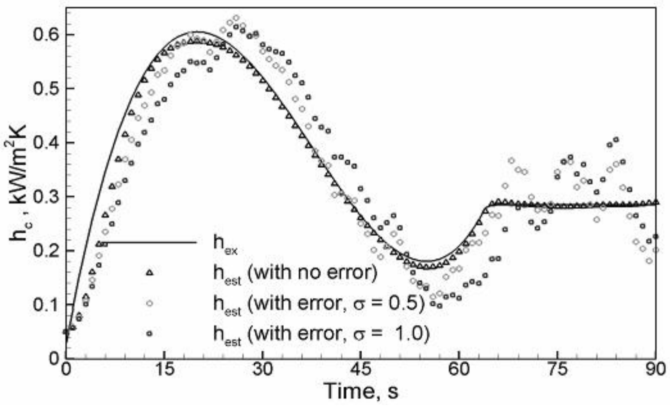

Figure 4: Variation of estimated thermal contact conductance for case C-2 with different random errors

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*Case D: Constant value of h ft*A

In this case a constant value of TCC (17104.8 W/m^2-K) has been utilized to obtain the simulated measured data. The TCC value has been selected on the basis of the steady state experimental result for brass-brass contacts [21]. A typical plot of estimated and input temperature data with time for ± X0.5 has been shown in Figure 5. While Figure 6 shows the estimated TCC with time for errorless measurements as well as measurements with random errors. In Figure 5, T_{1Zi}, T_{2Zj} refers to estimated temperatures while Y_{1Zi}, Y_{2Zj} indicates the simulated measurements in specimen-1 and specimen-2.

Figure 5 shows good agreement between the input simulated measurements with random errors (± X0.5) and estimated temperature data. Whereas from Figure 6, it may be noted that during first 5-8 seconds, the estimated values of TCC are not accurate for errorless measurements as well as measurements with random error. This is due to the effect of initial guess value, as explained in the earlier section in detail. Further, the effect of random error is evident from Figure 6 that an increase in the standard deviation of measurement error '± ' (i.e. increase in measurement error) leads to the decrease in the accuracy of the prediction. Therefore, the estimate is found to be fairly good with no error and remain very close to the exact value; however the difference from the exact value is greater for ± X1.0 than ± X0.5, which is quite logical.

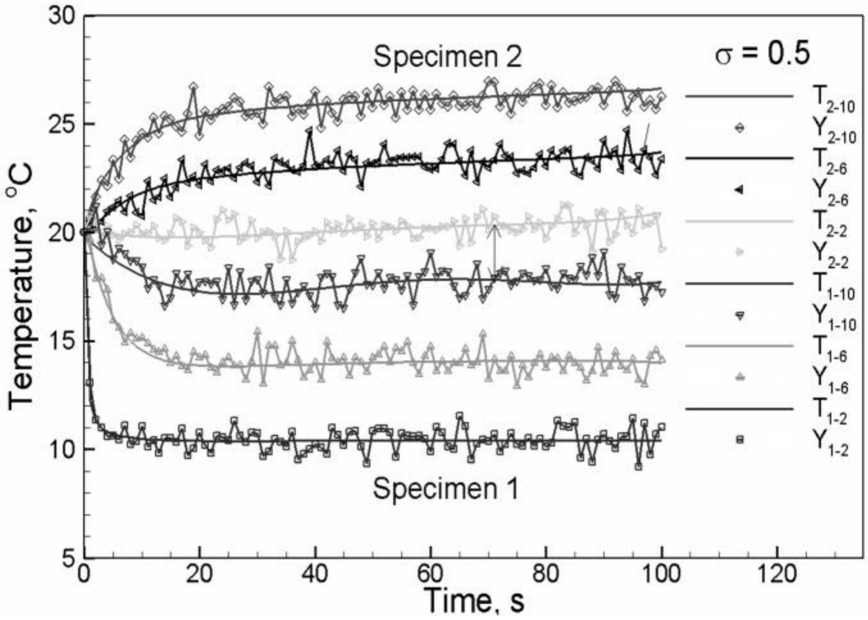

Figure 5: Estimated and input simulated temperature (σ = 0.5) with time for case D

It is further evident from the root mean square (rms) errors and relative percentage deviation. It has to be noted that rms error and relative percentage deviation, both were minimum for errorless measurements e_{rms} X0.7 and ℵ = 9 %, while errors increase for the measurements with random errors. It has been found that errors in the results for ± X1.0 (e_{rms} X0.78 and ℵ = 14.2 %) are more than for ± X0.5 (e_{rms} X0.76 and ℵ = 13.4 %). Further, it has been observed that the estimated temperatures show better agreement with input temperatures as against the comparison of estimated TCC for standard deviation of measurement, ± X0.5 with exact function, (Figure 5 and 6). This may be due to errors introduced in the inverse solution steps.

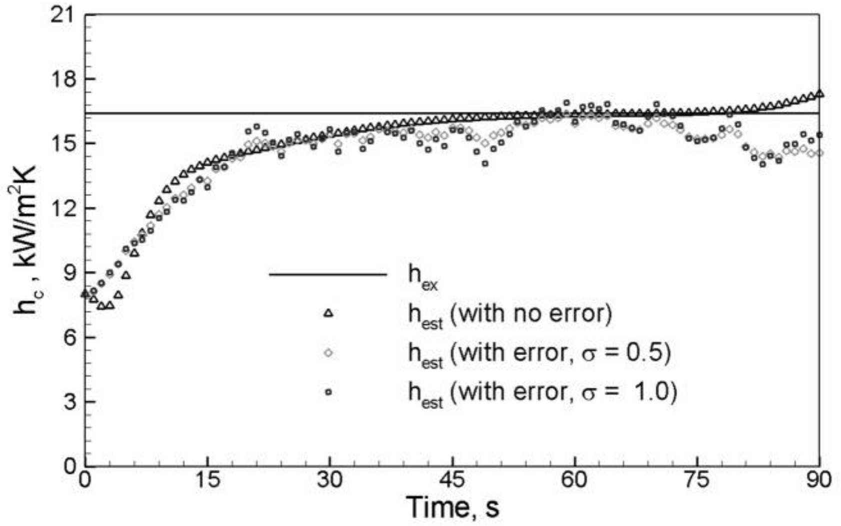

Figure 6: Variation of estimated thermal contact conductance for case D with different random errors

6. Conclusions

The inverse technique based on the conjugate gradient method with adjoint problem has been employed to estimate the unknown function i.e. thermal contact conductance with time for two bodies in contact. Simulated temperature data have been generated for different levels of random errors assuming one-dimensional heat conduction. Four types of test functions based on their practical relevance have been selected for thermal contact conductance $h(t)$, namely; step function, triangular jump, polynomial function and constant value. Constant heat flux has been imposed at one of the non-contacting end while constant temperature at the other non-contacting end. It has been concluded that for all the test functions, estimated thermal contact conductance is found to be agree very well with the exact one with deviations at the starting time step and sharp corners which is obvious with the inverse analysis. However, for simulated data with random errors, the estimated thermal contact conductance shows increased deviations with the exact one, for all the test functions of thermal contact conductance. The results are also confirmed by calculated the corresponding root mean square and percentage relative error. Hence, the results demonstrate the suitability and applicability of inverse technique employing conjugate gradient method with adjoint problem.

7. References

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